ELECTRICAL CIRCUITS AND MACHINERY

BY FREDERICK W. HEHRE
AND GEORGE T. HARNESS

Volume I. Direct Currents. *6 × 9; 513 pages; 388 figures.*
Volume II. Alternating Currents. *6 × 9; 635 pages; 477 figures.*

ELECTRIC CIRCUIT AND MACHINE EXPERIMENTS

BY FREDERICK W. HEHRE
AND J. ARTHUR BALMFORD

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ELECTRICAL CIRCUITS AND MACHINERY

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VOLUME I
DIRECT CURRENTS

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PREFACE

In writing this book on direct currents, which is Volume I of a two-volume treatment of the subject, *Electrical Circuits and Machinery*, we have followed the general plan and method of a textbook of the same title, also in two volumes, written in 1933 by John H. Morecroft and Frederick W. Hehre. This book, as was that, is designed both as a general textbook for non-electrical engineering students and as an introductory textbook for electrical engineering students.

In this book we have retained the unrationalized absolute system of units because we believe that it should first be decided whether a rationalized or unrationalized mks system of units is preferable. The system, too, should have the adoption of all teachers of physics. We believe, also, that the engineering student of today must be familiar with the old system of units to use the reference books available. Furthermore, with a complete understanding of the old system of units, the student is able to grasp easily the mks system.

Our new book, however, is much more than a revision of the old Morecroft-Hehre textbook. Although not adopting the mks system, we have introduced many relationships in terms of the unrationalized form of it. An important new chapter has been written on electronics, a subject becoming of more and more interest to non-electrical engineering students, in which it is fully discussed and is carried as far as possible without the introduction of any alternating-current theory or applications. Although electronics, because of its more numerous applications to alternating currents, may be considered to belong in a textbook treating alternating currents, which will be the subject of Volume II, we felt that by the inclusion of this chapter the total subject matter—direct current and alternating current—would be more evenly divided between our two volumes. In Volume I there are enough direct-current applications to interest the student in what is to follow in Volume II.

In an introductory chapter we have presented the electron theory in sufficient detail to enable the student to grasp its fundamentals. Here we have paid particular attention to the transfer of electric charge as applied to ordinary circuits and to electronic circuits. In the chapter on self-induction and mutual induction there is an extended mathematical discussion of both phenomena. The chapter on losses, efficiency, heating, and rating has been written from the viewpoint of the A. S. A. and N. E. M. A. standards, with numerous arithmetical applications.

We have included a very great number of problems, so that from year to year complete sets of illustrative problems may be chosen without duplication.

We wish to acknowledge our indebtedness to our associate, Professor Morton Arendt, who offered many valuable suggestions in connection with the chapter on batteries.

We wish also to acknowledge our indebtedness to the companies that have furnished cuts and photographs and special data regarding their products. And we wish to thank the National Fire Protection Association for permission to publish the new tables for allowable current-carrying capacities of conductors and the new classification of conductor insulation, both of which are part of the National Electrical Code of 1940.

Columbia University
August 1, 1940

F. W. H.
G. T. H.

CONTENTS

CHAPTER I

INTRODUCTION

1. Nature of Electricity. Everyone is familiar with many applications of electricity, yet the real nature of electricity is unknown and promises to remain so for some time.* In general, the electrical engineer does not need to know the exact nature of electricity; his observations of electrical relations and phenomena are complete enough to enable him to formulate and solve most of the problems that confront him.

The study of the nature of electricity is the task of the physicist, and his experiments have resulted in the establishment of many facts which give some idea as to what the ultimate nature of electricity will prove to be. Many of the relations found by the physicist are useful to the engineer.

2. Nature of Matter. It is considered that all matter is composed of atoms and molecules. In a gas the molecules are widely separated and are in violent agitation. The individual molecule is continually colliding with its fellows and as often changes its speed and direction of motion. These properties can best be considered by treating the atoms and molecules as small elastic bodies. The exact mechanism of collision of molecules is not important; it is only necessary that the resultant motion be correctly defined or that the total momentum and total kinetic energy be unaltered by the encounter. To an observer a long way off, a comet coming from outer space and passing around the sun and going off again will behave as though comet and sun were two elastic bodies in collision.

It has been shown as the result of much experimentation that every atom of matter is made up of minute particles, a number of which have been definitely identified.

The simplest conception of the normal or neutral atom (i.e., an atom charged neither positively nor negatively) is that it consists of a number of positively charged and negatively charged particles, such that the resultant charge is zero. The term "positive nucleus" has been used to group the positively charged particles, although its exact nature is unknown.

3. Electrons. The negatively charged particles are called *negative electrons*, or merely *electrons*, and their existence and properties have been definitely proved. The negative charge of an electron is 1.590×10^{-19}

* It might be pointed out in this connection that the statement that the real nature of matter is unknown is equally true; but this lack of knowledge does not seriously limit our use of materials.

coulomb and its mass is 9.038×10^{-28} gram. An electron, when detached from an atom of matter, shows none of the properties of ordinary matter; it does not react chemically with other electrons to produce some new substance. Moreover, all electrons are similar, and act in precisely the same way no matter from which substance they have been extracted. As yet the electron has not been subdivided. It is known that there are one to a large number of negative electrons associated with each atom of the different elements being held or bound to the atom with different degrees of firmness. It is also known that the number of electrons associated with the atom defines the element so formed. In the neutral atom the combined charge of all the electrons is exactly equal to the positive charge of the nucleus.

4. Structure of the Atom. It is convenient to have a mental picture of the structure of the atom which would embody the various known physical and chemical facts. At one time it was believed that the atom was a miniature solar system with the electrons rotating about the central nucleus in a number of different orbits. It was an idealized attempt to picture the atom in a way which has the virtue of explaining many of the simpler phenomena, and in this respect it is still useful. However, we know now that this picture will not explain all the properties of matter and is probably much simpler than the actual atom.

The simplest atom, that of hydrogen, is pictured as a nucleus with one electron rotating about it. The next element in order, helium, is pictured as having two electrons rotating in orbits close to the nucleus. To account for the fact that the electrons are held to the nucleus with different degrees of firmness, it is assumed that some electrons are in orbits close to the nucleus where they are held firmly, while others are in further orbits and are held less closely. The electrons that are held most closely are said to be in an "inner shell," the others in "outer shells." Furthermore, the number of electrons that can be placed in any shell seems to be definitely limited. The innermost shell, for example, can accommodate only two electrons and is filled when helium is reached.

The next element, lithium, has two electrons in the inner shell and a third electron in an outer shell. Such an arrangement, one or two electrons in an outer shell, gives an element that is chemically active and that can be easily deprived of one electron. In the succeeding elements, beryllium, boron, etc., the two electrons of the inner shell are present and each adds successively one electron to the outer shell, so that beryllium has two, boron has three, etc., until neon is reached, which has eight electrons in the outer shell, or a total of ten in two shells. Neon is the tenth element in the atomic table. Succeeding elements have the two electrons of the first shell, eight in the second, and begin a third shell, which builds up to and becomes complete with a total of eight electrons in the case of argon.

Argon, the eighteenth element in the atomic table, has therefore eighteen electrons revolving about its nucleus.

The next element, potassium, places its additional electron in a fourth shell, and so on, the pictures becoming more and more complex. The property of some substances to part rather easily with one or more electrons is one of the reasons for placing the electrons in orbits or "shells." When a substance loses an electron it usually comes from the outer shell; the electron is easily removed provided the shell is not complete. Thus the first shell is complete with two electrons in the case of helium, the second is complete with eight electrons in the case of neon, and the third is complete with eight electrons in the case of argon. These three elements will be recognized as inert gases which do not easily part with an electron. Where the shells are not complete an electron may be extracted more easily.

The idea of having the electrons revolve in orbits is suggested by the known fact that they are moving, but it may be that they are only vibrating in some manner. It must be remembered that the picture of orbits, etc., is only imaginative and, while it may represent some laws and facts, it fails in other respects.

5. The Nucleus. The positive nucleus, about which electrons are imagined as revolving, is not, like the electron, a simple body, but is complex and contains more fundamental particles. It is believed that the unit of positive charge resides in a particle called a *proton.* This particle is greater in mass than the electron, but contains exactly the same amount of electricity, of opposite polarity, as the electron.

Another particle found in the nucleus is known as the *neutron;* it has the same mass as the proton but has no electrical charge at all.

The nucleus of all elements except hydrogen is made up of protons combined in some manner with at least an equal number of neutrons. Hydrogen has a nucleus of one proton and an atomic weight of one, and has one electron associated with this proton nucleus.* Helium, the next element in complexity, and the second in the atomic table, has a nucleus composed of two protons and two neutrons. The atomic weight is thus four, and there will be associated with this nucleus, in the normal or neutral condition, two electrons just balancing the nuclear charge of the two protons. Lithium would require three protons and three neutrons to form its nucleus, about which three electrons move in two shells.

Other fundamental particles have been observed such as the *positive*

*A second kind of hydrogen, or hydrogen isotope, has been identified. It has for its nucleus a particle, known as a deuteron, which has a greater mass than the proton. The deuteron is in turn made up of a proton and a neutron. The nucleus of this "heavy" hydrogen has only one electron associated with it, as in the first kind of hydrogen, but the greater mass of the nucleus results in a higher atomic weight. Usually the two types of hydrogen occur mixed, the heavier type occurring in extremely small quantities.

electron or *positron.* The positron is a small positively charged particle which corresponds to the negative electron, having the same mass and containing the same amount of positive electricity. It seems to be a short-lived particle and its place in the structure of matter is not evident. Still other particles have been identified or are suspected to exist.

Since the electron is going to prove to be the usual carrier of charge and by its motion along the circuit constitute the electric current, and since no other of the known particles as yet finds application in electrical engineering, attention need be paid only to electrons and the atoms.

A most important fact to be considered is the relative mass of the electron compared to that of an atom. The mass of the lightest atom, that of hydrogen, is 1.6617×10^{-24} gram. The mass of the electron is 9.038×10^{-28} gram, so that the mass of the hydrogen atom is 1839 times that of the electron. It is thus evident why the light and hence mobile electron should prove to be the carrier of charge rather than the heavy sluggish nucleus, particularly in the heavier conducting mediums.

6. Free Electrons. In some substances, notably the metals, the atom does not seem to hold all its electrons in rigid control. It may be that, in the solid state, the atoms are so closely packed together that the outer shell electrons of one atom are about as close to the nuclei of adjacent atoms as to their own nuclei. This tends to weaken the already loose attachment found for these electrons in many substances, and they become attracted to the adjacent nuclei. Thus electrons migrate from atom to atom throughout the volume of the solid and are known as *"free" electrons.* Moving with relatively little constraint and with random motion, the free electrons attach themselves to an atom and displace another electron, which in turn migrates. While the free electrons are able to move about rather freely within the solid material, they do not, under ordinary conditions, leave the boundaries of the solid itself or of other solids in contact. Materials in which there are many free electrons are called *conductors.*

In other solid substances, notably the non-metals, the electrons are so closely held that very few are able to reach this semi-detached state under ordinary conditions. Such materials are called *insulators.*

7. Charged Bodies. Everyone is familiar with the experiments showing phenomena associated with charged bodies. Charges induced by friction on a glass rod rubbed by a cloth produce sparks and attract small light bodies.

An electrified or charged body, by the electron theory, is one that has an excess or deficiency of one or the other kinds of electricity. The atom, in its normal state, has equal amounts of electricity, positive electricity associated with the nucleus and negative electricity divided among its electrons. Since the electron is the light mobile particle, it is the one that is always added to or taken from a body that is charged.

The removal or addition of electrons is made by allowing the electrons to flow or pass into another body *in contact.* This removal or addition of electrons does not change the chemical nature of the material since the nucleus is not changed. When the occasion presents itself, the nucleus will recapture an electron and return to its neutral state. The transmutation of elements, changing one into another, is done by breaking up the nucleus, but it is not a simple process and finds no engineering use.

A positively charged body, then, is one that has had some of its electrons removed, leaving a net positive charge. A negatively charged body possesses an excess of electrons. Electrons may be removed from a body only by doing work against the force of attraction of the nucleus for its electrons. If the electrons are allowed to go back to the body from which they were removed, work will be done, the balance will be restored, and the body again becomes neutral or uncharged. We can thus look upon the charging process as one of separation of negative charges from their associated positive charges and the passing of the former onto or into another body. If not held upon the second body by removal of the first, they will flow back and the system again become uncharged.

Although there are a large number of electrons per atom and an extremely large number of atoms in a body of even small volume, only a few of these will be separated. The charge usually represents the removal of only one electron from each of only a fraction of the atoms present. The greater the charge the greater the number of atoms that will have lost but one of their electrons.

It can then be seen that charge may be measured in discreet units, the number of charge units being the number of electrons added or removed. However, since the total number of electrons present is such an enormous number, and the gain or loss of so many electrons is necessary to make an appreciable charge, the significance of the discreet unit may become lost.

8. Coulomb's Law. The phenomenon of charged bodies was known to the Greeks probably as early as 600 B.C., when they observed that amber, after being rubbed with cloth, attracted light objects, but it remained only an interesting curiosity until much later. In order for a phenomenon to be well enough understood to be useful in science it must not only be described qualitatively, but measured quantitatively.

It remained for Coulomb to find, in 1785, that two small charged bodies, separated by a distance, r, attracted or repelled each other with a force expressed by

$$F = \frac{Q_1Q_2}{kr^2} \tag{1}$$

where F is the force of attraction or repulsion in dynes,
Q_1 and Q_2 are the electrifications or charges on the bodies in statcoulombs,*
k is the force-reducing effect of the intervening matter (for vacuum $k = 1$),
r is the separating distance in centimeters.

The force-reducing effect of the intervening medium can be understood when we consider that, although it is an insulator and contains electrons, none of these are in the "free" state but are strongly held by the atoms. The positively charged body will attract and the negatively charged body repel the electrons of the medium, but, since these electrons are not free to move through the medium, they are only displaced from their average position. This displacement will place a sheath, containing an excess of electrons above the normal or neutral number, in the medium about the positively charged body, so that the sheath has a resultant negative charge. There will be a similar sheath about the negatively charged body having a deficiency of electrons and a resultant positive charge. The effect of these sheaths is to neutralize partially the effect of the charges on the charged bodies and decrease the force between them. This phenomenon in insulating materials is known as *polarization.*

It was known that the force between two charged bodies would be one of repulsion if both bodies were similarly charged; if the bodies are oppositely charged the force is one of attraction.

Equation (1) serves to define the charge or degree of electrification. The unit of charge in statcoulombs is that value which will just repel a similar charge with a force of one dyne when placed at a distance of one centimeter from the second unit charge, in vacuum. This unit charge comes out equal to the charge on 2.09×10^9 electrons.

9. Methods of Separating Charges. There are a number of methods by which charges may be separated, the most familiar of which is the method of rubbing; the separation is caused by friction. A second method of separating charges is by chemical action; this is the method used in batteries. The third method of separating charges is by generator action. This is produced by moving a conductor through a magnetic field, and is, from the engineering standpoint, the most important. It will be discussed later in detail.

A fourth method is the separation of charges by induction. If a charged body is brought near a conductor, the conductor will act as if it too were charged, but in a peculiar way. If the body is positively charged, that portion of the conductor nearest the charged body becomes negatively charged and the part farthest away from the charged body becomes posi-

* Systems of units are described in the Appendix and their interrelations are shown in Table I of the Appendix.

tively charged (Fig. 1–1). As a whole the conductor is not charged and when the charged body is removed the charge disappears. It can be seen that this is merely a separation of the charges in the conductor, the negative charges, the electrons, being attracted toward the positively charged body, leaving a deficiency at the distant part of the conductor and thus a resultant positive charge. If, however, the conductor can be separated into two parts as along the dotted line in Fig. 1–1, the charges will be trapped and cannot recombine when the positively charged body is removed. Provided that the two parts of the conductor are supported by insulators so that the charges cannot leak through the supports, they will then retain their charges until again brought into contact.

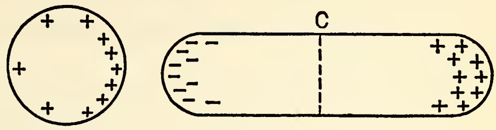


FIG. 1–1. If an uncharged conductor is brought near a positively charged body, the conductor will become charged by induction; although uncharged as a whole, the rod exhibits a negative charge at the end near the charged body and an equal positive charge on the other end. If the charged body is itself a conductor, its positive charge will not be uniformly distributed, when conductor C is near, as indicated above.

A fifth method of separating charges is known as polarization (mentioned before) and is the name given to induction in insulating materials. Since the electrons of an insulator are more closely held and there are no free electrons, the separation consists of a mere displacement of electrons from their usual average position. When the charged body inducing the effect is removed, the polarization disappears and the electrons return to their normal positions.

A sixth method of separating charges is by applying mechanical pressure to certain crystals. There is a slight separation of charges.

A seventh method is the separation of charges by thermoelectric action. If the junction of two dissimilar metals is heated, there is a slight separation of charges.

Another method of separation of charges is by photoelectric action. Certain substances when in contact with metals will develop a slight separation of charges when the junction is illuminated.

10. Electric Fields. Since a force is exerted when two charges are separated by some distance, it is evident that the charge exerts some kind of effect in the space surrounding it. This space surrounding a charged body, in which another charged body is acted upon by a force tending to move it, constitutes an *electrostatic*, or *electric, field*. It is always associated with a charge, and, although it extends to infinity, it is significant only in the immediate vicinity of the charge.

In diagrams, the electric field is conveniently represented by drawing lines from the charged body into the surrounding space. These lines are

so drawn that the direction of the lines indicates the direction of the electric force on a test body of unit positive charge, and the relative closeness of the lines in various parts of the diagram show the relative strength or intensity of the field at these points. The closer the lines at a point, the more intense is the field and the greater the force on the test body. A line of electric force will then begin on a positive charge and end on a negative charge.

Figure 1–2 shows how lines may be used to represent a section of the electric field of a positively charged metal sphere, supposedly far enough away from other bodies to be considered as by itself. The lines of electric force originate at the positive charges on the surface of the sphere and ex-

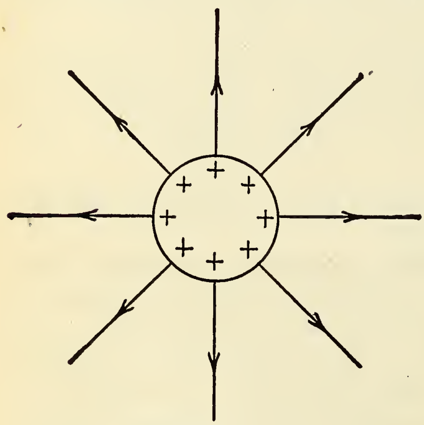


Fig. 1–2.

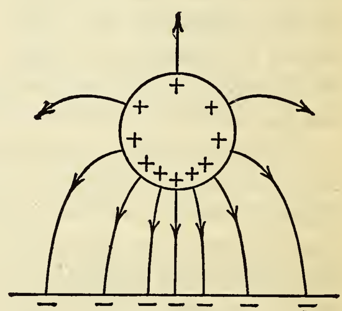


Fig. 1–3.

Fig. 1–2. Around an isolated positively charged sphere the electric field is properly represented by radial lines extending outward in all directions. Since the sphere is isolated, the charge is uniformly distributed on the surface and the lines are thus uniformly distributed. If the sphere were negatively charged, the direction of the lines would be reversed. The lines show the direction in which a positively charged test body would move.

Fig. 1–3. Actually the electric field about a charged sphere is not symmetrical as shown in Fig. 1–2, but owing to the proximity of other bodies, is more or less unsymmetrical. If the sphere is near the earth's surface the lines emanating from it will all bend around and finally end on the earth's surface, as indicated here.

tend as radii in all directions. The arrow heads on the lines indicate the direction in which a unit positively charged test body would be urged if placed in that part of the field.

The lines are closest together at the surface of the sphere, indicating that the force is greatest at these points, a fact in agreement with Coulomb's law. Although the lines are shown as discontinuous, ending in uncharged space, each line in reality extends in some direction until it encounters a negative charge. In the case of a positively charged metal sphere, sus-

pended in the air at a distance from other bodies, all the lines should be shown as ultimately ending on the earth's surface as suggested in Fig. 1–3. In this case the earth's surface in the region under the sphere is negatively charged by induction. In Fig. 1–4 is represented the electric field between two parallel metallic plates, one of which is charged positively and the other negatively. That all the lines originating at the positive plate are shown as ending on the negative plate indicates that the two plates are equally charged, i.e., as many electrons have been deposited on the negative plate as have been taken away from the positive plate. The field is properly shown as very intense between the plates, weaker at the edges, and very weak in the space not included between the plates.

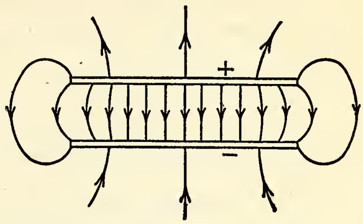


Fig. 1–4. When two plates having equal and opposite charges are placed parallel and close, as shown, the field between them is intense and uniform, except near the edges of the plates. Outside the plates the field is very weak and non-uniform. The lines coming from outside the plates do not end but pass in wide arcs around to the opposite plate.

11. Nature of an Electric Current. An electric field is always associated with separated charges, the separation having been effected by any of the means previously described. If now a free electron is inserted into or happens to be in the field, for example between the plates of Fig. 1–4, it will be attracted by the positively charged plate where there is a deficiency of electrons and repelled by the negative plate where there is an excess of electrons. As a result of the forces exerted, the electron will start to move in the direction of the positive plate. This direction is opposite to that indicated by the arrows on the lines of force as these arrows indicate the direction of the force on a *positive* test body.

It has been said before that in an uncharged medium the free electrons would be rapidly moving with random or haphazard motion, continually colliding with the molecules of the medium. In a charged medium, such as that between two charged plates, there is now a force tending to cause the electron to move toward the positive plate. In addition to the random motion of the electrons, they now will have a coordinated motion or drift in a prescribed direction. It is this coordinated motion or drift of the electrons in a prescribed direction which constitutes the *electric current.*

By analogy with the motion of water, we can think of charge flowing past a point as being similar to the motion of water particles. In a brook filled with obstructions the water will have a random motion, there will be eddies and whirls and cross-flow; the water at certain points may be actually flowing upstream. It is, however, the coordinated or downstream flow which measures the amount of water flowing past a point. The stream

flow in this analogy is misleading in that for most streams the downstream component of velocity is by far the largest, the other components being usually small. For the electron current the "downstream" or coordinated flow of electrons is of very much lower velocity than the random motion.

The name current, then, is given to the flow of charge, or electrons, past some point in the conductor or system. If the flow of charge or electrons is uniform, the current, measured in amperes, will be simply the charge in coulombs passing a point in one second.

If the current flow is not uniform but varies from time to time in some known manner, the current at any instant can be expressed mathematically as

$$i = \frac{dQ}{dt} \tag{2}$$

where i is the current in amperes,

$\frac{dQ}{dt}$ is the rate of flow of charge past a point in the circuit,

Q is the quantity of electricity expressed in coulombs,

t is time in seconds.

The amount of electricity on one electron is so small that the current produced by one electron in motion could not be detected by the most sensitive current-measuring instrument. To produce currents of the magnitudes used in ordinary applications requires the motion of electrons measured in billions of billions per second. An ordinary incandescent lamp requiring a current of exactly one ampere would require that

$$\frac{1 \text{ ampere} = 1 \text{ coulomb per sec}}{1.59 \times 10^{-19} \text{ coulomb per electron}} = 6.3 \times 10^{18} \text{ electrons}$$

flow past any point in one second. This large number per second could be brought about by a comparatively small number of electrons moving swiftly around the circuit or a comparatively large number moving more slowly. Contrary to what one might think, the coordinated motion of the electrons is very slow. To produce a current of one ampere in a copper wire one millimeter in diameter requires that the coordinated velocity of the electrons be only about 0.01 cm per sec.

Although the coordinated motion of the electrons is slow, as indicated above, the actual velocity of the electrons must not be so thought of. When no current is flowing in the wire, the electrons have a random motion in all directions as high as 10^7 cm per sec, but the net motion of all of them in any one direction is zero. Indeed this must be true, otherwise some separation of charge would take place within the conductor.

The reason for the slow progress or drift of the electrons along the conductor can be seen if we remember that a conductor is made up of nuclei

and electrons. Electrons in motion along the conductor must encounter many obstacles. There will be many collisions which, because of the high speed of the electrons, will be violent. Every collision will deflect the electron, and the length of the path followed by an electron will be many times the distance traveled in the direction of coordinated motion.

Thus an accurate concept of the coordinated motion shows it to be an almost inappreciable drift of the electrons, which have, owing to their random motion, velocities millions of times as great as the velocity of drift, which constitutes the current.

12. Direction of Current Flow. Before the nature of an electric current described by the electron theory, as given above, was fully known, it was recognized that the electric current was a flow of charge along the conductors. It is evident that the flow of negative charge in one direction will produce exactly the same sort of excess or deficiency of charge at the terminals, and hence represent the same transfer of charge, as the flow of positive charge in the opposite direction. Since the exact nature of the mobile charge was not known, the positive direction of current flow was arbitrarily taken as the direction of motion of positive charges along the wire. This convention is now firmly established and we shall always take our currents as being in the direction of motion of positive charges, even though the electrons (which really constitute the current) flow in the opposite direction, *and there is nothing flowing in the direction assumed for the current.*

13. Effects of Current Flow. The flow of current along a wire is always accompanied by certain phenomena, the most familiar being the production of heat within the wire. The reason for this heating is evident when it is recalled that the wire is made up of atoms, and that the electrons in their motion are continually colliding with the atoms of the conductor. When the free electrons are made to move along the conductor the coordinated or drift velocity is added to the velocity they already had because of their random motion. The effect is to increase the velocity of impact when they collide with an atom and thus increase the agitation of the atoms and molecules, which agitation is a measure of the temperature. The temperature of a wire when carrying current thus rises above that it had before the current started to flow.

A second effect, evident in the space around the conductor, is the appearance of a magnetic force tending to move magnets placed near the wire. The next chapter will be devoted to a discussion of this phenomenon.

Another effect is the production of light when the electric current is made to flow through gases. Many such types of lighting devices using mercury or sodium vapor, neon, argon, or similar gases are in use.

The flow of current will also produce chemical action. If the current is made to flow through certain solutions, chemical changes can be pro-

duced in the solutions or on bodies immersed in the solution. This effect is used in the electroplating of objects and the electrolytic refining of metals. The secondary or storage battery is another example of this effect where a chemical change is produced by a current.

14. Difference of Potential. It has been pointed out that whenever a charge is placed in an electric field it will experience a force tending to move it. If the electric field is between the ends of a conductor the free electrons within the conductor will move and we have an electric current. Such a condition would exist if two bodies, one charged positively and the other charged negatively by an equal amount, were joined by a conductor. When a sufficient number of electrons had left the negatively charged body and entered the conductor, and an equal number had left the conductor and entered the positively charged body, the charge would be completely neutralized at every point and no further current would flow. This charge distribution would take place very rapidly but not instantaneously.

Some devices such as electric generators and batteries have the ability to maintain a charge separation between two terminals even if the terminals are connected by a conductor and a current is flowing. Naturally, the amount of charge separation is not as great when the current is flowing, since the current is constantly removing the accumulation of separated charge, but enough charge is accumulated at the terminals despite this removal to maintain very high currents in many cases. There is a period of adjustment usually of very short duration, when the connection is first made called the *transient period*, during which the charge adjusts itself from the higher value, when no current flows, to the lower value when current is flowing. After this adjustment has taken place the system is said to be in a *steady-state* condition, until some further change is made in the connections.

The electric current flows as a result of the electric field produced along the conductor between the terminals of the machine due to the separated charges at the terminals. It is usual to measure this field, not in terms of the force it exerts upon unit charge, but in terms of the work done in moving a unit positive charge from one terminal to the other. It is evident that this gives a measure of the force acting, and the work done will always be proportional to that force. *If an amount of work equal to one joule is done in moving a positive charge of one coulomb between two points, the points are said to be at a difference of potential of one volt.* Thus difference of potential indirectly measures the electric force existing between these two points. In devices that have the ability to maintain the charge separation, even when a current flows, this difference of potential is known as an *electromotive force.* This term has led to much confusion. It is evident that it is *not a force at all*, but its use as such is firmly established.

When an indicating instrument, known as a voltmeter, is connected to

these points it will indicate the difference of potential between them. This reading, the work per coulomb, or the voltage, is easy to obtain, and, while it is not the force acting between these points, it gives an indication of its magnitude. To determine the magnitude of the electric field and the force acting is not a simple procedure and is practically never done.

A further element of confusion is added when we say, as is customary, that we impress a voltage on a conductor. Actually we impress forces which produce an electric field which is measured by its voltage, but the abbreviated form is usually used and the student must realize what is meant. It is evident that the sources of electromotive force, or emf, as it is usually abbreviated, are devices that will produce a separation of charges as previously described.

The flow of current has by convention been taken as the direction of motion of positive charges. This would take place in the conductor connecting the terminals of a generator from the terminal on which the positive charge accumulated to that on which the negative charge accumulated. The terminals are thus conventionally designated as positive and negative respectively, and the current is said to flow from the positive terminal to the negative terminal of the generator.

15. Conductors and Insulators. We have indicated earlier that some bodies have many free electrons which facilitate the transfer of charge, or the flow of current, through the body; such bodies were designated as *conductors*.

Actually the degree of attachment and the number of the "free" electrons seem to vary considerably in different materials. There seem to be all degrees, from extremely loose attachment and large numbers in some metals to practically no free electrons in other substances. One would expect to find conduction to some extent in almost all substances. Indeed such is the case and we find that the degree of conduction or the current carried through a strip of material of a certain size, connected to the terminals of a given electric generator, varies widely with the material of the strip.

For example, if strips of various substances are made up, all of the same dimensions, and connected to the terminals of a generator, the current that flows would be as follows:

Strip made of copper.......	100 units of current
Strip made of iron.........	16 units of current
Strip made of mercury.....	1.6 units of current
Strip made of carbon......	$\frac{1}{2500}$ unit of current
Strip made of selenium.....	$\frac{1}{40{,}000{,}000{,}000}$ unit of current

All these materials are called conductors. If other strips, made of non-metals, are used, still smaller currents will flow; these substances are called *insulators*.

The division line between conductors and insulators is rather broad. A given material such as selenium, in comparison with another substance such as copper, may be an insulator, but in comparison with a third material, such as glass, will be a conductor. Furthermore, a substance may be a good insulator at one temperature and a good conductor at another. Glass is the most striking example of this change of character with change in temperature; at ordinary temperatures it ranks high among the very best insulators, but if it is heated in some way to a red heat it becomes a fair conductor and will permit the passage of enough current to cause it to melt.

16. Disruptive Strength of an Insulator. If high voltage is applied to an insulator it will sometimes become a conductor and carry enough current to destroy it. For low voltage, which represents a weak electrical field, the force tending to move an electron is not sufficient to do more than displace it from its average position. If the voltage is made sufficiently high, which means that the electric field is strong, electrons may be torn out of the atoms and form a current. Such excellent insulators as glass and mica break down and carry current when a great enough voltage is employed. The only real insulator at all voltages is a perfect vacuum in which there will be, of course, no free electrons and no atoms or molecules to break up and yield electrons to carry current. This fact is made use of in vacuum switches and in high-voltage rectifiers.

17. Effect of Temperature on the Disruptive Strength of an Insulator. Imagine that a good insulator is heated by some means. The rise in temperature increases the random motion of its particles, with the result that the collisions between the various particles become more frequent and more violent as the temperature is raised. As these collisions occur, the resultant disturbances in the atomic structure tend to weaken the hold of the atom on its electrons. Hence, if an electric field is impressed and maintained, as an insulator is heated, the combination of electric force and weakening of the atomic holding power will finally result in some electrons leaving their atoms. The electric field will then urge them along through the substance of the insulator, resulting in a small current. This appears as a weakening of the insulating ability of the substance; such a current is called a leakage current through the insulator.

Generally, the partial breakdown of an insulator, as described above, is rapidly followed by its complete breakdown; the current, even though small, flowing through the insulator generates more heat, thus still further decreasing the disruptive strength.

18. Resistance. In a conductor in which the electrons are "free," their progressive motion is continually hindered by collisions with the atoms of

the material. This hindrance to their free passage constitutes the *electrical resistance* of the conductor. It differs, as previously indicated, for different metals, and it varies with the temperature. As the temperature of a material increases the agitation of its atoms or molecules increases, and this results in more hindrance to the progressive motion of the electrons because of the more frequent collisions between the electrons and the atoms.

This effect accounts too for the electric power used when a current flows through a conductor. The collisions, which the electron experiences, continually take from the electron the energy acquired as a result of its motion under the influence of the electric force, and give this energy to atoms of the substance. This results in a loss of energy in the electric system and a gain of energy of motion of the atoms and a corresponding increase in the temperature of the substance.

PROBLEMS

1-1. What is the charge on an electron or proton in terms of the esu, emu, and pu system of units? (See Appendix.)

1-2. How many electrons are necessary to give a charge of one coulomb?

1-3. If the number of electrons necessary to give a charge of one coulomb were placed along a line and spaced one inch apart, how far would the line extend? If the diameter of the earth is taken as 8000 miles, how far would it reach around the equator?

1-4. If 10^{23} electrons pass a point in a circuit in one second, what current in amperes is flowing? How many electrons must pass a point in one second to have a current of 100 amperes flowing?

1-5. It is often assumed that there are 10^{23} free electrons per cubic centimeter in copper. What would be the average electron velocity in the direction of current flow in order to have a current of 25 amperes in a No. 10 wire (0.102 inch in diameter)?

1-6. A charge of 1 coulomb passes a point in a wire in 0.01 second. What average current in amperes is flowing?

1-7. Express the current in problem 6 in statamperes and abamperes.

1-8. The charge on the plates of Fig. 1-4 has been determined at a series of different times and plotted against time in Fig. 1-5. What current was flowing at the end of 0.25 second? What current at the end of 0.75 second? What current at the end of 1.5 seconds? What observations can be made as to the direction of current flow in the three cases?

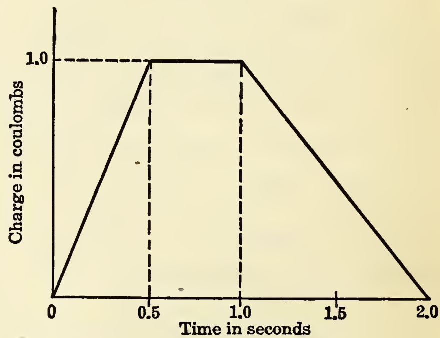


Fig. 1-5.

1-9. The charge on the plates of Fig. 1-4 has been determined and is represented by the equation $q = 1.5\,\epsilon^{-25t}$ coulomb (t is in seconds). What is the equation of the current in amperes as a function of time? What is the significance of the negative sign?

1-10. Plot the charge and current in problem 9 as a function of time.

1-11. If 120 joules are required to move 10^{20} electrons between two points on a wire, what is the difference of potential between those points?

1-12. The work required to move a unit charge between two points on a wire 25 cm apart is 300 joules. What is the voltage between these points? How much work would be required to move 2 coulombs between these two points? What is the current in amperes if the transfer is made in 0.1 second?

1-13. If 30 joules are required to transfer 10^9 statcoulombs from one body to another, what is the difference in potential between the two bodies? What energy is stored in the system while the charges are separated? If the transfer is made in 1 second, what average power is required?

1-14. If the bodies of problem 13 are connected by a conductor and the discharge takes place as given by the equation

$$q = 10^9\,\epsilon^{-20t} \text{ statcoulomb}$$

what is the current in amperes at the end of 0.01 second? (t is in seconds).

CHAPTER II

ELECTROMAGNETISM

1. Fundamental Facts. The property possessed by lumps of certain iron ores or lodestones of attracting bits of iron was early known. The best samples coming from Magnesia in Lydia led the Greeks to call them *magnetis lithos* (Magnetian stones), whence is derived our term *magnet* and the name *magnetite* for the ore. It has also long been known that this property of attracting bits of iron could be imparted to other pieces of iron, by rubbing with a lodestone. But it was not until after Oersted announced the connection between magnetism and currents of electricity, in 1819, that it was found that iron could be magnetized by current through a surrounding coil.

Whenever a soft iron bar is magnetized, it retains an appreciable portion of its magnetic properties only so long as it is in the presence of a magnetizing agent; the iron is thus only a temporary magnet. When, as in the case of certain alloys, properly heat-treated, the bar retains a considerable portion of its magnetic properties, it is said to be a *permanent* magnet.

From the action of a freely suspended magnet on the earth, which was itself recognized as a huge magnet by Gilbert in 1600, we obtain the compass and a means of distinguishing between the two ends of a magnet; by convention, the north-seeking end, or that pointing to the geographical north, has been called the north pole of the magnet, and its south-seeking end, the south pole.

Early experiments showed that if two freely suspended magnets were brought close together, there was a force of attraction between a north and a south pole and a force of repulsion between two north or two south poles. This action resulted in the law that like poles repel each other and that unlike poles attract. In general it was observed, in the case of bar-shaped magnets, that the magnetic effects usually appeared to be concentrated near the ends of the bar, one end forming a north pole and the other a south pole.

In Fig. 2–1 is shown a slender magnet, pivoted at its center and placed between two large magnetic poles, the forces which are exerted on the poles of the small magnet by the large poles being represented by the arrows. This figure really indicates all that our senses can realize, namely, that between the poles, forces exist which have direction and intensity. This fact

must be carefully borne in mind as it constitutes about all we know about the nature of magnetism.

2. Forces between Magnets. By experiment and by analogy with the laws of gravitation, it was discovered that the force between two magnets varied inversely as the square of the distance of separation between the poles. For example, if two north poles were placed two centimeters apart, the force of repulsion would be four times as great as if they were placed four centimeters apart. To perform this experiment it is usual to take long thin magnets, so that the accompanying south poles will be far enough away to have practically no effect.

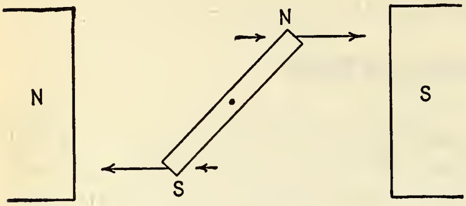


FIG. 2–1. Forces acting on a slender magnet pivoted in a magnetic field.

It was also known that different poles varied in their magnetic effects, some attracting iron with greater force than others. This difference is accounted for by assigning a *pole strength* to each pole.

Combining these effects, it was observed that the force exerted between two magnetic poles of strength m_1 and m_2 varied directly as their product, inversely as the square of their separation, r, and also with the medium. Expressed as an equation

$$F = k\,\frac{m_1 m_2}{r^2} \tag{1}$$

where k is a constant depending upon the medium. For air, k is essentially unity.

3. Unit Pole. All measurement being a matter of comparison, in order to measure pole strengths or compare magnetic fields it is necessary to assume and define a unit magnetic pole. It is derived by placing each term in Eq. (1) equal to unity, so that a unit pole is a point of such magnetic pole strength that, when placed in air a distance of one centimeter from an exactly similar point pole, the force of repulsion will equal one dyne.

The idea of a point pole is, of course, an artificial one, the magnetism never being concentrated at a single point. It is, however, nearly approximated in the case of long, slender magnets. In speaking of unit north poles, it is always assumed that the accompanying south pole is so far away as to exert negligible force; this south pole is always required, for there cannot be a north pole without its companion south pole.

By comparison with the unit pole, other poles of unknown strength are measured; for, if such an unknown pole is placed a distance of one centimeter in air from a unit pole, and the force between them is m dynes, the unknown pole has a strength of m units.

4. Direction of the Field of a Magnet. When a magnet is placed near other magnets or between the poles of a large magnet, as in Fig. 2–1, there will be forces of attraction between the unlike poles of the system, and forces of repulsion between the like poles. Forces are vector quantities, that is, they have direction as well as magnitude or intensity, and we may represent such magnetic forces by arrows or vectors, the direction of which indicates the direction of the forces and the length of which indicates the intensity of the forces. The total force acting upon a magnetic pole is then the resultant of all the forces due to all the other poles of the system.

By the *field of a magnet* is meant the space surrounding a pole, or the space in which magnetic forces act on poles. Every field in space is set up between poles; the field and poles cannot be separated, but one may be thought of as substituted for the other. We may say that the bar magnet of Fig. 2–1 is placed between the poles of the large magnet, or is placed in a magnetic field without mentioning the poles which cause it.

The unit north pole becomes a convenient means of investigating magnetic fields. By convention, the direction of the field at any point is the direction of the force acting upon a unit north pole placed at that point. If a unit north pole is introduced at some point in the field of a bar magnet, as in Fig. 2–2, the direction of the force acting on the unit pole may easily be determined. The force of repulsion between N and n, along the line a, is represented by the vector nR, and the force of attraction between n and S, along b, is represented by the vector nA. From Eq. (1),

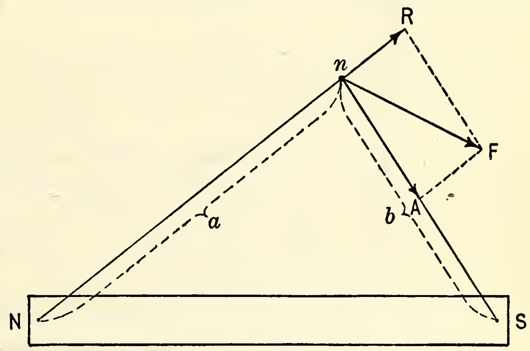


FIG. 2–2. Resultant force on a unit north pole placed in the field of a bar magnet, a distance a from the north pole and a distance b from the south pole.

$$nR : nA = b^2 : a^2$$

The resultant force, nF, acting on n, is the vector sum of nR and nA.

In Fig. 2–3, assume that a unit north pole is placed at n, near the north pole, N, of the bar magnet. The distance from n to N is a and from n to S is b. Assume each pole of the magnet as of strength m, and the medium to be air. From Eq. (1) the forces of attraction and repulsion between the unit pole n and the poles of the magnet are calculated and represented to scale by the vectors nA and nR. The resultant force acting on the unit pole is then given in magnitude and direction by the resultant vector nF.

If the unit north pole is permitted to move a very short distance along the resultant, nF, and the diagram reconstructed for this new point, the new distances nN and nS may be measured and the calculation repeated, giving the intensity and direction of a new resultant. Again the unit pole may be moved and the process repeated until the entire curve is obtained by joining points. The force acting on the unit pole at the top of the

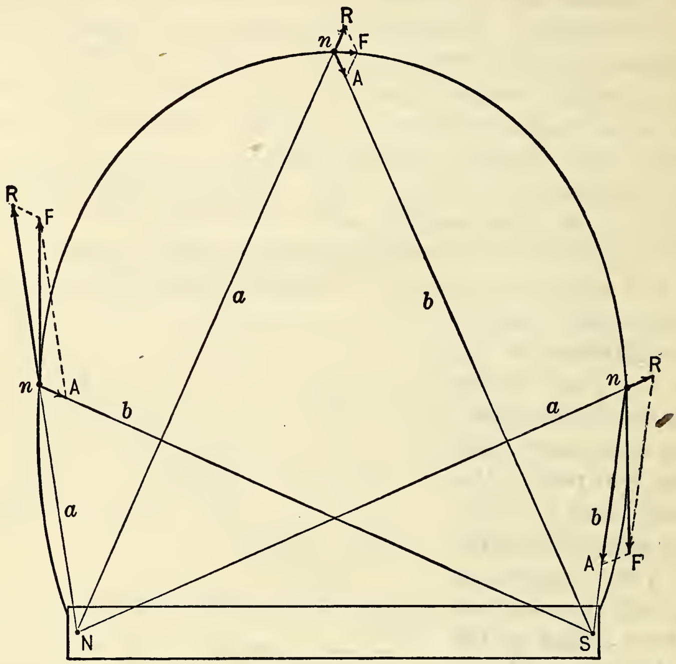


FIG. 2–3. Path described by a small, slowly moving north pole in the field of a bar magnet. The direction of the resultant force acting on the small north pole at any point on the curve is tangent to the curve at that point. If the small north pole was started from another part of the pole of the bar magnet it would describe another curved path somewhat similar to that shown here. This path has been constructed accurately, to scale.

curve is seen to be very small compared with the force at points near the magnet poles.

In mapping a field any one particular curve is determined by the starting point chosen. If many starting points are chosen, the entire field may be represented by many curved lines drawn from the north to the south pole; the direction of the force at any point in the field is obtained by drawing a tangent to the curved line drawn through the point. Any one curved line, of course, represents the path that a freely moving north pole would follow if allowed to move slowly.

One may obtain an idea of the magnetic field by the familiar experiment in which iron filings are sprinkled over a paper placed over a bar magnet. When the paper is tapped, the iron filings arrange themselves in filaments or lines similar to those shown in Fig. 2–3. The field of the magnet, however, is greatly disturbed by the presence of the iron filings, each iron particle becoming a tiny magnet in the presence of the large magnet. The picture obtained is not a true picture of the field before it was disturbed by the iron filings.

5. Unit Field. We have defined a magnetic field as the region surrounding a magnet, or any region in which magnetic forces act. It follows from this that whenever a unit pole is introduced into a magnetic field it will be acted upon by a force, and we can then define the intensity of a magnetic field at any point as the *force in dynes exerted on a unit north pole placed at that point,* provided, however, that the introduction of the unit pole does not alter the original intensity or distribution. When the force acting on the unit north pole is one dyne, the pole is situated in unit field intensity.

By this definition, when two unit poles are placed one centimeter apart, the force acting between them being one dyne, each unit pole is situated in unit field produced by the other pole.

6. Representation of Magnetic Fields. A vector quantity, such as a force, is ordinarily represented by a line, the direction of which indicates the direction of the force, and the length of which (to some arbitrary scale) measures the intensity of the force. Thus, a square piston, with an area of 16 square inches and with a pressure of 2 pounds per square inch acting on it, bears a total force of 32 pounds. This might be represented, as in Fig. 2–4, by a single vector drawn to the center of the piston and 32 units long, one unit representing one pound.

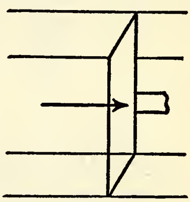


Fig. 2–4. Representation of a uniform force over the area of a square piston by a single vector, the length of which represents the total force on the piston.

Another way of representing the same force is to draw as many arrows or vectors per unit area as the force per unit area. In the case above, there would be two arrows per square inch over the entire area of the piston, as in Fig. 2–5. Or, if the force dropped to half a pound per square inch, and the same units were used, there would be one arrow in each two square inches.

In this method, direction of the force is shown as before, but intensity of the force is represented by the number of arrows per unit area; the length of the arrows has no significance in this method of representation.

The latter method of vector representation is of special value where both the direction and intensity of the force vary from point to point. Imagine a stream of air passing out of a pipe through a funnel, with a num-

ber of obstacles in the small end which cause the air to swirl and eddy as it passes through the funnel. Let us assume, however, that there are no pressure losses, due either to these eddies or to friction on the sides of the funnel.

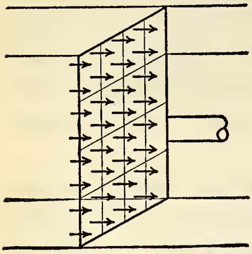


Fig. 2–5. Another scheme of vector representation of uniform force over the area of a piston, the number of arrows per unit area indicating the intensity of the force.

If we wish to map out the pressure due to the air currents over some test area aa', we may conveniently do so by having one arrow represent a force of one ounce per square inch. If the force at certain regions were only half an ounce per square inch, we would draw one arrow in each two square inches, and so on. In Fig. 2–6, this has been done for sections aa' and bb' of the funnel, and it follows that, if the test area is gradually moved over the entire length of the funnel, the arrows become lines, the direction of which at any point indicates the direction of the pressure, while the number of lines per square inch indicates the intensity of the pressure.

From our assumption of no pressure losses due to eddies or wall friction, it follows that the lines are continuous, the product of average pressure per square inch and area remaining constant. It so happens that in this particular case the continuous lines also show the stream lines of the air, but this is only because there is something actually moving.

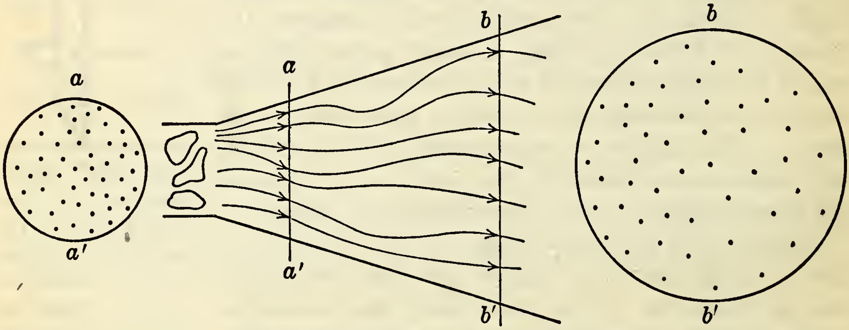


Fig. 2–6. Representation of the velocity air-pressure gradient in a funnel carrying a stream of air. By using the scheme of Fig. 2–5, for closely adjacent sections at right angles to the axis of the funnel, and connecting the force arrows in the adjacent sections, the center portion of the figure is obtained. The side diagrams show the density of the force arrows over sections aa' and bb'.

7. Lines of Force. We may use for magnetic fields the same method of vector representation as that given in Fig. 2–6. Having defined unit field intensity as that field strength which will exert a force of one dyne on a unit north pole, we might map out the field of a bar magnet by placing

a unit north pole at every point in the field and noting the force exerted on it. We then draw as many arrows per square centimeter as there are dynes of force at that point on the unit north pole, as is represented in Fig. 2–7. In this case, likewise, as the positions mapped out come closer and closer together, the arrows tend to form continuous lines. It also follows that there must be as many lines leaving a north pole as there are entering a south pole.

Unit field is thus by definition represented by one line per square centimeter. Having a field mapped out according to these ideas, if we desire to determine the force at any point, we need only insert at that point a test

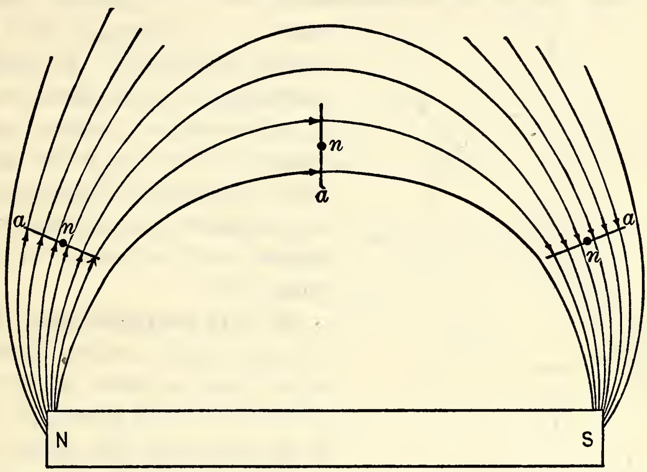


Fig. 2–7. Vector representation of the force on a unit north pole n, at the center of a square centimeter a. The number of arrows normal to the square centimeter represents the force, in dynes, acting on the unit pole. As the points mapped out in the field come closer together, the arrows become lines of force.

area of one square centimeter at right angles to the direction of the lines and count the number of lines traversing it.

It must be carefully borne in mind, however, that the lines of our diagram, which in the case of magnetic fields are called lines of force, are but a vector representation of the direction and intensity of the force of the field on a unit north pole, and nothing more. They have no physical existence and can therefore have no properties of their own; nor do they represent the flow of anything, any more than they would if used to represent intensity and direction of gravitational forces.

However, lines of force serve as a means of determining or measuring field intensities and are universally used; we might speak of a field of 100,000 lines of force per square centimeter, really meaning a field of such strength as would exert a force of 100,000 dynes on a unit north pole.

8. Flux and Flux Density. The Maxwell, the Weber, and the Gauss. The total number of lines of force leaving or entering a magnetic pole is called its flux, or total flux. It is generally spoken of as so many lines of force, or maxwells, a *maxwell* being one line of force. The flux of a magnet is also expressed in terms of a larger unit, the *weber*, a weber being 10^8 maxwells. The symbol Φ (Greek letter phi) is generally used to represent total flux.

The flux density of a field, for which the symbol B is generally used, is the number of lines of force per unit area, and is thus a measure of field intensity. Flux density may be expressed in gausses where a *gauss* is a density of one line per square centimeter, or in kilogausses, where a kilogauss is a density of 1000 lines per square centimeter. If total flux is expressed in webers, flux density may be expressed in webers per square meter, so that one weber per square meter is equal to 10^4 gausses. American designers of electrical machinery express flux densities in lines per square inch.

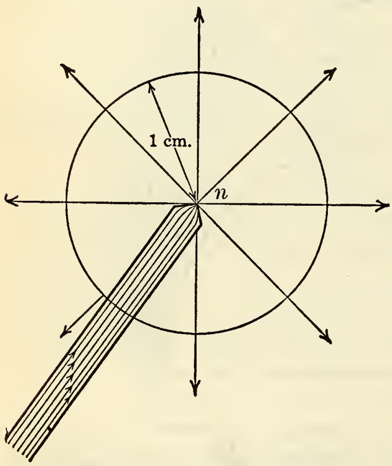


FIG. 2–8. Flux emanating from a unit north pole. According to the conventional method of representing the field strength by the number of lines per square centimeter, and the definition of unit pole strength, there are 4π lines emanating from a pole of unit strength.

9. Flux from Unit Pole. If a sphere of one-centimeter radius is drawn about a unit pole as center, as in Fig. 2–8, a similar unit pole placed at any point on the surface of this sphere would be repelled by a force of one dyne. The second pole is, therefore, by definition, situated in unit field, and there must be one line of force per square centimeter of surface of the sphere, the area of which is $4\pi r^2$ square centimeters. Accordingly, since $r = 1$, the total flux from the unit pole is 4π lines. At a distance $r/2$ cm from the pole, the force, from Eq. (1), is 4 dynes, and the area of this sphere is $4\pi r^2/4$, and again there are 4π lines; the lines are thus continuous as has been previously stated. For a pole of strength m units, the flux emanating is $4\pi m$ lines. In general the flux density due to an isolated unit pole, in a homogeneous medium, is equal to $1/r^2$, and that due to any pole of strength m is m/r^2 lines per square centimeter.

It must be remembered that a unit south pole must accompany every unit north pole, and, if the unit north pole is considered at the end of a long, slender magnet, the 4π lines leaving it must enter the sphere through the substance of the magnet, as shown in Fig. 2–8.

10. Applications. The dyne being a very small unit and commercial magnetic fields being generally very powerful, we find such fields reaching flux densities of thousands of lines per square centimeter. The air-gap flux densities of modern direct-current generators and motors may reach 60,000 lines per square inch or 9300 lines per square centimeter; a unit pole in such a field would be acted upon by a force of 9300 dynes or 9.5 grams.

The field of the ordinary direct-current ammeter or voltmeter (discussed in Chapter IX) of the permanent magnet type, as shown simply in Fig. 9–1, has a flux density of about 1200 lines per square centimeter.

The earth's magnetic field may be considered in terms of its horizontal and vertical components, the values of which vary, particularly with latitude. At Washington, D. C., the horizontal and vertical components are roughly 0.2 and 0.55 line per square centimeter, respectively.

11. Field Intensity or Magnetizing Force. We have defined the intensity of a magnetic field at a point as the force in dynes acting on a unit pole. This is also called the magnetizing force at that point and is usually represented by the symbol H.

Consider a unit pole at a distance r from a pole of strength m. From Eq. (1) the force in dynes due to pole m, acting on unit pole in air, is

$$F = \frac{1 \times m}{r^2}$$

Hence,

$$H = \frac{m}{r^2} \tag{2}$$

If a pole of strength m' were substituted for the unit pole, the force in dynes acting on pole m' would be

$$F = \frac{m' \times m}{r^2} = m'H \tag{3}$$

From this it follows that the force on m' can be computed either by considering it in proximity to another pole m, or placed in a certain field of intensity H, the origin of which does not concern us. It is thus possible to consider either the pole or its field; generally in electrical engineering, the field, and not the pole itself, is considered.

The definition given for field intensity suffices also for the flux density *in air*, as they are here equal, that is $B = H$. We say that unit magnetizing force or unit field intensity produces a flux density of one line of magnetic flux per square centimeter, *in air*.

Another important view of field intensity, and more particularly of magnetizing force H, is that it represents the work done in moving a unit pole a distance of one centimeter along the path of the lines of force. The force due to a field of intensity H being H dynes, the work done per centi-

meter becomes H ergs. The conception that magnetizing force is the work done per centimeter per unit pole is, in general, the most convenient one in considering the various relations existing between the different magnetic quantities.

12. Magnetic Field Surrounding a Conductor Carrying Current. In 1819, Oersted discovered that a magnetic needle is disturbed when a conductor carrying electric current is brought into its vicinity. Further experiment showed that there was a magnetic field around such a conductor, that the direction of the field was circular about the conductor, as in Fig. 2–9, and its intensity was proportional to the current.

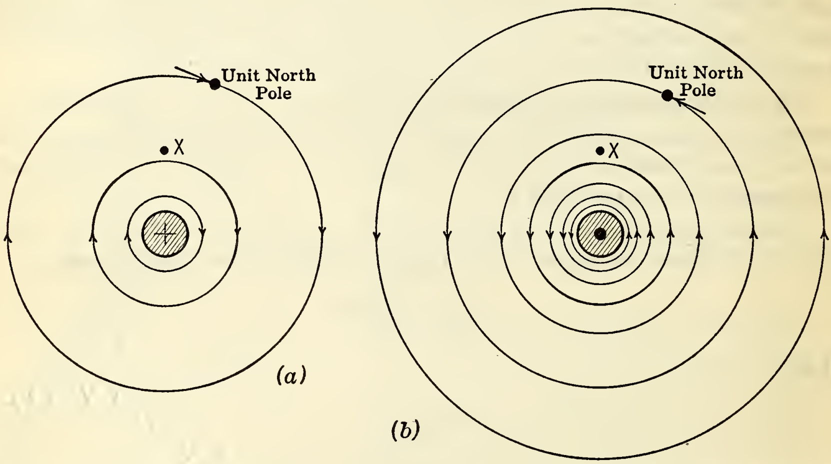


FIG. 2–9. Maps of the magnetic field surrounding a conductor which is carrying current. The current in (a) is assumed as flowing into the paper and out of the paper in (b), so that the direction of the fields is opposite. The direction shown for the magnetic fields is that in which a north magnetic pole would be urged. The current in (b) is assumed as double that in (a). The intensity of the field will thus be doubled, so that the lines of force will everywhere be twice as dense.

Throughout this book a current flowing out of the paper will be indicated by a dot within the conductor to represent the end of an arrow coming up out of the paper; a current flowing into the paper will be indicated by a cross to represent the feathered end of an arrow going into the paper.

Using conventional directions of current, when current flows into the paper the direction of the field is found to be clockwise, a unit north pole being so urged, and counter-clockwise when current flows out. Two simple rules are available for determining this relation. If a right-handed screw is screwed into the conductor in the direction of the current flow, the screw is turned in the direction of the lines of force; or if the conductor is grasped by the right hand, with the thumb extending in the direction of the current, the direction of the fingers indicates the direction of the lines of force.

In Fig. 2–9, the field is correctly mapped out for the current the conductor is carrying, the number of lines per square centimeter indicating the intensity of the field. In (*b*) the current is supposed to be double that in (*a*), and the field intensity at any point X must then be doubled; there are therefore twice as many lines per square centimeter.

If the increase in current were a gradual one, the lines would appear to be spreading out from the conductor as the current increases. The lines,

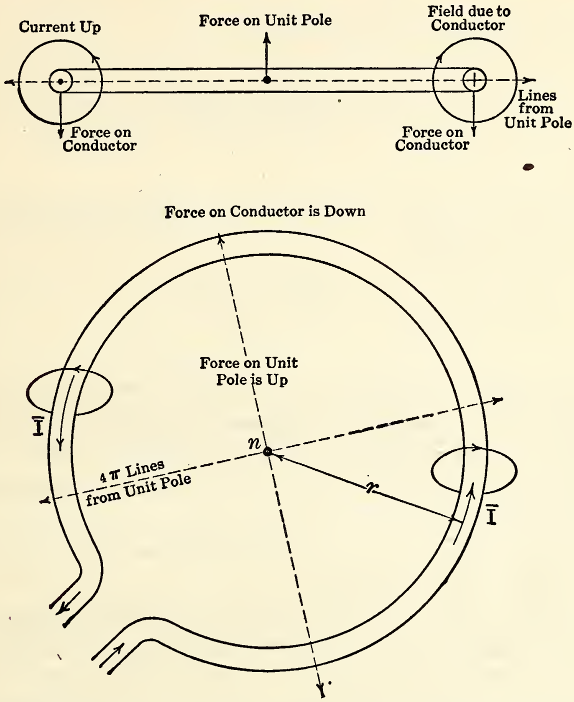


Fig. 2–10. Force acting on a unit north pole at the center of a circular loop of wire carrying current. If the loop is one centimeter in radius, unit current (one abampere) will produce a force of 2π dynes on the unit pole.

having no physical existence, are not really doing anything of the sort; what actually happens is that the field intensity at any point, X, is increasing. It is convenient, however, to speak of the lines as spreading out, and we often conceive the growing magnetic field to be established in this fashion.

13. Field at Center of a Circular Loop Carrying Current. In Fig. 2–10 is shown a circular loop of wire of radius r centimeters, and carrying a current $\bar{I}$; at the center of the loop is placed a unit north pole. From the ideas presented in the previous section, it is evident that there will be a force acting upward on the unit north pole, due to an element of the wire

of length ab. It is found by experiment that the force on the pole varies directly as the length of the element of the wire and the strength of the current, and inversely as the square of the distance between the element of wire and the pole, or

$$F \propto \frac{ab \cdot \bar{I}}{r^2}$$

and the total force due to the entire loop,

$$F \propto \frac{2\pi r \bar{I}}{r^2} \propto \frac{2\pi \bar{I}}{r}$$

Evidently, by proper choice of units, this proportionality may be written in the form of an equation

$$F = \frac{2\pi r \bar{I}}{r^2} = \frac{2\pi \bar{I}}{r} \tag{4}$$

14. Unit Current. If the force on the unit north pole placed at the center of a loop of one centimeter radius is 2π dynes, the current value is taken as unity. Accordingly, unit current will exert a force of 2π dynes on a unit pole placed at the center of a loop of one centimeter radius. Such unit current is taken as the absolute unit and is called the *abampere*; but as this is a fairly large unit, the practical unit, the *ampere*, has been taken as one-tenth of the abampere.

The ampere may also be defined from the standard ohm and the standard volt by the relation known as Ohm's law, discussed in Chapter III.

In this book currents and voltages in absolute units will be designated by a bar over the symbol, i.e., $\bar{I}$, $\bar{E}$, etc.; currents and voltages in practical units (pu) will be designated by symbols without the bar, i.e., I, E, etc.

The determination of the ampere by the method just suggested is difficult and subject to error. A much easier and more accurate way is to determine the ampere in terms of its electrolytic effects; according to standard specifications, the ampere is that unvarying current, which, when passed through a standard silver nitrate solution, under specified conditions, deposits silver at the rate of 0.001118 gram per second.

15. Force Acting on a Conductor Carrying Current in a Magnetic Field. In the conditions shown by Fig. 2–10, it is evident that there must be a force acting on the loop of wire, due to the pole. Or, by considering the field of the pole instead of the pole itself, it is evident that a conductor placed in a magnetic field has a force acting on it.

In the expression $F = 2\pi r\bar{I}/r^2$ (Eq. 4), the quantity $2\pi r$ is the length of

the conductor, l. The flux density B, at any point in the field of the pole is

$$B \text{ (at distance of } r \text{ cm)} = \frac{\text{flux emanating from the unit pole}}{\text{area of surrounding sphere of } r \text{ cm radius}}$$

$$= \frac{4\pi}{4\pi r^2} = \frac{1}{r^2}$$

so that

$$F = Bl\bar{I} \text{ dynes} \tag{5}$$

If, as in Fig. 2–11, the current-carrying conductor is a straight wire placed at right angles to the direction of a field of uniform flux density, B, the force is dependent on the same quantities.

The equation, $F = Bl\bar{I}$, also serves as a means of defining the abampere, for, if the conductor is placed in unit field and the force on the conductor is one dyne per centimeter length of conductor, the current flowing is one abampere.

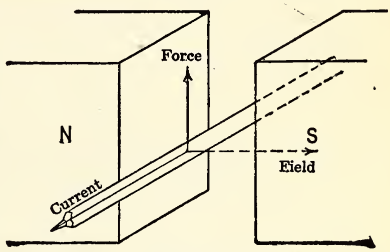


FIG. 2–11. The force acting on a conductor, which is carrying current and is placed in a magnetic field, other than its own, is at right angles to the length of the conductor and to the direction of the field.

The relation between the directions of the current, field, and force are given by *Fleming's left-hand rule*. When the thumb, forefinger, and middle finger of the left hand are held mutually perpendicular to one another, the forefinger pointing in the direction of the field and the middle finger in the direction of the flow of the current, the thumb will point in the direction of the force on the conductor. If the conductor is not perpendicular to the direction of the field, but makes some angle θ with it, the force will vary as the sine of the angle θ.

16. Magnetic Field of a Solenoid, Carrying Current. A solenoid is a conductor wound in the form of a long cylindrical helix of constant cross-section. When carrying current, a solenoid has a magnetic field and, generally speaking, has all the properties of a bar magnet. When mapped out with a unit pole, the magnetic field of a solenoid with an air core appears as in Fig. 2–12*a*. The flux lines are found to be closed, leaving at one end, passing around outside, and entering at the other end. The field at the center is found to be quite uniform, but all the lines do not leave at the ends, some leaking out before they reach the ends. If the turns comprising the solenoid are not close together, there will be much more flux leaking out between turns, and, if the turns are far enough apart, closed lines will exist to an appreciable extent around the individual turns. as shown in Fig. 2–12*b*.

If the solenoid is long in comparison to its diameter (say length = 10 times diameter), the field intensity is practically uniform within the coil, but falls off very rapidly as the end is approached.

17. Action of a Conductor Carrying Current in a Magnetic Field, from the Standpoint of Interlinkages. Every electric circuit carrying current forms one or more closed loops, and every line of force also constitutes a closed loop. When a line of force links with a closed loop in the same way that two links of a chain interlink with each other, one *interlinkage* results. The flux lines may be set up by the coil itself, or by some neighbor-

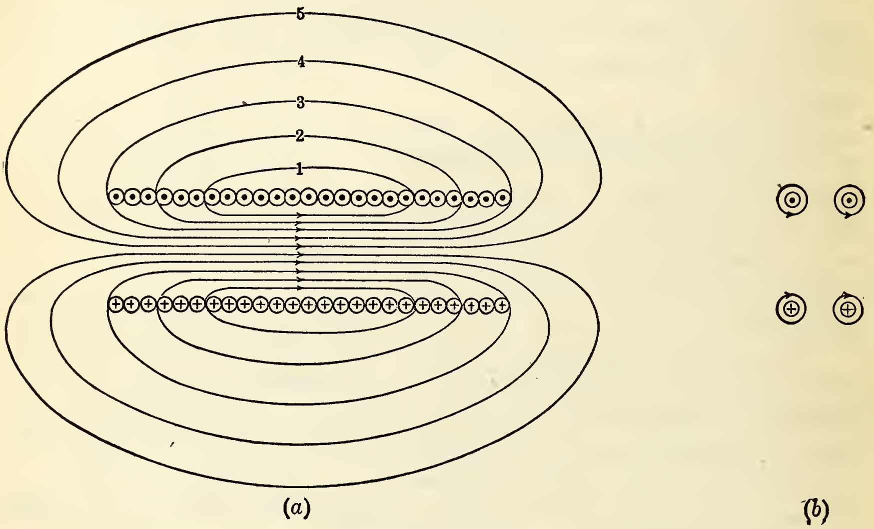


Fig. 2–12. Map of the field set up in a solenoid, having an air core; the actual field is caused by the resultant effect of all the circular loops, each acting as though the others were not present, as suggested by the two extra turns shown at (*b*).

ing coil or magnet. If several lines link with a number of turns, the number of interlinkages is equal to their product, as in Fig. 2–13, where 3 lines link with 2 turns, producing 6 interlinkages. In Fig. 2–12 we have 2 lines linking with 13 turns, 2 lines linking with 19 turns, and 6 lines linking with 25 turns, resulting in a total of 214 interlinkages.

As stated in the previous paragraph, a coil of wire carrying current has all the properties of a bar magnet, and it is therefore always possible to replace a permanent magnet by one or more coils. In Fig. 2-14 is shown a field between the poles of a permanent magnet.

Now, two coils separated from each other as in Fig. 2-15 can be made to produce a field of the same intensity and distribution as that given by the above magnet, by properly adjusting the current passing through them; the coils can therefore be used to replace the magnet of Fig. 2–14.

Consider a system as in Fig. 2-16, made up of a movable rectangular coil of, say, 3 turns, with its magnetic field, and a field set up by a perma-

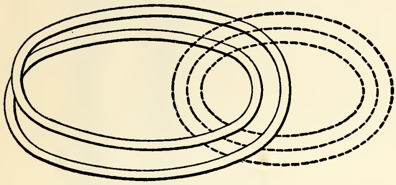


Fig. 2–13.

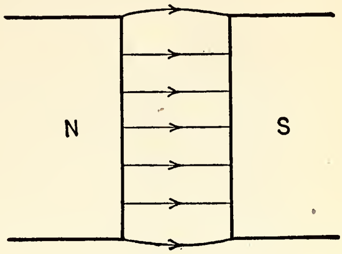


Fig. 2–14.

Fig. 2–13. This figure gives the idea of interlinkages; there are shown three lines of magnetic flux, each linking with the two turns of wire shown. One line of flux linking with one turn of a circuit constitutes one interlinkage, so in this diagram there are six interlinkages.

Fig. 2–14. Field between the poles of a permanent magnet. If the pole faces are plane, as shown here, the lines of flux are parallel to each other except at the edges, where they curve out. The lines leave the pole faces perpendicular to the surface.

nent magnet, the coil and the magnet being far enough away not to influence each other. The current flowing through the coil is of such value that 6 lines of force are set up, giving 18 interlinkages. If we replace the

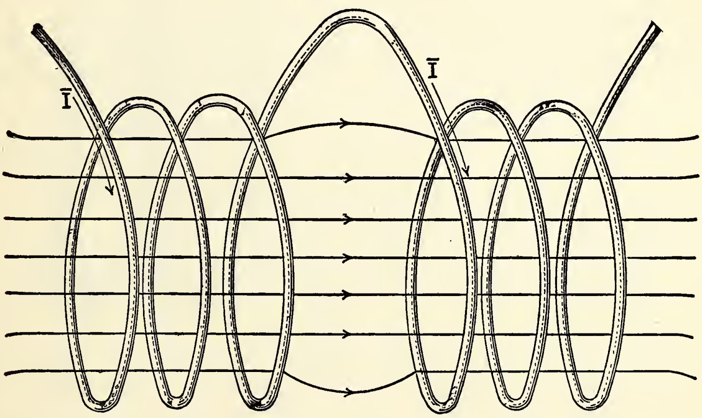


Fig. 2–15. The field of the permanent magnet shown in Fig. 2–14 may be replaced by the field between two coils. If the coils are properly placed, and have the proper number of turns and current, the field set up between them will be exactly the same as that shown in Fig. 2–14.

permanent magnet by 2 fixed coils, each of 2 turns, and pass through them a current of such value as to produce a field between the coils of the same density as between the poles of the magnets of Fig. 2–16, we find condi-

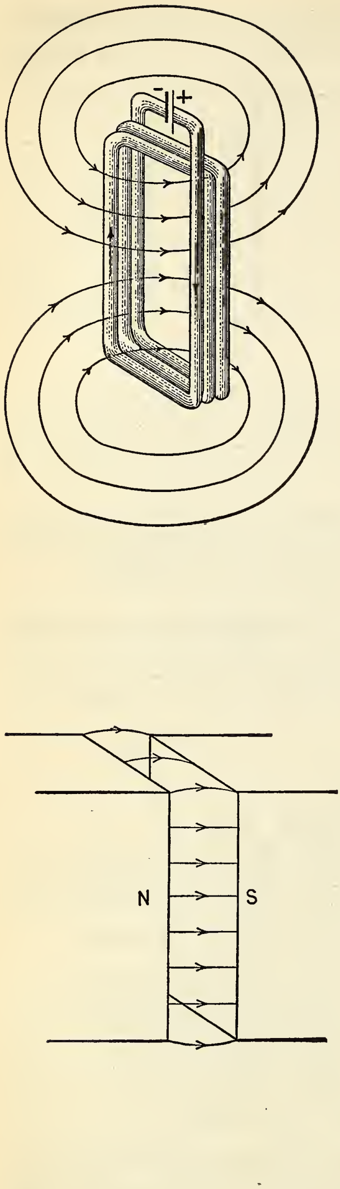


Fig. 2–16.

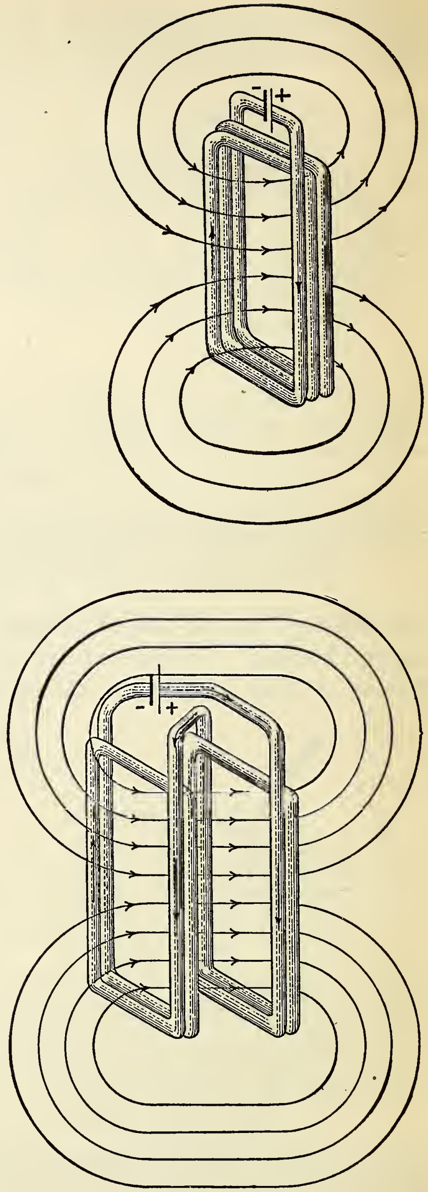


Fig. 2–17.

Fig. 2–16. Coil carrying current approaching the field due to a permanent magnet. The two magnetic fields are not, as yet, appreciably interlinked.

Fig. 2–17. The permanent magnet of Fig. 2–16 replaced by two coils. The system, composed of the two fixed coils (lower) and the single movable coil (upper), has a total of 50 interlinkages, 32 for the lower coil and 18 for the upper coil.

tions as in Fig. 2–17, assuming that all the lines pass completely through all the turns of both coils. If the current sets up 8 lines, we have 32 interlinkages for the fixed coil and 18 interlinkages for the movable coil, or 50 in the entire system.

The action of a current-carrying conductor in a magnetic field may now be regarded from the standpoint of interlinkages, for every such conductor is but part of a closed loop. If the rectangular movable coil of Fig. 2–17 is brought into the field of the fixed coils, as indicated simply in Fig. 2–18, application of Fleming's left-hand rule indicates that the lower coil-side L,

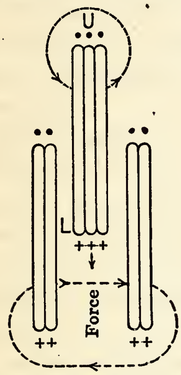


Fig. 2–18.

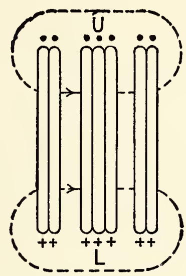


Fig. 2–19.

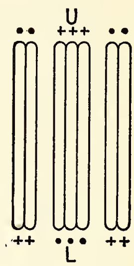


Fig. 2–20.

Fig. 2–18. The movable coil of Fig. 2–17 is here shown, in simplified form, partly in the field of the two fixed coils of Fig. 2–17. The force exerted between the movable coil and the field of the fixed coils tends to draw the movable coil farther into the field of the fixed coils, thereby increasing the total interlinkages of the system.

Fig. 2–19. The movable coil of Fig. 2–18 is now entirely in the field of the fixed coils. The interlinkages of the combination are now a maximum, and the movable coil is therefore in a position of stable equilibrium.

Fig. 2–20. The current in the movable coil has been reversed, so that the magnetizing effects of the two coils tend to neutralize each other. The interlinkages of the system are now a minimum and any motion of the movable coil will result in an increase in the total interlinkages; the movable coil is therefore in a position of unstable equilibrium.

in which we are imagining current to be flowing into the paper, will develop a force tending to move the coil down into the field of the fixed coils. It will be seen that the direction of the field of the fixed coils and that set up by the movable coil across its own plane are both from left to right (as shown by one dotted line for each of the coils); by moving into the field of the fixed coils (or the magnet) the movable coil acts to include within itself the greatest possible number of lines.

When the movable coil has moved so as to include as much of the fixed field as possible, as in Fig. 2–19, the number of interlinkages of the system is a maximum. Assuming that the medium is air, the lines threading

both coils will now be the sum of those individually set up, or $6 + 8 = 14$. Since there are altogether 7 turns in the system, $7 \times 14 = 98$ interlinkages result. Thus, the action of a coil carrying a current (which is maintained constant), placed in a magnetic field, is to move so *as to increase the total number of interlinkages of the system.*

The assumption that when the movable coil has moved completely into the field, a total of 14 lines will be set up, or that all the lines set up will completely thread all the turns of the system, is not necessarily correct; but it is evident that the total number of interlinkages of the system is increased when the movable coil comes into the field, as in Fig. 2–19.

When the coil is in the position shown in Fig. 2–19, where the total number of interlinkages of the system is a maximum, it is in a position of stable equilibrium, a force tending to move the coil up being developed by the upper coil-side U, and an equal downward force by the lower coil-side, L. Any dislodgment of the movable coil, tending to reduce the total number of interlinkages of the system, results in unbalancing the forces developed by the coil-sides, thus tending to restore the coil to its position in Fig. 2–19. This follows because whichever coil-side of the movable coil moves out of the fixed field will evidently develop less force than the other coil-side, which has moved up into a position of more intense field.

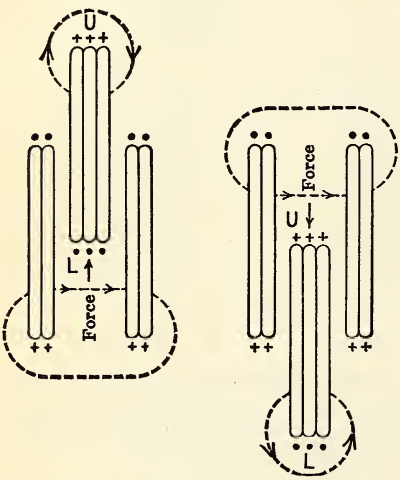


FIG. 2–21. If the movable coil of Fig. 2–20 is dislodged in either direction, a force is developed which tends to make it move farther in the same direction, until it is entirely outside the field of the fixed coils. Increase of interlinkages results.

However, if the direction of the current in the movable coil is reversed, as in Fig. 2–20, the coil is in a position of unstable equilibrium and tends to move out of the field, as application of Fleming's left-hand rule will show. It may move out of the fixed field in either an upward or a downward direction, depending upon which way it started to move, as indicated in Fig. 2–21.

With the current reversed, as in Fig. 2–20, and with the same assumptions as to the lines threading all the coils, the total lines set up through both coils will now be $8 - 6 = 2$, since the movable coil now tends to set up its lines in a direction opposite to that of the fixed coils. There are now 2×7, or 14, interlinkages in the system. When the movable coil has moved completely out of the fixed field, *the total number of interlinkages*

of the system will have increased to 50, the conditions being as shown in Fig. 2–17.

Thus, the action of the coil has again been to move so as to increase the total number of interlinkages of the entire system, and we can conclude that, whenever a coil carrying current is situated in a magnetic field other than its own, it will always tend to move so as to make the total interlinkages of the system a maximum.

18. Applications of the Principle of a Current-carrying Conductor in a Magnetic Field. The most important of the numerous applications of this principle is the motor, whose armature carries many coils. Suppose a pivoted coil, carrying current out from the paper along the lower conductor and into the paper along the upper, is placed in a magnetic field, as in

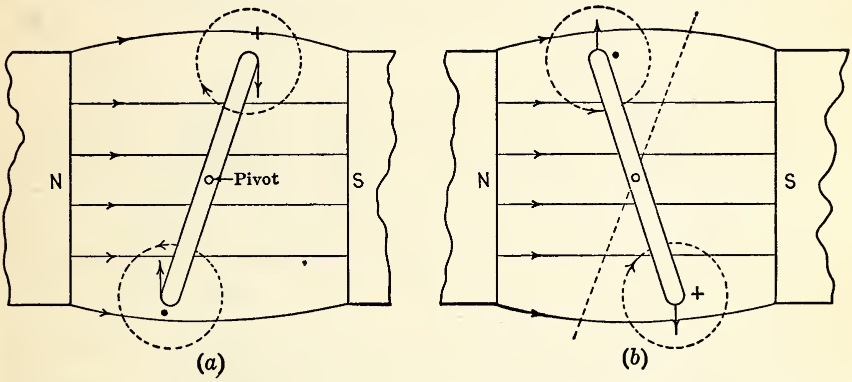


Fig. 2–22. In a coil carrying current, pivoted in a magnetic field, the forces developed tend to rotate the coil. Evidently in position *b* the total interlinkages is greater than in position *a*, because in *a* the current in the coil tends partly to neutralize the field of the magnet.

Fig. 2–22*a*. Applying our first line of reasoning, we see that forces will act on both conductors, tending to rotate the coil in a clockwise direction until the coil reaches approximately its position in Fig. 2–22*b*. The action has also been to make the coil include a greater number of flux interlinkages than before. It is interesting to note that, if the coil had sufficient momentum to carry it from its position in Fig. 2–22*b* about to the dotted line, and the current in the coil were reversed at about the same time, the coil would continue to rotate. This is the principle of the motor, the reversal of the current being effected by the commutator.

In the D'Arsonval, or moving-coil, type of instrument, the same idea is utilized by attaching a pointer to the coil and making the coil work against a spring. The two limiting positions of the coil are shown in Fig. 2–23.

As the simplest illustration of "motion such that flux interlinkages increase" it may be found by experiment that, if a single turn of flexible

conductor, such as lamp cord, is made to have the form of a narrow rectangle, and a heavy current is passed through it, the sides of the rectangle will at once jump apart, so as to give a turn of circular form, this being the shape of turn which gives the most flux for a given current and length of wire.

If, in Figs. 2–22 and 2–23, we had used, instead of a coil of a single turn, one having two turns very close together, each turn carrying the same current, the forces would have been doubled in each case. But in the case of the one-turn coil the force would also have been doubled if we had doubled the current; or, with two turns and doubled current, the forces would have been quadrupled. In other words, the effects are proportional to the product of amperes and turns, or *ampere-turns*, for a given value of

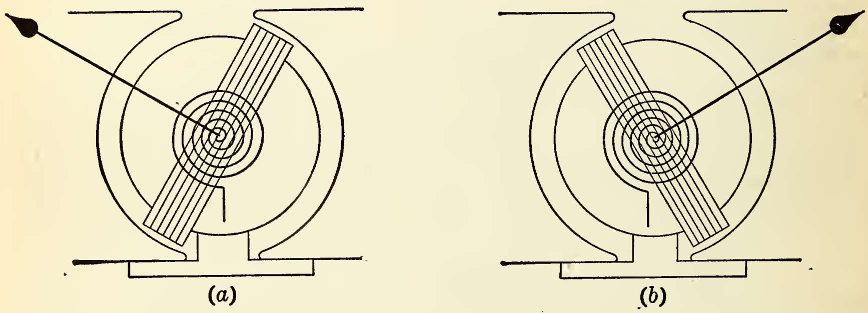


FIG. 2–23. A pivoted coil in the field of a permanent magnet is used in the D'Arsonval, or moving coil, type of d-c instrument. The limiting positions of the coil are shown above, as the coil evidently cannot pull itself farther than position *b*; the force acting on the coil becomes zero when it moves past the pole tip of the magnet.

flux density; the same product of amperes and turns gives the same effect, no matter what the separate values of current and turns may be.

19. Laws of Induction. The simplest method of generating a voltage in a circuit is by means of chemical action, such as that which takes place in the dry cell or storage battery; but in the method much more frequently employed, voltage is generated by relative motion between a magnetic field and a conductor (the motion being in such a direction that the conductor "cuts" flux); such voltage is called an induced voltage.

The general laws of induced voltages, first discovered by Faraday in 1831, and more completely summed up by Lenz about 1834, are as follows:

First Law. Whenever a conductor moves so as to cut flux, or whenever there is a change in the number of lines of force passing through or linking with a closed circuit, voltage is induced, which tends to set up current in such a direction as to produce a field which opposes the *change* in the flux threading the circuit.

Second Law. The voltage induced in a single turn of wire is proportional to the rate of change of the lines of force passing through, or linking with, that turn.

20. Unit EMF. The absolute (cgs) unit of electromotive force is the electromotive force (emf) induced when an inductor is cutting flux at a rate of one line of force per second, or when the flux threading a coil of one turn is changing at a rate of one line per second. This unit is called the absolute volt, or *abvolt*, but is, for ordinary purposes, an exceedingly small unit. The practical unit, the volt, is taken as 10^8 absolute volts, and is then the emf generated when the flux threading a coil of one turn is changing at a rate of 10^8 lines of force, or one weber, per second.

We have, then,

$$\bar{E}_{av} = \frac{\Phi}{t} \text{ abvolts, or } E_{av} = \frac{\Phi}{t \cdot 10^8} \text{ volts} \tag{6}$$

where Φ is in lines of force or maxwells.

Of course, it may be that the flux, Φ, cut in time t is not cut at a uniform rate; perhaps more rapidly at first and then more slowly at the end of time t. In such a case the voltage will not be the same throughout the time t; and Eq. (6) gives the average value for this interval. The voltage can be stated accurately only by expressions from the calculus involving rates of change, as

$$\bar{e} = \frac{d\Phi}{dt} \text{ abvolts, or } e = \frac{d\Phi}{dt} \times 10^{-8} \text{ volts} \tag{7}$$

where e is the instantaneous voltage.

If total flux cut is measured in webers, the equation reduces to

$$e = \frac{d\Phi'}{dt} \text{ volts} \tag{8}$$

where Φ' is in webers.

Working standards of emf are provided by certain standard cells, the voltages of which are very definite and constant, on open circuit. The Clark cell has an open-circuit emf of 1.4328 volts at 15 C., and the Weston cell, the one principally used, has an emf of 1.01830 volts on open circuit at 20 C.

21. Conductor Moving in a Magnetic Field. Experiment shows that when a conductor moves in a magnetic field at right angles to both the direction of the magnetic field and itself, so as to cut the lines of force, an electromotive force is induced in the wire as long as the cutting of the lines continues. The voltage generated is found to be equal to the product of the flux density in which the conductor is moving, the length of the wire cutting flux, and the velocity of the conductor in a direction at right angles

to the field. This follows from the definition of unit emf as may be seen from Fig. 2–24.

Let B = flux density in gausses in which conductor is moving;
l = the length in centimeters of conductor cutting flux;
s = the distance in centimeters the conductor moves at right angles to the direction of the field, in time t.

The velocity of the conductor, v, is then s/t, and the product $l \cdot s$ is evidently the area passed over by the wire. Hence, the total flux cut is

$$Bls$$

and the flux cut, per second,

$$\frac{Bls}{t} = Blv$$

The average voltage generated is then

$$\bar{E}_{av} = Blv \text{ abvolts,}$$

or

$$E_{av} = Blv \times 10^{-8} \text{ volts} \qquad (9)$$

If the distance, s, is passed over at a non-uniform rate, we must again use the calculus. For any motion, uniform or not, we have

$$\text{Rate of cutting of flux} = Bl\frac{ds}{dt}$$

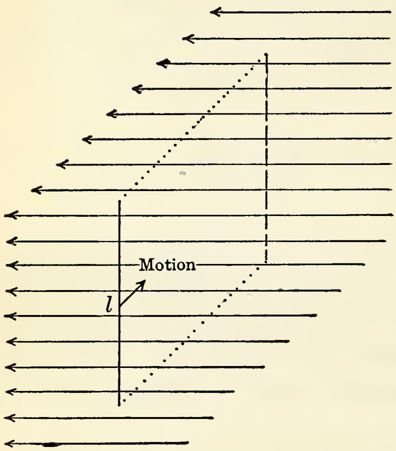


FIG. 2–24. A conductor moving through a magnetic field, so as to cut lines of force, generates an electromotive force.

The instantaneous voltage generated is then

$$\bar{e} = Bl\frac{ds}{dt} \text{ abvolts,} \quad \text{or} \quad e = Bl\frac{ds}{dt} \times 10^{-8} \text{ volts} \qquad (10)$$

In case Eq. (9) is used when the rate of cutting is not uniform, the voltage so calculated will be the average value of the voltage generated during the motion.

If flux densities are expressed in webers per square meter and the length of the conductor and the distance it travels in time t are expressed in meters, the second portions of Eqs. (9) and (10) become

$$E_{av} = Blv \text{ volts} \quad \text{and} \quad e = Bl\frac{ds}{dt} \text{ volts} \qquad (11)$$

If the conductor does not move at right angles either to itself or to the field, the voltage generated is proportional to the sine of the angle it makes with either. It will be remembered that the force acting on a con-

ductor carrying current in a magnetic field involves the same angle in a similar manner.

The relation between the directions of voltage, field, and motion is given by *Fleming's right-hand rule,* which is similar to his left-hand rule; if the thumb and two fingers are held mutually at right angles to each other, the thumb in the direction of motion, the forefinger in the direction of the field, the second finger points in the direction of the induced voltage (Fig. 2–25).

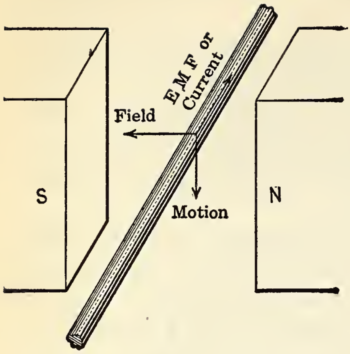
22. Use of Fleming's Right-hand Rule. It is necessary, however, when using Fleming's right-hand rule to remember that the field is supposed to be stationary and the conductor moving. If the conductor in Fig. 2–25 were stationary and the field moving up, the condition would be the same as if the field were stationary and the conductor moving down, as represented. If the conductor were stationary and the field moving down, the direction of the generated emf would be reversed, as if the conductor moved up in a stationary field.

The fact to be emphasized is that the direction of the generated emf depends upon the relative motion of the conductor and field. This will be clear when this action is considered from the viewpoint of flux interlinkages in the next paragraph, the conductor of Fig. 2–25 being but a part of a turn of wire.

23. Coil Moving in a Magnetic Field. Considering the conductor as part of a closed circuit, or coil, we find in Fig. 2–26 that the coil is moving to enclose the flux from the poles, or "fill itself" with flux. The voltage induced in the lower coil-side L, in Fig. 2–26a, as it moves downward, is, by the application of Fleming's right-hand rule, up from the paper. In Fig. 2–26b, the coil is moving upwards, and the voltage in the upper coil-side, U, is into the paper. Thus, as the flux passing through the coil is changed, emf is generated. The current which will flow in the closed circuit as a result of the generated voltage will be in the same direction as the voltage generated, and, in both cases considered, tends to set up a magnetic field (as shown in dotted lines) which is in *the opposite direction* to that of the main field and therefore tends to prevent the flux through the coil from increasing.

If the coil starts to move from a position in which it encloses all the flux from the field, as represented in Fig. 2–27, and so tends to empty itself of flux, the direction of the current will be opposite to what it was when the coil was moving into the field. In Fig. 2–28a, the voltage in the upper coil-side, U, is now towards us, and in Fig. 2–28b, in the lower coil-side, L, it is away from us. The direction of the field which the resultant current tends to set up is in the same direction as that of the main field; the action of the coil thus tends to keep the flux through itself from decreasing.

The conclusion to be reached from the above analysis is that, when the



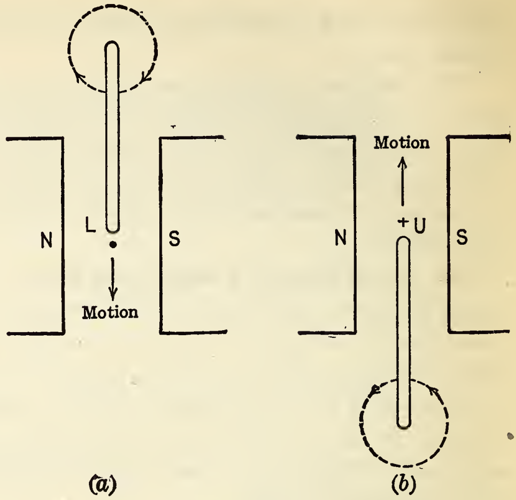
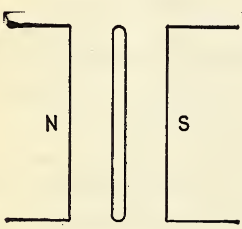


FIG. 2–25. FIG. 2–26.

FIG. 2–25. This diagram gives the relative directions of the motion, field, and induced voltage set up by the conductor's motion.

FIG. 2–26. A coil moves into a magnetic field; current is set up in the coil which tends to set up a magnetic field in the opposite direction to that of the field into which the coil is moving, that is, the current set up in the coil acts to prevent the flux linking the coil from increasing.



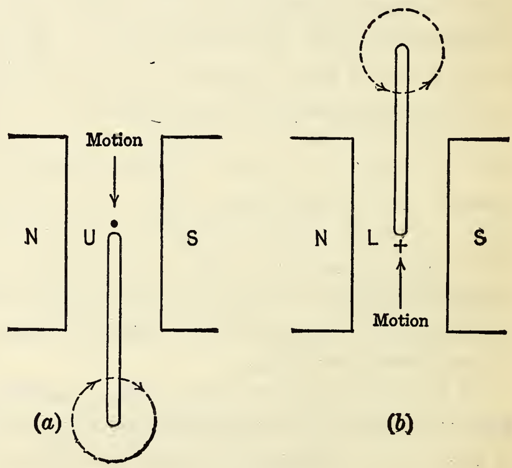


FIG. 2–27. FIG. 2–28.

FIG. 2–27. The coil is now in such a position that it links with all the flux from the magnet.

FIG. 2–28. Coil moving out from the field of the magnet; the current set up by the motion now acts to prevent the flux threading the coil from decreasing, so that the current is in the opposite direction to that it had when the coil was being moved into the field of the magnet.

flux threading a coil is changing, voltage is induced in amount proportional to the rate of change of the flux threading the circuit or coil, and that the action of the resulting current is always such as to tend to prevent the change of flux which is the cause of the induced voltage. In other words, the current acts to maintain constant the flux linking the circuit.

If the coil comprises more than one turn, so as to have a number of inductors in series, the voltage induced will be equal to the product of the voltage induced per turn and the number of turns. Thus, if N turns cut a total flux of Φ lines in t seconds, the average voltage induced is

$$E_{av} = \frac{N\Phi}{t} \text{ abvolts,} \quad \text{or,} \quad E_{av} = \frac{N\Phi}{t \cdot 10^8} \text{ volts} \tag{12}$$

The product $N\Phi$ is evidently the number of interlinkages between flux and turns, and the voltage generated is therefore equal to the rate of change of interlinkages.

If the flux is cut at a non-uniform rate, as is generally the case, the instantaneous generated voltage is given by

$$e = N\frac{d\Phi}{dt} \times 10^{-8} \text{ volts} \tag{13}$$

In addition to the voltage and current effects just analyzed, it is to be noted that if current flows in the conductor, as a result of its motion in a magnetic field, it must also be regarded as a current-carrying conductor in a magnetic field and therefore there are mechanical forces to be considered. The application of Fleming's left-hand rule then shows that the force developed by the conductor, due to currents set up by its motion, is always such as to oppose the motion to which the current is due. Therefore, when a conductor which is part of a closed circuit moves so as to cut a magnetic field, there are two separate reactions to be considered, one electrical and the other mechanical. The electrical reaction tends to oppose any change in the magnetic field linking the circuit, by tending to set up a field which opposes this change; and the mechanical reaction tends to stop the motion by the development of a resisting mechanical force.

Considering now the case of a current-carrying conductor in a magnetic field, we find that, as the conductor or coil of which it is a part is allowed to move into the magnetic field, reactions are set up which tend to stop this action. In Fig. 2–29 the movable coil of Fig. 2–18 is reproduced, carrying current, supplied by a battery, up from the paper in the upper coil-sides and into the paper in the lower coil-sides, as indicated by the dots and crosses *outside* the wires. As the coil moves down into the field, we must consider that its lower coil-side, L, is cutting the flux of the poles, or that the coil is filling itself with flux. As we have shown in Fig. 2–26, such motion, or flux change through the coil, results in the generation of voltage

which, as indicated by the dots and crosses *inside* the wires, will be in a direction opposite, or counter to, that of the current. We then have two emf's acting in the coil, the impressed emf, E, which is necessary to maintain the current which we have supposed flowing in the coil, and the induced, or counter, emf, E_c, which opposes the impressed emf and *lasts as long as the flux through the movable coil is changing.* The current, I, through the coil will then be

$$I = \frac{E - E_c}{R} \tag{14}$$

where R is the resistance of the coil.

Thus, by reducing the current, the generation of a counter emf acts to decrease the mechanical force causing the motion of the coil. Furthermore, as this decreased current will reduce the flux caused by the coil itself, it follows that the counter emf tends to prevent an increase of the interlinkages of the system.

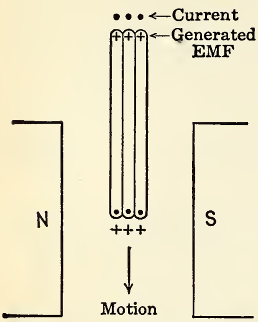


FIG. 2–29. Coil carrying current, moving into a magnetic field; the motion induces a voltage in such a direction that the current flowing in the coil is decreased. This evidently decreases the force which is pulling the coil into the magnet's field, so that the induced voltage thus tends to prevent the interlinkages from increasing, as before.

When the coil is forcibly removed from its position of stable equilibrium in Fig. 2–19, being moved out of the field as in Fig. 2–30, and thus cutting flux, the voltage induced will be in the same direction as the current, so that the latter will be momentarily increased. The induced emf therefore acts to cause an increase in the force developed by the coil, which force must be overcome in removing it from the field. Furthermore, as this increased current will increase the flux set up by the coil, it follows that the counter emf tends to prevent a decrease in the interlinkages of the system.

In general, we may then conclude that whenever a coil moves so as to cause the flux interlinkages to change, reactions, both electrical and mechanical, at once result which tend to oppose this change.

24. Eddy Currents. In the construction of electrical machinery, iron is used extensively as a path for the flux and a support for the copper conductors. Whenever the flux through iron changes, voltages are induced, and, as iron itself is a conductor, currents flow in the volume of the iron. The action of these currents (generally called eddy, or Foucault, currents) is exactly the same as that of those just discussed, although their presence is usually objectionable, and every effort is made to reduce them to reasonable values.

Suppose a sheet of iron to be pushed edgewise into a magnetic field, as in Fig. 2–31. As the edge moves into the field, a voltage will be induced

from B to A, and, there being no voltage induced elsewhere, current will circulate or eddy in the iron, as shown by the dotted lines.

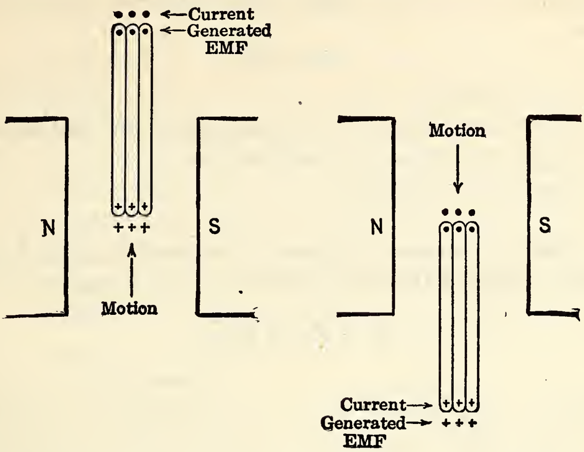


Fig. 2–30. Coil carrying current as before, now being moved out of the magnetic field; the generation of emf by the cutting of the flux now causes an increase of current, thus increasing the force by virtue of which the coil tends to retain its position. This action evidently tends to maintain the interlinkages constant, as before.

The sheet of iron being pushed into a field as in Fig. 2–31 is similar to the coil being inserted into the field of Fig. 2–26. While motion continues eddy currents are produced in the iron which react exactly as the current of the coil. The eddy currents tend to set up a magnetizing force in the opposite direction to that of the poles between which the iron is being inserted, thus weakening the actual field existing. They also develop a mechanical force opposing the agent moving the iron into the field. Further, as a result of the eddy currents, heat is produced, consuming the energy of the system.

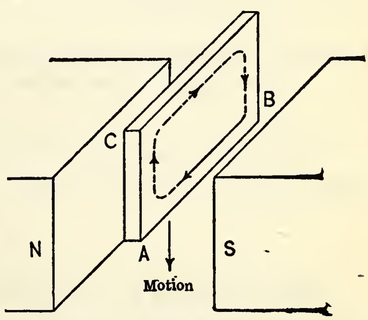


Fig. 2–31. So-called "eddy currents" generated in a sheet of iron moving into a magnetic field; because of this effect all iron cores, in which the magnetic flux changes when the apparatus is in operation, are made laminated, being built up of thin sheets of iron, insulated from one another.

To reduce the eddy currents, it is necessary to build up the volume of iron with thin sheets or laminations parallel to face CA, and to provide some degree of insulation between them, so that current cannot pass freely from one lamination to another. Eddy currents will be discussed further later in this book.

25. Work Done in Moving a Conductor Carrying Current in a Magnetic Field. We have already shown that when a conductor carrying current is placed in a magnetic field (Fig. 2-11) the force exerted on the conductor is

$$F = Bl\bar{I}$$

where F = force in dynes;
B = flux density in gausses or lines per square centimeter;
l = length of conductor in centimeters;
$\bar{I}$ = current in abamperes.

If the conductor is moved a distance x centimeters (Fig. 2–32), at right angles to both itself and the field, the work done in ergs is

$$W = Fx = Bl\bar{I}x \tag{15}$$

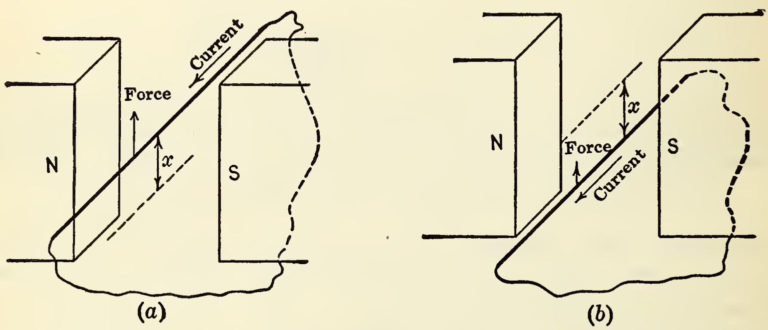


FIG. 2–32. Conductor carrying current, moved in a magnetic field; the work done is proportional to the product of current and flux cut by the conductor.

The quantity Blx is evidently the total flux cut in moving the conductor over the distance x and may be represented by Φ. Hence,

$$W = \bar{I}\Phi \text{ ergs} \tag{16}$$

This expression is an important one, indicating that whenever a current-carrying conductor moves so as to cut flux, work is done in cutting this flux. Whatever work has been done in setting up either the current in the coil or the magnetic flux through which the coil is being moved is not considered here, but merely the work done in moving the current-carrying conductor through the constant flux Φ, the current $\bar{I}$ being held constant.

If the conductor is forced through the field, the work is done *by the agency* pushing the conductor, as in the electric generator. But if the conductor is allowed to move by itself (in the opposite direction), the work is done *by the conductor,* as in the electric motor.

The above expression for work was derived for a single conductor. If there were N conductors in series, the work done would be

$$W = \Phi N \bar{I} \tag{17}$$

and we find that the work is equal to the change in the product of current and flux interlinkages.

If we imagine the conductor in the position shown in Fig. 2–32a, but with the field not set up, and if the field is then built up to the same value it had in the foregoing discussion, the work that would have to be done *in building up the field* in the circuit, of which the conductor forms a part, would be the same as is done *in moving the conductor into the field* so as to enclose all the lines of force. This becomes more apparent if we look at it from the standpoint of flux interlinkages. Work must be done in changing their number, whether it be by moving the coil or building up the interlinkages within the coil.

From section 21, page 37, it is evident that whenever the current-carrying conductor moves, voltage is generated. By the use of Eq. (16) we can evidently derive the relation

$$W = \bar{I}\Phi = (\bar{I}t)\frac{\Phi}{t} = \bar{Q}\bar{E}_{\text{av}} \tag{18}$$

The expression $\Phi/t = \bar{E}_{\text{av}}$ is evidently the average voltage generated by the moving conductor, and $\bar{I}t = \bar{Q}$ (current times time) is the total quantity of electricity. Hence, the work done is that necessary to raise a quantity of charge or electricity through a certain potential difference. This follows too from the definition of potential difference previously made.

As before, if the flux is cut at a non-uniform rate, we must write

$$w = \bar{\imath}\Phi = \bar{\imath}dt\frac{d\Phi}{dt} = \bar{q}\bar{e} \text{ ergs} \tag{19}$$

where all terms are in instantaneous values.

Thus, when a conductor carrying current is moved through a magnetic field, the mechanical energy supplied must be used up in doing the electrical work represented by raising a quantity of electricity through a difference of potential. This is the action of the generator.

26. Power. Unit of Power. If the work done in moving a conductor carrying current through a magnetic field is represented, as before, by

$$W = \bar{I}\Phi \text{ ergs}$$

the average rate of work done or power, P, is

$$P = \frac{W}{t} = \bar{I}\frac{\Phi}{t} \text{ ergs per second}$$

But $\Phi/t = \bar{E}_{av}$, the average voltage generated by the moving conductor. Hence average power is

$$P = \bar{I}\bar{E}_{av} \text{ ergs per second} \tag{20}$$

Here again we may write more accurately that instantaneous power, p, is

$$p = \frac{dw}{dt} = \bar{\imath}\frac{d\Phi}{dt} = \bar{\imath}\bar{e} \text{ ergs per second} \tag{21}$$

Expressed in instantaneous amperes and volts, we have

$$p = i \cdot 10^{-1} \cdot e \cdot 10^8 = ei \cdot 10^7 \text{ ergs per second.}$$

The *watt* being taken as 10^7 ergs per second, instantaneous electrical power in watts is the product of instantaneous amperes and volts, or

$$p = ei \tag{22}$$

If the voltage and current are constant, the electrical power in watts is given by the product of volts and amperes, or

$$P = EI \tag{23}$$

The unit of electrical power used probably more than any other is the *kilowatt*, which is equal to 1000 watts.

The *joule*, a unit of work much used, is taken as 10^7 ergs. A watt is thus a joule per second, or a joule is one watt-second, i.e., a watt for a second (not a watt per second). (It is convenient to remember that a joule of work is about three-quarters of one foot-pound of work.) The unit of energy most frequently used in electrical measurements is the kilowatthour, which is equal to the energy supplied by one kilowatt for one hour; it is equal to 3.6×10^6 joules.

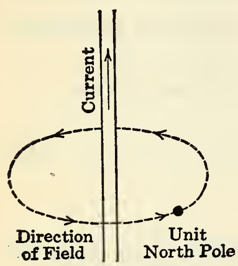


FIG. 2–33. Field about a straight wire carrying current can be calculated from the force which must act on a unit north pole in the vicinity of the conductor.

27. Field Intensity Produced by Current Flowing in a Straight Wire. If a unit north pole is moved around a conductor in air, as in Fig. 2–33, describing a circle of radius = r centimeters against the field intensity H (r centimeters from the wire), the amount of work done is

$$W = H\,2\pi r$$

The work done is also equal to the product of current flowing in the conductor and the flux cut by the conductor as the pole is moved around. There being 4π lines emanating from the unit pole, we have

$$W = \Phi\bar{I} = 4\pi\bar{I}$$

Equating, we have

$$H\, 2\pi r = 4\pi \bar{I}$$

or

$$H = \frac{2\bar{I}}{r} \tag{24}$$

If the force on the unit pole is $H = 2\bar{I}/r$ dynes, there must be this number of lines of force per square centimeter a distance of r centimeters from the wire, in air, due to the current in the wire, or

$$B = \frac{2\bar{I}}{r}$$

If current is expressed in amperes,

$$H = \frac{2I}{10r} \quad \text{and} \quad B = \frac{2I}{10r} = \frac{0.2I}{r} \tag{25}$$

28. Force between Two Parallel Conductors Carrying Current. In Fig. 2–34, either conductor may be considered as lying in a field of flux

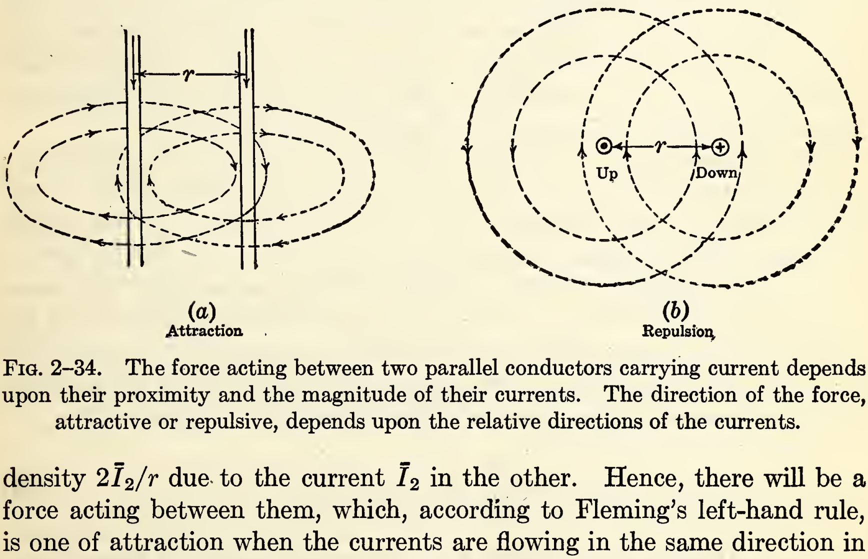


FIG. 2–34. The force acting between two parallel conductors carrying current depends upon their proximity and the magnitude of their currents. The direction of the force, attractive or repulsive, depends upon the relative directions of the currents.

density $2\bar{I}_2/r$ due to the current $\bar{I}_2$ in the other. Hence, there will be a force acting between them, which, according to Fleming's left-hand rule, is one of attraction when the currents are flowing in the same direction in the two conductors (Fig. 2–34*a*), and one of repulsion when the currents are in opposite directions (Fig. 2–34*b*).

The force on the first conductor, due to the field of the second, is

$$F = Bl\bar{I}_1$$

Since $B = 2\bar{I}_2/r$, we have

$$F = \frac{2\bar{I}_2}{r} l\bar{I}_1 = \frac{2\bar{I}_1\bar{I}_2 l}{r} \tag{26}$$

The force in dynes per centimeter length of conductor is then

$$F = \frac{2\bar{I}_1\bar{I}_2}{r} \tag{27}$$

If the currents in the two conductors are equal, the last equation obviously reduces to

$$F = \frac{2\bar{I}^2}{r} \tag{28}$$

An application of this principle is found in the Siemens dynamometer, in which two coils at right angles to each other, one stationary and the other movable, are used to measure current. An instrument which operates on the principle of the dynamometer is shown in Chapter IX.

It is convenient to remember that two conductors, carrying current, act on one another, and if they are free to move, they will always move so as to increase the total magnetic field or interlinkages set up by the combination of turns of which the conductors are but a part. If the two conductors are carrying current in the same direction, more field will be produced if they come close together; if in the opposite direction, it is evident that the two currents are tending to neutralize each other magnetically, so that the force between them must tend to push them apart, thereby increasing the sum of the magnetic effects of the two.

29. Permeability. In vacuum, unit field intensity or unit magnetizing force produces one gauss or one line of force per square centimeter, or unit flux density; from this it follows that $H = B$ in vacuum. In this respect air is practically the same as vacuum, so we have $B = H$, even for air.

In other mediums, unit field intensity or unit magnetizing force sometimes produces more than one line per square centimeter; the ratio of the number of lines in the medium to the number which would be produced in air by the same magnetizing force is termed the *permeability* of the medium, and is designated by the symbol μ (Greek letter mu). We have then,

$$\mu = \frac{B}{H} \quad \text{and} \quad B = \mu H \tag{29}$$

For such mediums as iron and its compounds, and to a lesser degree cobalt and nickel, μ is considerably greater than unity, which means that, for a given magnetizing force, a greater flux density will result through a volume of iron than through an equal volume of air. For all other mediums, permeability is practically equal to unity.

Permeability is then the ratio of the flux density produced in a volume of any medium to the flux density produced in an equivalent volume of air, by the same magnetizing force.

For iron, permeability is not a constant, but varies greatly with the flux density, from perhaps 275 for very low densities to perhaps 4500 at densities of several thousand gausses.*

30. Permeability of Iron Alloys. The construction of motors and transformers requires that certain portions be built up of thin sheets or laminations. The iron used is usually alloyed with silicon in amounts up to 4.5 per cent, the result being known as silicon iron or silicon steel. The effect of silicon is to improve the permeability at moderate flux densities, but impair it at high flux densities (see magnetization curves of Fig. 2–38). The most important effect of silicon is to reduce the hysteresis and eddy current losses that occur with varying flux density, and this is the reason for its use. As commercially rolled into sheets the maximum permeability of silicon steel varies between 5000 and 8000, depending upon composition and heat treatment.*

There are available, commercially, alloys of iron and nickel which show remarkable properties. One of these, styled Hipernik,† is a refined alloy, 50 per cent iron and 50 per cent nickel, which has a maximum permeability of 90,000.*

Permalloy ‡ is an iron-nickel alloy, containing 78 per cent nickel, that has a maximum permeability of 100,000.*

31. Magnetomotive Force. Consider a toroid (solenoid bent into the form of a ring) of N turns, with an air core (permeability = 1) of mean radius r, and carrying a current of $\bar{I}$ abamperes, as in Fig. 2–35. The magnetizing force or field intensity in the core is H lines per square centimeter. This field intensity H will be nearly constant over the cross-section of the core, providing the radius of the cross-section of the ring is small compared to r, the average radius of the ring.

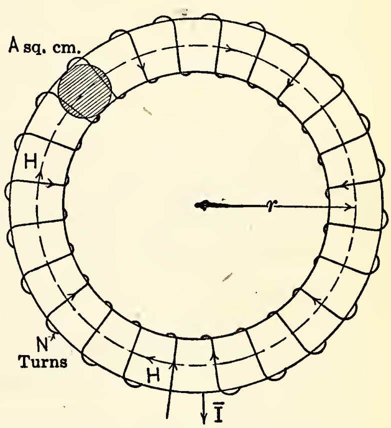


Fig. 2–35. Toroidal coil carrying current; if the inner and outer radii are nearly the same this is a very simple magnetic circuit to analyze.

If a unit north pole is carried around the magnetic circuit, the work done will be equal to

$$W = 2\pi r H \text{ ergs}$$

* Values from *Electrical Engineers' Handbook*, by Pender and Del Mar, John Wiley & Sons, 1936.

† Westinghouse Electric & Manufacturing Company.

‡ Western Electric Company.

The 4π lines emanating from the unit pole, however, in moving at a uniform rate around the circuit in a time, t seconds, and cutting the N turns, will generate in the toroid an average voltage

$$\bar{E}_{av} = \frac{4\pi N}{t} \text{ abvolts}$$

For t seconds, then, the toroid acts as a generator, inducing $\bar{E}_{av}$ abvolts and supplying $\bar{I}$ abamperes. It thus furnishes an amount of power $\bar{E}_{av}\bar{I}$ ergs per second, or a total amount of work of $\bar{E}_{av}\bar{I}t$ ergs.

The mechanical work done in moving the pole is thus transformed into electrical work, and these must be equal. Hence,

$$2\pi r H = \bar{E}_{av}\bar{I}t = \frac{4\pi N\bar{I}t}{t} = 4\pi N\bar{I} \text{ ergs}$$

The length of the magnetic path is $2\pi r = l$ cm. Hence,

$$lH = 4\pi N\bar{I}$$

or

$$H = \frac{4\pi N\bar{I}}{l} \tag{30}$$

The quantity $lH = 4\pi N\bar{I}$, which represents the work needed to move the unit pole around the toroid, is called the *magnetomotive force* (abbreviated mmf) of the coil. The symbol for magnetomotive force is $\mathfrak{F}$.

The magnetomotive force, like the electromotive force, is evidently not a force at all. It does, however, like the electromotive force, measure a force by measuring the work done when a unit test body is moved between two points. It is evident that this gives a measure of the force, and it happens that this quantity of work done is much easier to measure than the force itself.

The magnetomotive force thus measures the actual force existing along a path between two points by measuring the work done, in ergs, when a unit pole is moved between the two points.

32. Magnetizing Force. The Gilbert and the Oersted. Magnetizing force, H, is then from Eq. (30) the mmf per centimeter length (or work required to move unit pole one centimeter against the field), which agrees with the statement in section 11, page 25.

If the current is expressed in amperes,

$$\text{Mmf} = \mathfrak{F} = lH = 0.4\pi NI \tag{31}$$

where $\mathfrak{F}$ is expressed in *gilberts* and

$$H = \frac{0.4\pi NI}{l} \tag{32}$$

where H is expressed in *oersteds,* or gilberts per centimeter.

Magnetomotive force is thus proportional to the product of amperes and turns, or ampere-turns. Nor is it necessary that the turns be evenly distributed over the whole magnetic path; they may be concentrated in one or more places, but the mmf for the complete magnetic circuit is the same as though they were distributed uniformly.

In most published scientific data on magnetic materials, flux density is expressed in gausses and magnetizing force in oersteds or gilberts per centimeter. In the data used by designers of electrical machinery, flux density is usually expressed in lines per square inch and magnetizing force in ampere-turns per inch.

33. Practical Units of MMF. Designers of electrical machinery prefer to express magnetomotive force in ampere-turns, where one *ampere-turn* is the mmf produced by one ampere flowing around one turn; it is to be noticed that one ampere-turn is also given by 0.25 ampere flowing around 4 turns, 0.002 ampere around 500 turns, etc.

From Eq. (31)

$$\text{Ampere-turns} = \frac{\text{gilberts}}{0.4\pi} = \frac{\text{gilberts}}{1.2566} = \text{gilberts} \times 0.7958 \tag{33}$$

$$\text{Gilberts} = \text{ampere-turns} \times 0.4\pi = \text{ampere-turns} \times 1.2566 \tag{34}$$

From the ampere-turn there is derived as a unit of magnetizing force, H, the ampere-turn per centimeter, used by European designers, and the ampere-turn per inch, used by English and American designers. The reason for using ampere-turns per inch or centimeter will appear later.

$$\begin{aligned}\text{Ampere-turns per cm} &= \text{gilberts per cm} \times 0.7958 \\ &= \text{oersteds} \times 0.7958\end{aligned}$$

34. Reluctance and the Law of the Magnetic Circuit. The reluctance of a magnetic circuit may be defined as the capacity of a circuit for opposing the setting up of a magnetic field, and we may write that

$$\text{Flux} = \frac{\text{mmf}}{\text{reluctance}} = \frac{\mathfrak{F}}{\mathcal{R}} \tag{35}$$

Unit reluctance, for which no name has at present been adopted, is that of a centimeter cube of air, for when unit mmf is applied across a centimeter cube of air, one line per square centimeter is produced through a length of one centimeter. But if unit mmf is applied to a centimeter cube of a magnetic material of permeability μ, then μ lines per square centimeter will be set up and the reluctance will be $1/\mu$. The reciprocal of permeability, $1/\mu$, is called the magnetic reluctivity and is often represented by the symbol ν (Greek letter nu).

It is reasonably evident that the reluctance of a path will increase with the length, and decrease as the area of the path increases. Thus the reluctance of a path of l centimeters length and uniform area of cross-section, A square centimeters, will be

$$\mathcal{R} = \frac{l}{\mu A} \tag{36}$$

and hence

$$\Phi = \frac{\text{mmf}}{l/\mu A} = \frac{0.4\pi N I \mu A}{l} = H\mu A \tag{37}$$

$$\frac{\Phi}{A} = B = \mu H$$

as was proved before.

If the magnetic path is made up of several parts in series, as is generally the case, the expression for reluctance of the entire path will comprise several terms. We have then

$$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \cdots$$

and

$$\mathcal{R} = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_3}{\mu_3 A_3} + \cdots \tag{38}$$

where $\mathcal{R}$ is the reluctance of the entire circuit.

35. Reluctances in Series. We may then write that, for a number of reluctances in series, upon which a given total mmf is impressed,

$$\Phi = \frac{\text{mmf}}{\dfrac{l_1}{\mu_1 A_1} + \dfrac{l_2}{\mu_2 A_2} + \dfrac{l_3}{\mu_3 A_3} + \cdots}$$

and

$$\text{Mmf} = \Phi\left(\frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_3}{\mu_3 A_3} + \cdots\right)$$

Since

$$\Phi = H_1\mu_1 A_1 = H_2\mu_2 A_2 = H_3\mu_3 A_3 = \cdots$$

$$\text{Mmf} = H_1\mu_1 A_1 \frac{l_1}{\mu_1 A_1} + H_2\mu_2 A_2 \frac{l_2}{\mu_2 A_2} + H_3\mu_3 A_3 \frac{l_3}{\mu_3 A_3} + \cdots$$

$$\text{Mmf} = H_1 l_1 + H_2 l_2 + H_3 l_3 + \cdots \tag{39}$$

The last equation is a very important one; it states that the total mmf impressed upon a magnetic circuit of several reluctances in series is the sum of the several mmf's or the sum of the products obtained by multiplying the length of each reluctance by the magnetizing force required to set up the flux in that reluctance.

It is to be noted that neither area nor flux density appears in the last equation, and this must be true even if the flux density in the various reluctances in series is different.

36. Reluctances in Parallel. Consider a ring built up of three materials, with permeabilities, μ_1, μ_2, μ_3, as in Fig. 2–36. With a given mmf impressed on the entire circuit acting in such a way as to set up a flux around the ring, the flux produced in each section will be

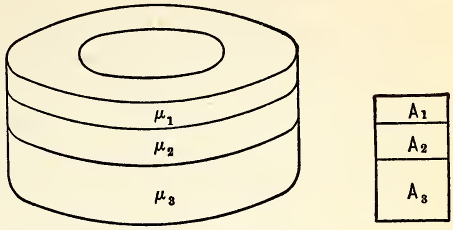


FIG. 2–36. Magnetic circuits in parallel; the reluctance of the whole path is evidently less than that of any one of its sections, so that the addition of sections permits more flux with a given magnetomotive force.

$$\Phi_1 = \frac{\text{mmf}}{\mathcal{R}_1}, \quad \Phi_2 = \frac{\text{mmf}}{\mathcal{R}_2}, \quad \Phi_3 = \frac{\text{mmf}}{\mathcal{R}_3}$$

Since the total flux $\Phi = \Phi_1 + \Phi_2 + \Phi_3$

$$\Phi = \frac{\text{mmf}}{\mathcal{R}} = \text{mmf}\left(\frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3}\right)$$

The reluctance of the entire circuit will then be determined for any group of parallel reluctances by the expression

$$\frac{1}{\mathcal{R}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3} + \cdots$$

or

$$\mathcal{R} = \frac{1}{\dfrac{1}{\mathcal{R}_1} + \dfrac{1}{\mathcal{R}_2} + \dfrac{1}{\mathcal{R}_3} + \cdots} \tag{40}$$

37. Permeability Not Constant. Not only is there a difference in permeability between various irons and steels and other magnetic materials, but also it is found that the permeability of even one particular sample of magnetic material is dependent upon the degree to which the material is magnetized, or upon the flux density.

If a completely demagnetized sample, say in the form of a ring, is gradually magnetized, and the corresponding values of mmf's per centimeter length and flux densities are plotted, a so-called *magnetization curve* is obtained, as in Fig. 2–37.

Since permeability is the ratio of flux density to mmf per centimeter length, or $\mu = B/H$, the values of permeability corresponding to the mag-

netization curve can readily be determined and plotted, as in Fig. 2–37. It will be seen that, as the flux density increases, the value of μ first increases to a maximum and subsequently decreases.

The relation between permeability and flux density, for an unknown sample, *can be determined only experimentally,* and it becomes at once evident that the use of formulae involving reluctance in dealing with magnetic problems is impracticable. If we deal with reluctance to get flux, and hence flux densities, we must assume values for permeability, but, as per-

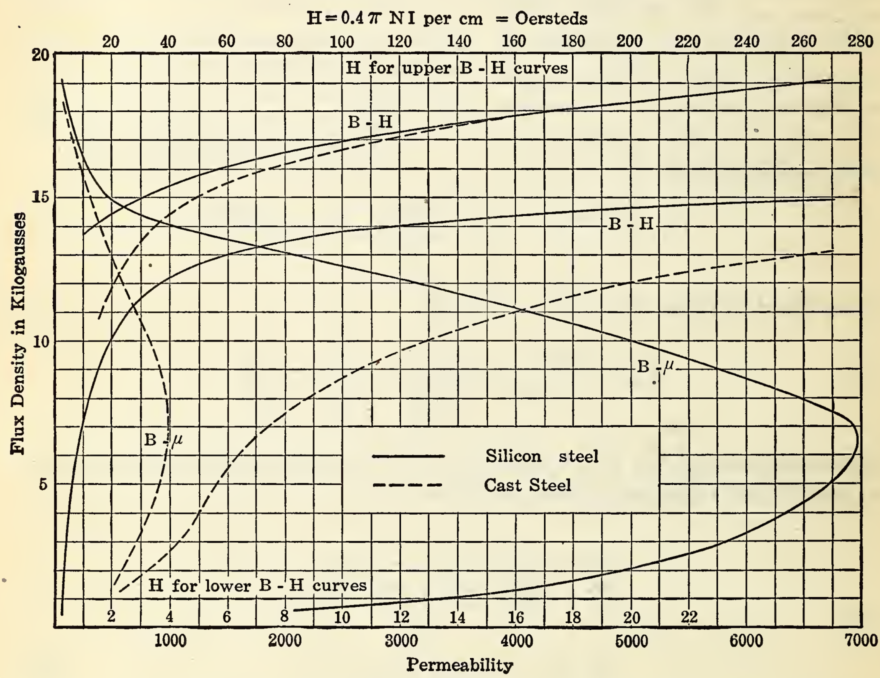


Fig. 2–37. Magnetization and permeability curves for rolled sheets of a grade of silicon steel, and for a grade of cast steel. Comparison of the permeability curves indicates how much better a magnetic material the silicon steel is than the cast steel.

meability depends upon flux density, we can arrive at a solution only after several trials.

38. B-H Curves. As said before, the designer of electrical machinery is ultimately concerned in knowing how many amperes must pass through a winding of a given number of turns; he is thus interested in ampere-turns required in field windings, etc. For the practical solution of problems involving magnetic circuits, use is therefore made of magnetization curves, experimentally determined, giving the relation between mmf per centimeter length and corresponding flux densities for the quality of iron to be used; such curves are shown in Fig. 2–38.

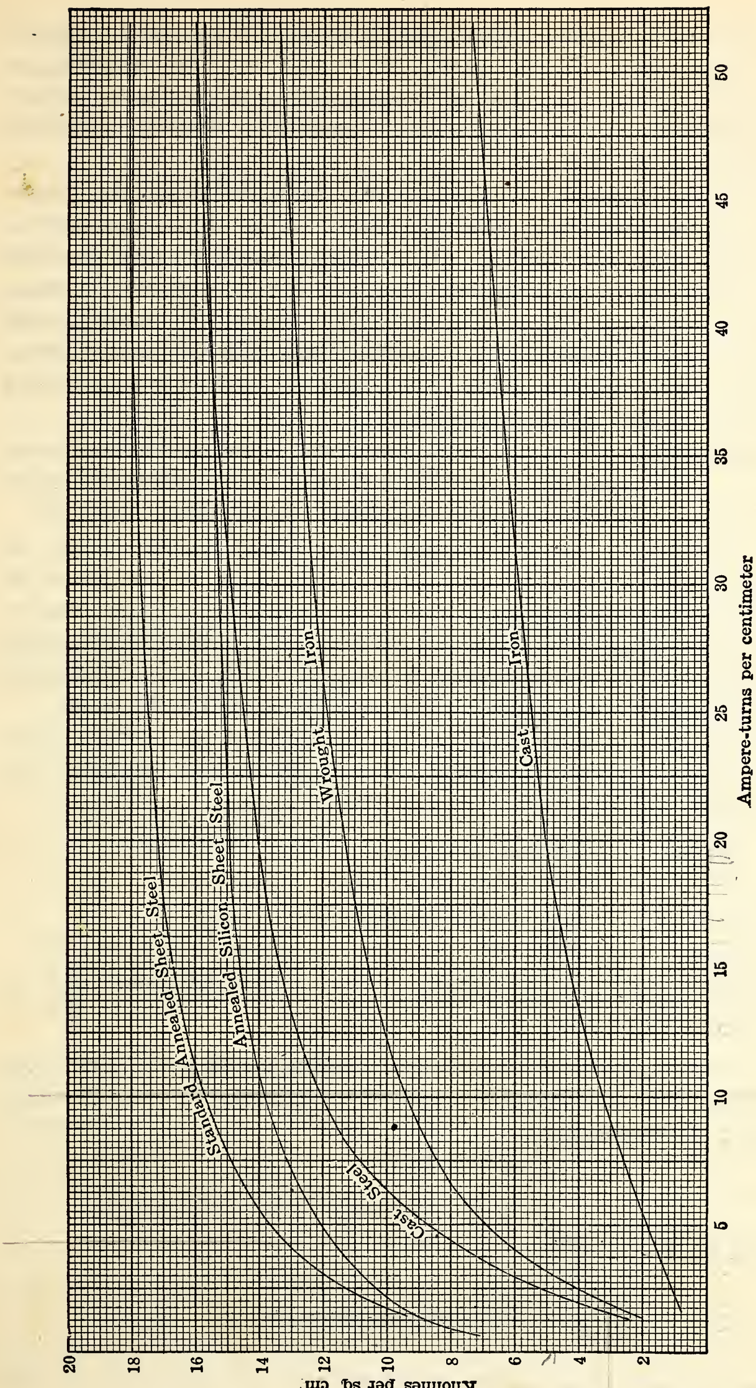


FIG. 2–38. Magnetization curves, or *B–H* curves, for various materials used in the construction of the magnetic circuits of electrical apparatus.

Therefore, if in a given magnetic circuit it is desired to set up a given flux, we first determine the flux density, $B = \Phi/A$, and then, referring to the B–H curve, determine the required ampere-turns per centimeter length; this figure multiplied by the length gives the necessary total ampere-turns to set up the required flux.

The electrical designer, whose business it is to determine the proper sizes of the various parts of the magnetic circuits of motors and generators, works entirely from B–H curves. The testing laboratory furnishes him with an experimentally determined curve, showing the relation between ampere-turns per centimeter or per inch and flux densities for the materials used in the magnetic circuit; he then properly proportions the frame, poles, etc., so that a reasonable number of ampere-turns suffices for magnetizing the field of the machine being designed.

39. Applications. *Example 1.* It is required to set up 40,000 lines in a cast-iron ring with a circular cross-section of 8 sq cm (Fig. 2–39). The ring has a mean radius of 10 cm.

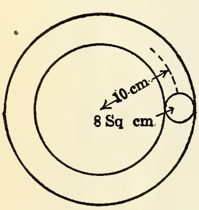


FIG. 2–39. The simplest type of magnetic circuit.

The flux density being 5000 lines per sq cm, we require, from the curves in Fig. 2–38, 19.5 ampere-turns per centimeter length, or (using Eq. (39)) a total of $2\pi \cdot 10 \cdot 19.5 = 1225$ ampere-turns. This can be accomplished by passing 1 ampere through 1225 turns wound on the ring, or 0.2 ampere through 6125 turns, etc.

Example 2. In a ring of the same dimensions, composed half of cast steel and half of wrought iron, it is desired to set up a total flux of 80,000 lines.

Flux density = 10,000 lines per sq cm		
Ampere-turns per centimeter for cast steel	= 6.3	
Ampere-turns per centimeter for wrought iron	= 12.2	
Total ampere-turns, cast steel	$= \pi \cdot 10 \cdot 6.3$	= 198
Total ampere-turns, wrought iron	$= \pi \cdot 10 \cdot 12.2$	= 383
Total ampere-turns		581

If the ring were cut with a hack saw, making an air gap 0.1 cm long, additional ampere-turns would be required to maintain the same flux density. From the expression

$$lH = 0.4\pi NI$$

when

$$\mu = 1, \quad B = H$$

we require, for $B = 10{,}000$

$$\frac{10{,}000}{0.4\pi} = 7958 \text{ ampere-turns per cm}$$

or 796 ampere-turns for an air gap of 0.1 cm.

Thus the total magnetic circuit would require $581 + 796 = 1377$ ampere-turns.

Example 3. A rectangular core is made of annealed silicon-steel sheets, and has the form and dimensions given in Fig. 2–40. The winding is on one leg of the core and contains 110 turns. How many ampere-turns are required in the winding to produce a flux of 700,000 lines or maxwells?

The average path of the magnetic flux is as indicated in the figure by the dotted line. At each of the four corners the mean flux path is assumed as being an arc of a circle, so that the mean length of path is $(4 \times 25) + 5\pi = 116$ cm. The flux density is equal to $7 \times 10^5 \div 50 =$ 14,000 lines per sq cm. By reference to Fig. 2–38, it is seen that such a flux density requires 10.8 ampere-turns per centimeter of length. The total ampere-turns required, therefore, are $116 \times 10.8 = 1252.8$; and as there are 110 turns in the coil, the required current must be $1252.8 \div 110 =$ 11.39 amperes.

40. Multiple Magnetic Circuits. Suppose now that the thickness of the core of Fig. 2–40 were doubled. This would double the area but

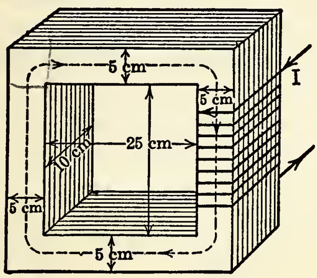


Fig. 2–40.

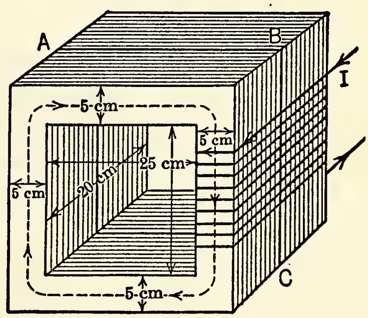


Fig. 2–41.

Fig. 2–40. A simple magnetic circuit with its winding on one leg.

Fig. 2–41. A magnetic circuit which has the same length but double the area of the circuit of Fig. 2–40.

would leave the mean length of the flux path the same. Such a core would appear as in Fig. 2–41. The mean length of the flux path being the same and the area of the path doubled, the same mmf would, from Eq. (35), give double the flux; the area having been doubled, the flux density remains the same. Or, to set up double the flux, 1,400,000 lines, with a density, therefore, of $14 \times 10^5 \div 10^2 = 14{,}000$ gausses, requires the same magnetizing force of 10.8 ampere-turns per cm.

Next consider that the rectangular core of Fig. 2–41 is split in half along the plane *ABC* and that the rear half is rotated about the axis *BC*, until the two halves are back to back. The two cores will then appear

as is shown in simpler fashion in Fig. 2–42, with the same winding of 110 turns about the middle leg. In the middle leg there will still be 14×10^5 lines, which total flux divides, one half going around into each outer leg.

If the problem is given directly that, in the middle core of Fig. 2–42 we desire 14×10^5 lines, since the total flux will divide, the flux density in all parts of the circuit is the same, so that the magnetizing force of 10.8 ampere-turns per cm is still to be used. The length of the average path of the flux being still the same, 116 cm, the same number of ampere-turns is required.

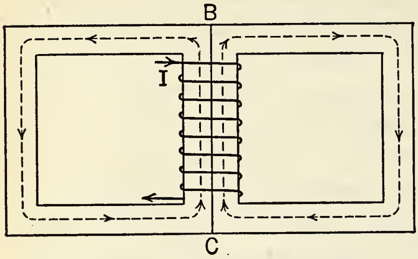


FIG. 2–42. The core of Fig. 2–41 has been split in half along the plane ABC and the rear half rotated about the axis BC. The flux density in each part of the figure is still the same as in Fig. 2–41.

In Fig. 2–43, the rectangular core of Fig. 2–41 is reproduced with the total winding of 110 turns divided into two halves, one half or 55 turns being placed on each outer leg. The conditions are still the same as in Fig. 2–41, provided the mmf's of the two windings act in the same direction around the magnetic circuit. The total mmf produced by the two windings is still $2 \times 626.4 = 1252.8$ ampere-turns, required to set up 14×10^5 lines.

Each winding in Fig. 2–43 therefore furnishes the mmf for one half the path. We may say that coil 1 furnishes the mmf for the path *zyx*, and

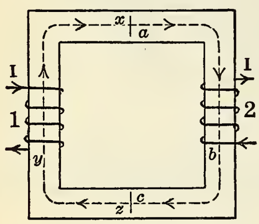


FIG. 2–43.

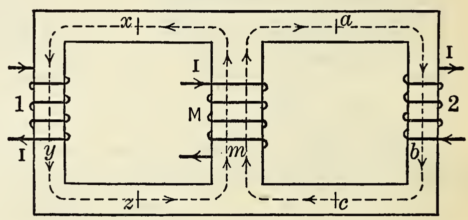


FIG. 2–44.

FIG. 2–43. The same core as in Fig. 2–41. The winding has been divided into two parts.

FIG. 2–44. The core of Fig. 2–42 with a winding on each leg.

coil 2 that for path *abc*. To furnish 10.8 ampere-turns per cm for a path 58 cm long requires $10.8 \times 58 = 626.4$ ampere-turns per coil.

In Fig. 2–44, the three-part core of Fig. 2–42 is again shown with a winding of 55 turns on each leg, each winding carrying a current of 11.39 amperes, as before. In the middle leg there is again to be set up a total flux of 14×10^5 lines, and this will divide so that in each outer leg there

will be 7×10^5 lines. Coil 1 may be considered as furnishing the mmf of $55 \times 11.39 = 626.4$ ampere-turns for the path *xyz* and coil 2 that for path *abc*. Similarly the middle coil provides the proper mmf of 626.4 ampere-turns to set up the flux in the middle part of the circuit between z and x and between c and a. The direction of the mmf's must, of course, be such that they act in the same direction around any one path. Thus the direction of the mmf of the middle coil, M, is up and that of coils 1 and 2 is down.

If the coil M of Fig. 2–44 were removed entirely and coils 1 and 2 each given 110 turns carrying 11.39 amperes, the magnetic conditions would be the same. The mmf per flux path would not be changed.

If a number of rectangular cores were merged as in Fig. 2–45, giving a cross-section of 100 sq cm per core, with the same flux densities as before, a coil of 55 turns carrying 11.39 amperes would be needed on each leg. The portion of the magnetic circuit which lies between vertical lines drawn from a to c, and from x to z, is identical in Figs. 2–44 and 2–45. Thus if a total flux of 14×10^5 lines is to be set up in each core, the flux density in each core will be $14 \times 10^5/100 = 14 \times 10^3$ gausses, requiring 10.8 ampere-turns per cm. The total flux divides in the upper and lower paths, so that the density in all parts is the same. Coil 2 will then be required to furnish the mmf for the flux paths *xmz* or *amc*, which are, as before, 58 cm long. There are needed $10.8 \times 58 = 626.4$ ampere-turns per coil, requiring $626.4/55 = 11.39$ amperes as before.

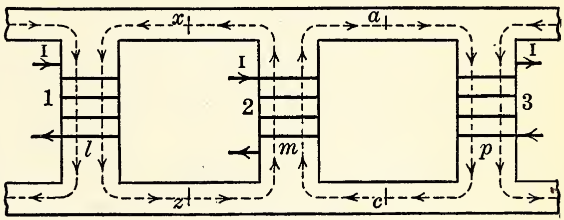


Fig. 2–45. An extended parallel magnetic circuit.

From another viewpoint we might also consider that coils 2 and 3 together furnish the mmf for the path *mapcm*, which is 116 cm long, and that coils 1 and 2 together furnish the mmf for the path *mxlzm*. Thus regarded, there is required *per pair of coils* $10.8 \times 116 = 1252.8$ ampere-turns, or again 626.4 ampere-turns per coil.

Example 4. In the magnetic circuit shown in Fig. 2–46, it is required that a total flux of 1,200,000 or 12×10^5 lines is to be set up in the poles. As noted in the figure, the poles are of wrought iron, the top of silicon-steel sheets, the base of cast iron. There is an air gap, 0.95 cm long, between each pole and the base.

The flux density in the poles and in the air gaps is $\dfrac{12 \times 10^5}{10 \times 12} = 10{,}000$ gausses; the total flux at points a and c of the top and at points b and d of the base is 6×10^5 maxwells. The flux density at a and c of the top is

$\frac{6 \times 10^5}{4 \times 10} = 15{,}000$ gausses. At the joint between the poles and the top, z, the flux enters the top at a density of 10,000 gausses; and as the flux bends around, the density rises to 15,000 gausses. These short distances where the flux density in the top and base changes might be subject to a refinement in the calculation, but it would be rather a complicated one, and, as we shall see later, unnecessary in this case, where there is an appreciable air gap. We shall, therefore, consider that the flux density over the entire length of the path in the top is 15,000 gausses. Similarly

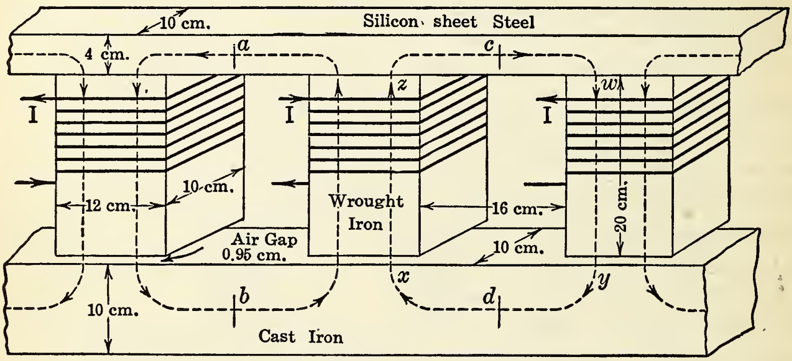


FIG. 2–46. A parallel magnetic circuit involving an air gap and parts of different materials.

we shall consider the flux density over the entire path in the base to be $\frac{6 \times 10^5}{10 \times 10} = 6000$ gausses.

Consider that the middle coil is to provide the mmf for that portion of the circuit between lines *ab* and *cd*. To set up a flux density of 10,000 gausses in the wrought-iron pole requires, from Fig. 2–38, a magnetizing force of 12.3 ampere-turns per cm. For a length of 20 cm there are needed $12.3 \times 20 = 246$ ampere-turns.

The magnetizing force needed in the silicon-steel sheets of the top, to set up 15,000 gausses, is 22.5 ampere-turns per cm. The distance from z to c being 12.5 cm, a total of $22.5 \times 12.5 = 281$ ampere-turns is needed.

For the cast-iron base, with an average flux density of 6000 gausses, a magnetizing force of 32 ampere-turns per cm and a total mmf of $32 \times 14.5 = 464$ ampere-turns is needed.

For an air-gap density of 10,000 gausses the magnetizing force is $10{,}000/0.4\pi = 7958$ ampere-turns per cm, and for a gap of 0.95 cm there are needed $7958 \times 0.95 = 7560$ ampere-turns. The above figures are grouped in Table A.

TABLE A

Part	Material	B in Gausses	H in Amp-turns per Cm	Length in Cm	Mmf in Amp-turns
Pole	Wrought iron	10,000	12.3	20	246
Top	Silicon-steel sheets	15,000	22.5	12.5	281
Base	Cast iron	6,000	32	14.5	464
Gap	Air	10,000	7958	0.95	7560
Total ampere-turns per pole.					8551

It will be seen that the reluctance of the comparatively short air gap is such as to require over 88 per cent of the total ampere-turns. For this reason the refinement in determining the flux density, where the flux enters and leaves the top and base, is unnecessary. A variation of 0.001 cm in the length of the air gaps will change the ampere-turns by $7958 \times 0.001 = 8$ ampere-turns.

In Table B the calculation is repeated to determine directly the ampere-turns required per pair of coils to set up the mmf for a complete flux path. From this viewpoint, there are two poles and two air gaps, and the full distances from z to w and from x to y must be considered.

TABLE B

Part	Material	B in Gausses	H in Amp-turns per Cm	Length in Cm	Mmf in Amp-turns
Poles	Wrought iron	10,000	12.3	2×20	492
Top	Silicon-steel sheets	15,000	22.5	25	562
Base	Cast iron	6,000	32	29	928
Gaps	Air	10,000	7958	2×0.95	15,120
Ampere-turns per pair of poles					17,102
Ampere-turns per pole					8551

41. Energy Stored in a Magnetic Field in a Medium of Constant Permeability. By Eq. (17), when the flux threading a coil of N turns, carrying I abamperes, was increased from zero to Φ, the work done was

$$W = \Phi N \bar{I} \text{ ergs} \tag{41}$$

This flux, it is to be remembered, was *not* the flux set up in the coil by the current $\bar{I}$, in the coil itself, but a flux maintained by some other coil or magnet. Thus when a coil carrying current is introduced into a field set up by some other agent there is finally, threading the coil, not only this introduced flux, Φ, but an additional flux set up by the coil itself, due to current $\bar{I}$.

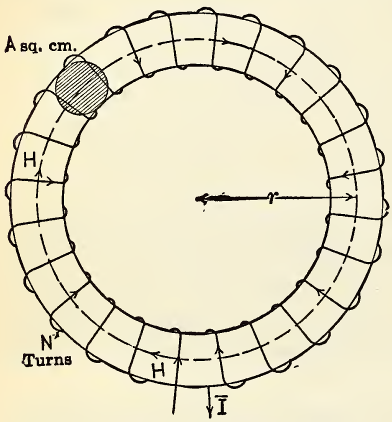


FIG. 2–47. Toroidal coil carrying current.

Consider again the toroid of Fig. 2–35 reproduced as Fig. 2–47. We have seen that when a steady current, $\bar{I}$ abamperes, is flowing, the value of the flux set up in the coil by the current is, from Eq. (37),

$$\Phi = \frac{4\pi N \bar{I} \mu A}{l} \tag{42}$$

This flux being produced by the current, it is obvious that, if the current is allowed to grow at a uniform rate from zero to a value $\bar{I}$, in time t seconds, the flux will also increase at the same rate from zero to its final value Φ. For the t seconds considered, the average value of the current is $\bar{I}/2$, and similarly the average value of the flux $\Phi/2$. We may apply Eq. (41) by considering that we have increased the flux threading the coil from zero to a value Φ while the coil was carrying an average current $\bar{I}/2$, and accordingly the work done is

$$W = \frac{\Phi N \bar{I}}{2} \text{ ergs} \tag{43}$$

and this must represent the energy stored in the field at the end of the t seconds.

It will be noticed that this is only one-half as much energy as was represented in Eq. (41). This is due to the fact that in obtaining this latter equation the field was supposed uniform, and independent of current strength, being set up by some other magnet, and not by the current in the coil.

If the current does not grow at a uniform rate, the validity of the argument by which Eq. (43) was obtained may not be apparent. However,

by the method of the calculus the proof may be made general. Using Eq. (21) we have, since power is the time rate of doing work,

$$\frac{dw}{dt} = \bar{\imath}\bar{e} = \bar{\imath}N\frac{d\Phi}{dt} \tag{44}$$

Equation (42) may be changed to the form, since μ is constant,

$$\frac{d\Phi}{dt} = \frac{4\pi N\mu A}{l} \times \frac{d\bar{\imath}}{dt} \tag{45}$$

Substituting this value of $d\Phi/dt$ in Eq. (44), we find

$$\frac{dw}{dt} = \frac{4\pi N\mu A}{l} N\bar{\imath}\frac{d\bar{\imath}}{dt} \tag{46}$$

From this differential equation the calculus yields the relation

$$W = \frac{4\pi N\mu A}{l} N \frac{\bar{I}^2}{2} \text{ ergs} \tag{47}$$

where $\bar{I}$ is the final value of the varying current $\bar{\imath}$, in abamperes.

We can now use Eq. (42) and thus change Eq. (47) into

$$W = \frac{\Phi N\bar{I}}{2} \text{ ergs}$$

which is the same as Eq. (43). This equation was derived without any limitation on the rate of increase in current $\bar{\imath}$, as it varies from its initial value of zero to its final value $\bar{I}$.

If we now substitute in Eq. (43), for Φ, its value as given by Eq. (42), we have

$$W = \frac{4\pi N^2\bar{I}^2\mu A}{2l} \text{ ergs} \tag{48}$$

Multiplying numerator and denominator by $4\pi l$ and substituting for $4\pi N\bar{I}/l$ its equivalent, H, we have

$$W = \frac{(4\pi)^2N^2\bar{I}^2\mu Al}{l^2 4\pi\cdot 2} = \frac{H^2\mu^2 Al}{8\pi\mu} \text{ ergs} \tag{49}$$

Since $B = \mu H$,

$$W = \frac{B^2Al}{8\pi\mu} \text{ ergs} \tag{50}$$

where B is in lines per square centimeter and A and l in square centimeters and centimeters respectively.

The product Al represents the volume of the magnetic field, and the

energy stored in the field per cubic centimeter in a medium of permeability μ is therefore

$$W = \frac{B^2}{8\pi\mu} \text{ ergs} \tag{51}$$

and for air,

$$W = \frac{B^2}{8\pi} \text{ ergs} \tag{52}$$

The fact that, with the same flux density, the amount of energy stored per unit volume in a medium of permeability μ is less than that stored in air per unit volume is accounted for by the fact that fewer ampere-turns are required for the former than for the same flux density in air.

42. Self-inductance. In Eq. (48) for the energy in a magnetic field

$$W = \frac{4\pi N^2 \bar{I}^2 \mu A}{2l} = \frac{4\pi N^2 \mu A}{l}\left(\frac{\bar{I}^2}{2}\right) \tag{53}$$

the part $4\pi N^2 \mu A/l$ is a constant; it is called the coefficient of self-induction, or merely the self-inductance of the circuit, being generally represented by the symbol L. It obviously depends upon the physical dimensions of the circuit and the value of μ. In the absolute system the unit of self-inductance is the *absolute henry* or *abhenry.*

Thus, the energy stored in a magnetic circuit may be represented by

$$W = \bar{L}\frac{\bar{I}^2}{2} \text{ ergs} \tag{54}$$

where $\bar{L}$ is in abhenrys and $\bar{I}$ in abamperes.

If it is desired to express the energy stored with the current stated in amperes, it is necessary to divide Eq. (53) by 10^2, since one abampere is equal to 10 amperes, so that

$$W = \frac{4\pi N^2 \mu A}{l \times 10^2}\left(\frac{I^2}{2}\right) \text{ ergs} \tag{55}$$

Since one joule equals 10^7 ergs, the last expression may be converted into joules by dividing it by 10^7, so that

$$W = \frac{4\pi N^2 \mu A}{l \times 10^9}\left(\frac{I^2}{2}\right) \text{ joules} \tag{56}$$

The first part of the last equation is again the coefficient of self-induction,

$$L = \frac{4\pi N^2 \mu A}{10^9 l} = \frac{0.4\pi N^2 \mu A}{10^8 l} \tag{57}$$

and is now measured in the practical unit called the *henry.* Thus one henry is equal to 10^9 abhenrys.

The energy stored in a magnetic field may thus be expressed in practical units by

$$W = L\frac{I^2}{2} \text{ joules} \tag{58}$$

where I is in amperes and L in henrys.

Self-induction will be discussed further in Chapter IV.

It is interesting to note at this time that, if we have a circuit consisting of two coils (or conductors) in series, and these coils are carrying current, they will act, if allowed to move with respect to each other, so as to increase the interlinkages of the system. If the currents have such direction as to produce augmenting fields, the coils will move together; if the currents produce opposing fields, the coils will separate. This is in accord with the ideas presented in section 17.

From Eq. (43) an increase in system interlinkages represents an increase in the stored magnetic energy of the system, and this increase in stored energy must come from the battery which holds the current constant as the coils move. Of the total energy drawn from the battery, one-half will be dissipated in heat as an I^2R loss (as will be proved in Chapter IV) and the other half stored, as shown by Eqs. (54) and (58).

As the two coils move, they will overcome whatever mechanical forces are tending to prevent relative motion, and the amount of mechanical work done in moving the coils against these resisting forces will be just equal to that by which the stored magnetic energy of the system has been increased.

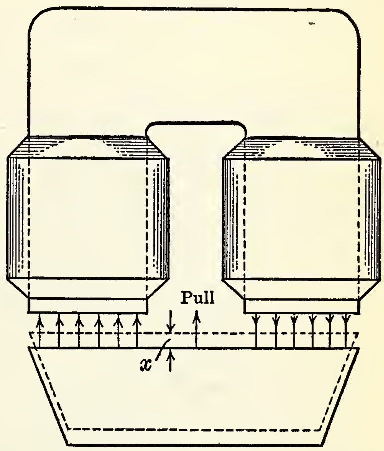


Fig. 2–48. The pull of an electromagnet on its armature can be calculated from the change in the energy stored in the air gap after the armature has been allowed to move through a distance so small that the flux density has not changed.

43. Pull of Magnets. Consider an electromagnet as shown in Fig. 2–48 the armature or keeper of which is separated from the magnet a distance D centimeters. Let A be the area of separation at right angles to the lines of force (in this case the area of the two pole faces), and let B represent the flux density in the air gap.

The total energy stored in the air gap will be, from Eq. (50),

$$W = \frac{B^2}{8\pi} DA$$

If the armature is allowed to move a very small distance, x, nearer the magnet, the change in the amount of energy stored in the air gap will be

$$w = \frac{B^2}{8\pi} xA$$

the distance x having been taken so small as not to change appreciably the value of B.

The armature must have done an amount of work equal to the change in the amount of energy stored in the air gap, and if P represents the pull of the armature,

$$w = Px = \frac{B^2}{8\pi} xA$$

from which

$$P = \frac{B^2}{8\pi} A$$

or, per unit area,

$$P = \frac{B^2}{8\pi} \text{ dynes per square centimeter} \tag{59}$$

At times the formula for the pull of a magnet is difficult to apply. With a given magnetomotive force and the keeper or armature in perfect contact with the poles of the magnet, the flux set up depends upon the reluctance of the magnet and the armature. Under these conditions the force tending to keep the poles and armature in contact is a maximum, and this force is often called the holding force or power of the magnet.

If the armature is separated from the magnet by a very short air gap, the reluctance of the whole magnetic circuit is increased by the reluctance of the air gap, so that the flux density throughout the entire circuit is reduced. Hence the pull is lessened. Further, with an air gap, fringing of the flux occurs, that is, at the edges of the gap, the lines, instead of passing straight across the gap, tend to bow out as is suggested in the lower part of Fig. 2–16. The effect of fringing is to increase the area of the gap and decrease the flux density. As the gap reaches lengths commensurate with the distance between the poles of the magnet, more and more of the flux passes directly from one pole to the other, without passing through the armature. This flux is called leakage flux and evidently serves no useful purpose; as leakage flux increases, the flux density in the gap decreases.

Therefore the distance x must be chosen reasonably small, so that the leakage flux is kept low and there is maintained a high flux density in the air gap.

The nearer the armature moves to the poles, the greater the flux density and the greater the force of attraction between the poles and the armature. If once the armature starts to move, the increasing force of attraction

accelerates the armature, so that it strikes the poles with a distinct blow.

A simple application of the pull of magnets is shown in Fig. 2–49, where a pivoted keeper is drawn upward and made to strike a blow against a latch or trigger *L*. The shorter the air gap, as controlled by the set screw, the smaller the current through the coil which will pull the keeper up against gravity. The latch, *L*, of non-magnetic material, may also act as a stop to keep the movable armature, *A*, from touching the pole, *N*, where it would otherwise stick by reason of residual flux, the flux which will remain after the exciting current has become zero.

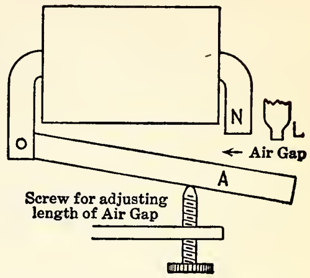


Fig. 2–49. The pull of an electromagnet, depending upon the current flowing through its coil, is utilized in all kinds of automatic switches.

In Fig. 2–50 is shown a commercial electromagnet used for lifting heavy iron masses. These magnets are particularly suitable for lifting scrap iron,

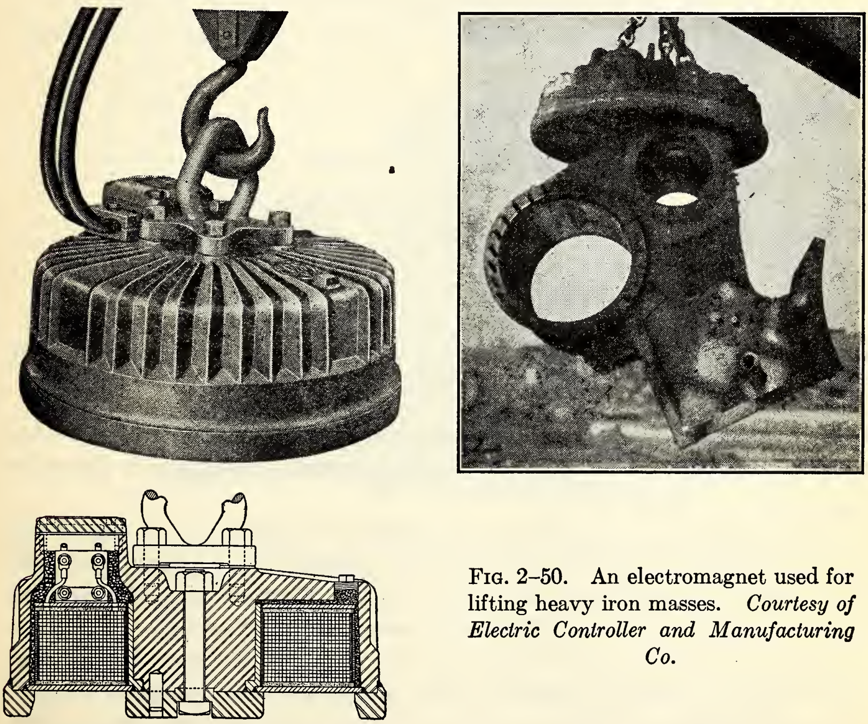


Fig. 2–50. An electromagnet used for lifting heavy iron masses. *Courtesy of Electric Controller and Manufacturing Co.*

turnings, and iron parts of irregular shape, which are difficult to handle by any other means.

44. Action of Solenoids with Iron Cores. If a soft iron core is introduced into a solenoid, as in Fig. 2–51*a*, because of the high permeability of the soft iron, the field set up within the coil will be much greater than would be the case without the core, and hence the total interlinkages of the system will be greatly increased. If the core were moved partly out of the solenoid, as in Fig. 2–51*b*, the average flux density within the solenoid, and therefore the total interlinkages of the entire system, would be less. From previous reasoning it follows that in Fig. 2–51*b* there must be a force acting between the iron core and the solenoid, tending to pull the core into the solenoid. The core then moves, if free to do so, until the

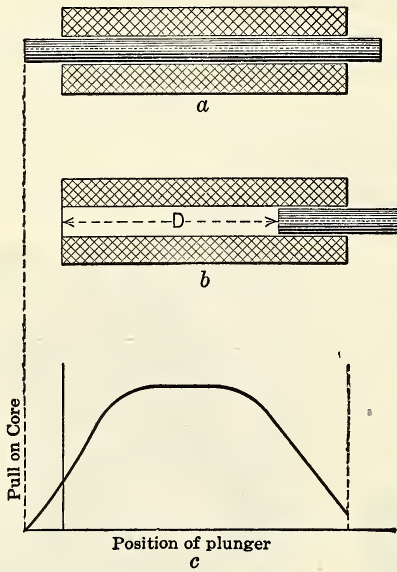


FIG. 2–51. For many purposes, a solenoid with a soft iron plunger is utilized to exert a pull.

total number of interlinkages of the system is a maximum, as in Fig. 2–51*a*. The variation of pull on the core will follow a curve, about as is shown in Fig. 2–51*c*.

By completely surrounding the solenoid with iron, as in Fig. 2–52, a so-called "iron-clad" solenoid is obtained, resulting in a greatly increased flux density within the core. Frequently the space ahead of the core up to the dotted line is also filled with iron, producing a stop. It will be realized that, with an appreciable air gap ahead of the core, the air gap will constitute the major portion of the reluctance of the magnetic circuit. As the core moves in and the air gap is shortened, the force on the core will increase gradually until the air gap becomes quite short, when the flux density, and therefore the force, will begin to increase very rapidly

toward the end of the stroke. The force on the core thus increasing steadily as the core moves in, the core accelerates and strikes its stop with a considerable blow.

Applications of solenoids with movable cores are very numerous, one

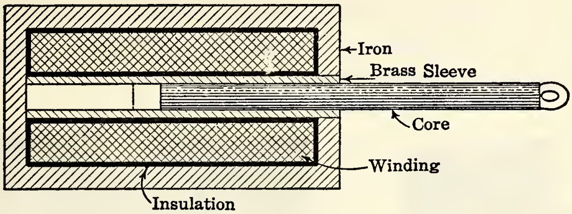


Fig. 2–52. The efficacy of the plunger type of magnet is much increased by making the solenoid "iron-clad," thus giving a nearly closed iron path for the magnetic flux.

of the most important being for the closing of electrical switches or contacts, the coils being energized manually or automatically. Solenoids are also used in the braking of cranes and elevators. On such lifting devices, brakes are applied by powerful springs while the hook or car is not moving.

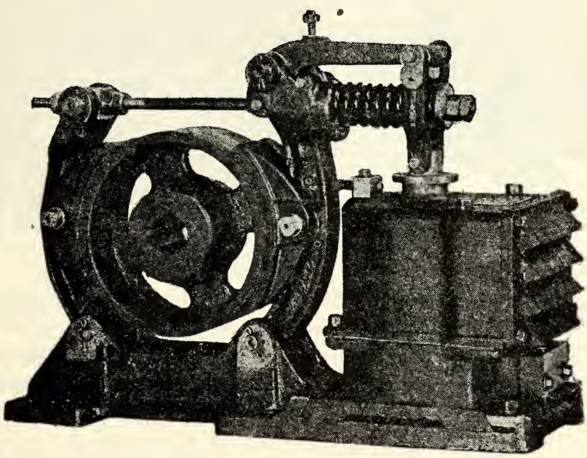


Fig. 2–53. Electromagnetically operated brakes are much used on hoisting devices. In the picture above springs hold the brake shoes against the pulley until the electromagnet is energized and pulls them away. *Courtesy of Westinghouse Electric and Manufacturing Co.*

As soon as power is applied to lift or lower, a solenoid pulls a plunger acting to remove spring pressure; at the moment power is removed, the brakes are again applied. Such a mechanism, as applied to an electromagnetic brake, is shown in Fig. 2–53.

45. Relays and Contactors. A relay may be defined as a device which is operative by a variation in the conditions of one system or circuit, to effect the operation of other devices in the same or other systems. A relay (Fig. 2–54) has an operative mechanism which will open or close one or more sets of electrical contacts, which in turn will control other relays or circuits. The operative mechanism may be a solenoid actuated by current, or any mechanical device actuated by temperature change, pressure, etc. In general the currents handled by relays are small.

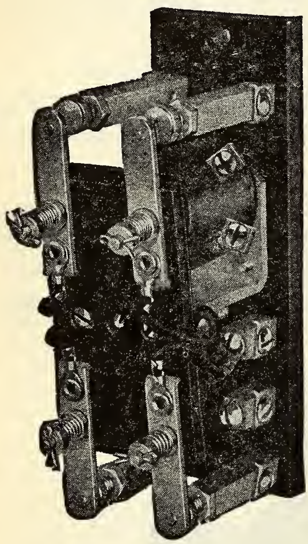


FIG. 2–54.

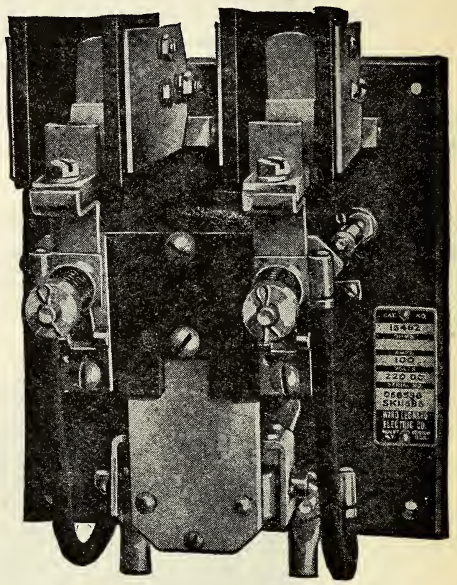


FIG. 2–55.

FIG. 2–54. A relay used to open and close contacts, which in turn control other circuits. This relay closes two circuits and at the same time opens two other circuits.

FIG. 2–55. This picture shows a contactor, used to control power circuits. This contactor may be energized originally by the closing of a relay or by the operation of a push button. *Courtesy of Ward Leonard Electric Co.*

A contactor (Fig. 2–55) is a device for repeatedly closing and opening an electric power circuit and consists of an operating mechanism and a set of heavy contacts across which the power circuit is opened and closed. The actuating mechanism may be a solenoid energized by current from a control circuit, in which case the contactor is said to be electromagnetically operated. Contactors may also be operated by compressed air, in which case a solenoid, actuated by a control circuit, opens an air valve allowing the air to operate the contactor; such a contactor is said to be electro-pneumatically operated.

46. Circuit-breakers. A circuit-breaker is a switch used to open an electric circuit automatically when certain conditions arise. In its simplest, or "overload," form, it acts if the current in the circuit in which it is placed exceeds a certain predetermined value. A very simple form is shown in Fig. 2–56. The current to be opened is passed through a set of flexible copper strips resting on contacts, CC, and then around a solenoid. The breaker shown is closed against the pressure of a spring coiled behind the plunger S, and held shut by the latch L. When the current rises to a value sufficient to lift the iron core P, the latter, driving a piece of non-magnetic material, N, ahead of it, delivers a

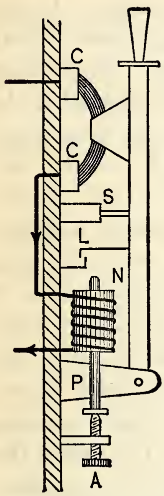


Fig. 2–56. A simple type of overload breaker. When more than a predetermined current flows through the circuit, the plunger, P, is lifted by the magnet; then by the trip and spring actions, the switch is opened.

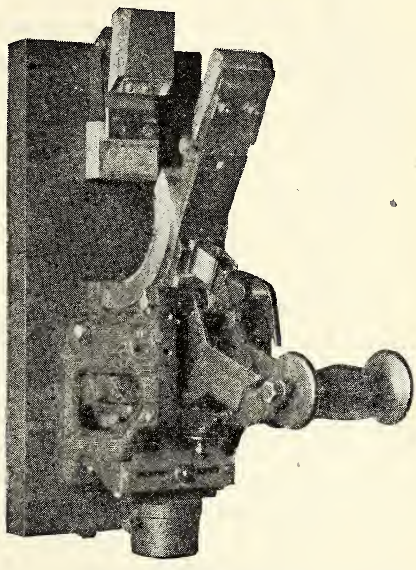


Fig. 2–57. A standard type of single-pole, overload, circuit breaker. This breaker is rated at 1200 amperes. *Courtesy of Westinghouse Electric and Manufacturing Co.*

blow to latch L, so that the breaker opens, rupturing the current at the contacts CC. The construction of commercial circuit-breakers is more complicated than that shown in Fig. 2–56, as may be seen in Fig. 2–57. In practically all modern types, the circuit is opened over several contacts, the main contacts, usually of leaf copper to insure good electrical contacts, opening first, followed by a smaller auxiliary copper contact, the circuit being finally ruptured across carbon blocks. The idea is to have any burning or arcing take place across carbon blocks, which are not much damaged by arcing and are readily replaceable. All breakers can be adjusted to open over a fairly wide range of current by suitable adjust-

ment, as in Fig. 2–56, where the initial position of the core can be adjusted by the screw A.

Circuit-breakers are built for many services, such as to open circuits whenever the load current exceeds a certain value, or flows in the reverse direction, or whenever the line voltage falls below a certain value.

47. Hysteresis. Consider again the toroid of Fig. 2–35, wound now on a cast steel core which has been thoroughly demagnetized. If the exciting current is gradually increased, and corresponding values of flux are measured by some convenient means, and the results plotted as in Fig. 2–58, the relation between magnetizing force and flux density will be shown by the curve, OA. If the exciting current is gradually reduced, it will be found that, when the current has reached zero, the value of the flux density is not zero, but some such value as OC, which is called the *residual magnetism.* In order to reduce the flux density to zero, a negative magnetizing force OD, called the coercive force, must be applied. Further increase in the exciting current, in the reverse direction, to the same maximum as before, gives the same maximum value of flux as before, but in the reversed direction. If the cycle is completed, the closed curve results.

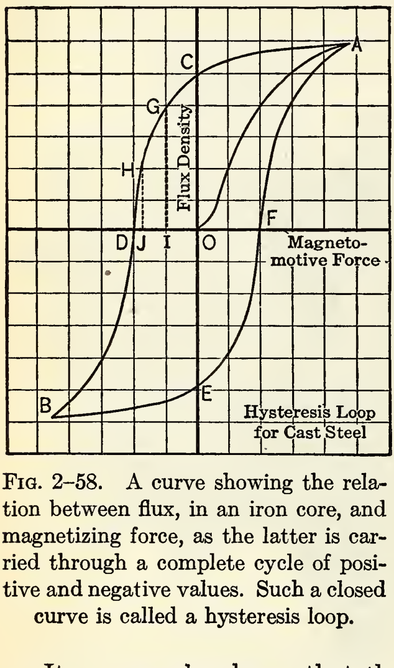


FIG. 2–58. A curve showing the relation between flux, in an iron core, and magnetizing force, as the latter is carried through a complete cycle of positive and negative values. Such a closed curve is called a hysteresis loop.

It can now be shown that the area of the loop represents an amount of energy dissipated as heat during the entire cycle.

Suppose a current $\bar{\imath}$ is flowing in a coil of N turns, having an iron magnetic circuit, and that the power used by the current in flowing through the resistance of the coil is neglected. The flux set up is Φ, and when the current increases a small amount this flux increases by the amount $d\Phi$. Then the work done in forcing the current $\bar{\imath}$ through the circuit, against the counter emf due to the increasing flux, is given by

$$dw = \bar{\imath}\bar{e}dt = \bar{\imath}N\frac{d\Phi}{dt}\,dt = \bar{\imath}Nd\Phi \tag{60}$$

Now $lH = 4\pi N\bar{\imath}$, so $\bar{\imath} = lH/4\pi N$; also $\Phi = BA$. So we may write

$$dw = \frac{lA}{4\pi}HdB$$

and the work done in increasing the magnetic field in one cubic centimeter of iron is given by

$$dw = \frac{1}{4\pi} HdB \tag{61}$$

But in Fig. 2–59 the area 1–2–3–4 is equal to the expression HdB; hence the shaded area of Fig. 2–60a is equal to $\int_0^{B_m} HdB$ and hence represents the work put into the iron (1 cc) while B was increasing from zero to B_m; the factor $1/4\pi$ of course must be considered in determining the actual quantity of work.

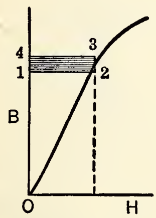


Fig. 2–59.

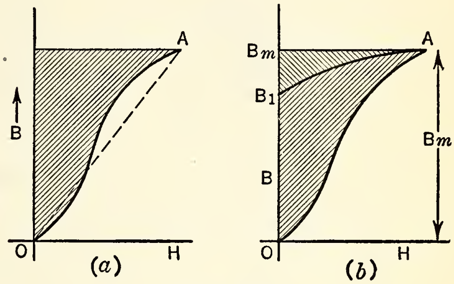


Fig. 2–60.

Fig. 2–59. The area 1–2–3–4, representing a small change in flux when the mmf is equal to H, may be considered as equal to an amount of work, HdB.

Fig. 2–60. In the magnetizing part of the cycle, work is put *into* the iron as in a; when the value of H is diminished to zero, the decaying flux in the iron causes energy to flow *out of* the iron into the circuit, as indicated by the area B_1–A–B_m of diagram b.

Now if the magnetizing current (hence H) is decreased to zero, the relation between H and B is shown by the curve A–B_1, of Fig. 2–60b. During this change in flux the value of the expression $\int HdB$ is shown by the shaded area B_1–A–B_m. However, as the value of H is positive during this part of the cycle and dB is negative, the area B_1–A–B_m must be considered as negative, so that the total work put into the circuit (excluding the I^2R loss which has been neglected) is given by the area O–A–B_1.

Now after the iron has been put through a few magnetic cycles the B–H relation follows the hysteresis curve of Fig. 58. In Fig. 2–61 the complete hysteresis cycle has been divided up into sections. In diagram a the mmf is increased from OM to a value which gives a flux density N; during this part of the cycle the expression $\int HdB$ is equal to the shaded area, and as both H and dB are positive this work is put into the circuit. In diagram b the shaded area represents work which the magnetic circuit

restores to the electric circuit, H being positive and dB negative. The shaded area of diagram c represents work put into the magnetic circuit; that of d represents negative work and that of e positive work.

The net work put into the magnetic circuit is then shown in diagram f, which is the hysteresis loop of the iron. If B is given in gausses and H in oersteds or gilberts per cm, with B and H plotted to the same scale, then the area of the loop divided by 4π gives the energy, in ergs, required to carry one cubic centimeter of the iron once through the magnetic cycle. Quite evidently material with a narrow loop is one with small hysteresis loss.

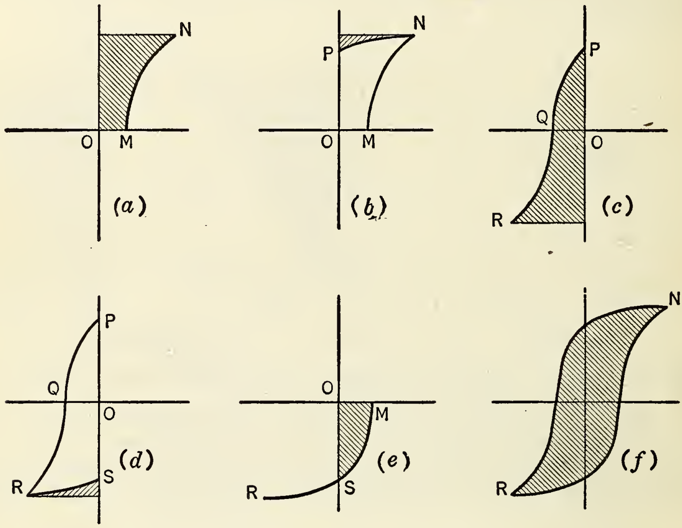


FIG. 2–61. During the complete magnetizing cycle, work is put into the iron during parts a, c, and e, and the iron gives out energy during parts b and d. The net work is shown in f and is equal to the area of the hysteresis loop.

It is interesting to note that if the permeability had been constant in Fig. 2–60a, the magnetization curve would have been the straight dotted line OA. The energy stored in the field per cubic centimeter would then have been equal to $1/4\pi$ times the area of the triangle, instead of the shaded area, i.e.,

$$W = \frac{1}{4\pi} \cdot \frac{BH}{2} = \frac{BH}{8\pi}$$

Substituting for H its equivalent B/μ (B in air), we have

$$W = \frac{B^2}{8\pi\mu} \quad \text{or, for air,} \quad W = \frac{B^2}{8\pi}$$

which checks Eqs. (51) and (52).

In Fig. 2–62 are given a number of hysteresis loops for the same material, but for different values of maximum flux densities. The maximum values of B all lie along the dotted magnetization curve, and the loops indicate that their area increases faster than the maximum value of B. It was found experimentally, by Steinmetz, that the hysteresis loss per cubic centimeter per cycle, for any material, varies approximately as the 1.6th

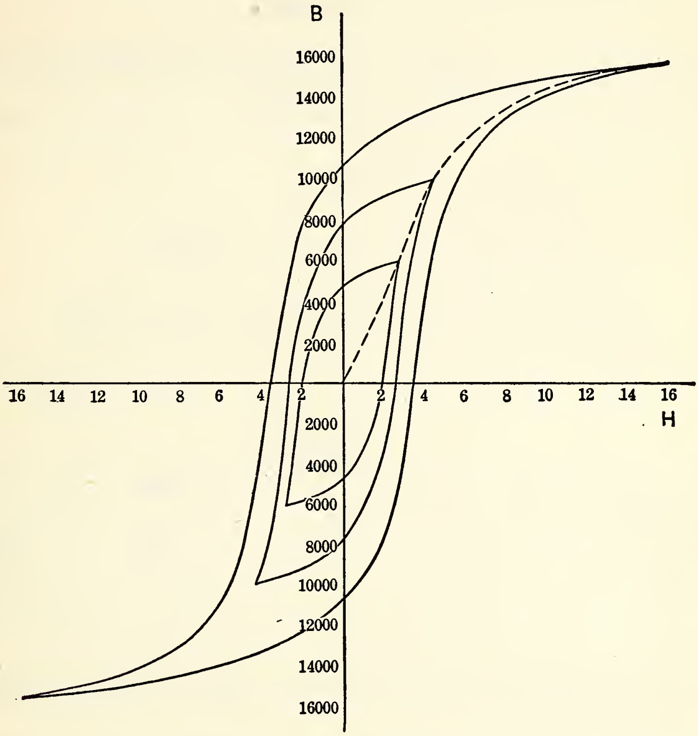


Fig. 2–62. The hysteresis loops for cast steel as the sample was carried through cycles of different intensities of magnetizing force. The maximum values of flux density lie on the normal magnetizing curve; the areas of the different loops vary approximately as the 1.6 power of the maximum value of B.

power of the maximum flux density. This exponent 1.6 is a fair average value for densities between 3000 and 10,000 gausses, but increases for higher flux densities. With low densities it does not seem to apply.

The hysteresis loss for any material may be expressed, according to Steinmetz, by

$$P_h = \eta f V B^{1.6} 10^{-7} \text{ watts} \tag{62}$$

where η = the hysteresis constant, or the loss per unit volume, per cycle, for $B = 1$ gauss;

f = cycles per second;

V = volume;

B = maximum flux density in gausses.

In practice, the hysteresis loss for materials is more often expressed by

$$P_h = w_1 W_t f B^{1.6} \text{ watts} \tag{63}$$

where w_1 = the loss per unit weight;
W_t = weight of iron;
f = cycles per second;
B = maximum flux density in gausses.

The value of w_1 is often given as the watts lost per pound, or kilogram, at 60 cycles for a flux density of 10,000 gausses.

Representative limits for hysteresis constants are given in Table C.

TABLE C

AVERAGE VALUES OF HYSTERESIS CONSTANT FOR SHEET STEEL USED IN D-C MACHINERY

η = ergs per cycle, per cubic centimeter, for B = 1 gauss
w_1' = watts per pound at 60 cycles, for B = 10,000 gausses

	Hot-rolled annealed soft steel sheets	Annealed silicon-steel sheets	
		From	To
Value of η	0.0016	0.0008	0.00105
Value of w_1'	1.42	0.73	0.96

In applying the lower row of constants of Table C in Eq. (63), the equation is taken as if written

$$P_h = w_1' W_t \left(\frac{f}{60}\right)\left(\frac{B}{10{,}000}\right)^{1.6} \text{ watts} \tag{64}$$

where f and B are the required frequency and density, respectively.

48. Permanent Magnets. In many pieces of electrical apparatus, the permanent magnet plays a very important part. Practically all direct-current instruments of the better class use movements of the D'Arsonval type, a coil movable in the field of a permanent magnet. All watthour meters, for both direct- and alternating-current power circuits, use permanent magnets in that part of their mechanism which serves as a speed control. All magnetos used for ignition purposes and as tachometers, use permanent magnets for their magnetic fields.

Of course, no magnet is really permanent, but magnets can be so made that their strength holds sensibly constant (within, say, less than 1 per

cent) for long periods of time. A special grade of steel is employed, and it is very carefully heat-treated to give permanence to the magnet. The magnet should not be subjected to vibration, and its armature (iron piece connecting or nearly connecting the poles) should always be left in position. Sometimes armatures are not supplied, but for such magnets the poles must be very close together if they are to retain much magnetic strength.

The general idea involved in the permanence of a magnet may be seen with the help of Fig. 2–58, which is the hysteresis loop of a piece of hard steel, having a completely closed magnetic circuit. After being properly shaped and tempered, the magnet is magnetized to point A; then, if the magnetizing force is taken off, the flux will decrease to point C on the loop, leaving a residual magnetization in the steel equal to OC. If there is an air gap in the magnet (as there always must be to make it of any use), this amount of flux will not stay in the magnet because of the demagnetizing effect of the air gap. The flux decreases, therefore, along the hysteresis loop, to some point G, fixed by the condition that with the flux IG through the air gap the mmf required across the gap is equal to OI. If the magnet is jarred, this equilibrium is destroyed, and more flux disappears.

Instead of leaving an amount of flux, IG, in the permanent magnet, it is customary to demagnetize it to some point H, at which the demagnetizing effect of the air gap, now less than before, because of the decreased flux density through it, is much less than OJ, which is the amount of demagnetizing action required before the magnet can be further weakened. This initial weakening of the magnet by the manufacturer is known as "aging" the magnet. In actual magnets used in meter construction, the normal flux density in the magnet is about 5000 gausses. This figure may safely be made higher if the ratio of magnet length to air-gap length is made greater; evidently the greater the air-gap length the greater is its demagnetizing action; also, the longer the piece of steel used in making the magnet the greater is its tendency to stay magnetized. The limit of the ratio of magnet length to gap length is generally fixed by economic reasons; magnet steel is expensive, and the commercial forms of magnets do not generally have a ratio greater than about fifty.

In modern magnets an alloy of cobalt and iron, in about equal proportions, is used. This cobalt-steel shows much greater coercive force than earlier types of magnet steel; even short bar magnets, with no keeper, hold a high flux density for years, with very little decrease in their strength.

PROBLEMS

2–1. Is the needle of the ordinary magnetic compass acted upon by a force tending to make it move as a whole, as well as by the turning force which normally makes it point north? Explain.

2–2. A north pole of a very long, slender magnet, of 40 units strength, is situated 7 cm from the north pole, and 12 cm from the south pole, of a slender bar magnet 15 cm long, the pole strength of the second magnet being 120 units. What force is exerted on the north pole, and what angle does it make with the axis of the bar magnet?

2–3. What is the flux density at a point 25 cm away from a supposedly isolated north pole of 65 units strength. With what force would it act on a test pole at this point, if the test pole is of 2 units strength?

2–4. What force in dynes acts on a test pole of 7 units strength when placed in a field of (*a*) 1500 gausses, (*b*) 15,000 lines per square inch?

2–5. What is the strength of a test pole, which when placed in a field of 9000 lines per square inch is acted upon by a force of 35.56 grams?

2–6. A test pole of 35 units strength is placed at the center of a loop of wire 15 cm in radius and carrying a current of 3 amperes. What force is acting on the test pole?

2–7. A slender magnet 12 cm long, suspended so as to swing in a horizontal plane, has a turning moment in this plane of 30 dyne-centimeters when at right angles to the earth's field at Washington, D. C. What is the strength of its poles?

2–8. The coil of a tangent galvanometer 1 foot in diameter has 10 turns and is carrying 5.2 amperes. A compass needle at the center of the coil is 3 cm long and has poles of 10 units strength. What is the turning moment when the needle is in the plane of the coil?

2–9. A conductor carrying a current of 30 amperes lies entirely in a uniform magnetic field of 12,000 gausses. What is the length of the conductor if the force tending to move it at right angles to both itself and the field is 900 grams?

2–10. A conductor, 10 inches long and carrying a current of 25 amperes, lies entirely in a uniform magnetic field. The force tending to move the conductor at right angles to both itself and the field is 100 grams. What is the flux density of the field?

2–11. A copper rod 12 inches long and weighing 0.2 pound is suspended at right angles in a uniform horizontal magnetic field of 10,000 gausses. How much current in amperes must be passed through the rod so that the force on the suspension due to the weight of the wire becomes zero?

2–12. A piece of No. 8 copper wire is suspended horizontally in the vertical air gap of a powerful magnet developing an air-gap density of 78,000 lines per square inch. The piece of wire is 10 inches long, and 60 per cent of its length is in the air gap. How much current must the wire be carrying if the force on the suspension (due to the weight of the wire) is to be reduced to zero? (See Table IV in the Appendix, for weight of wire.)

2–13. On a certain armature there are 60 conductors, each 8 inches long, carrying 15 amperes; average flux density in which the conductors are lying is 20,000 lines per square inch. What is the force, in pounds, tending to move the conductors in a direction perpendicular to the field?

2–14. On a certain armature there are 100 conductors each carrying 12 amperes. All the conductors lie in a field the average flux density of which is 60,000 lines per square inch. If the radius of the armature is 4 inches and the torque exerted by the armature is 20 pound-feet, how long is each conductor?

2–15. There are 80 conductors on the armature of a motor, each 9 inches long and each carrying 17.5 amperes. The density of the field in which all of them lie is 22,000 lines per square inch. What is the torque in pound-feet if the armature is 9 inches in diameter?

2–16. On a certain armature there are 120 conductors each carrying 14 amperes. All the conductors lie in a field of 30,000 lines per square inch average density. If the radius of the armature is 6 inches and the torque developed is 22 pound-feet, how long is each conductor?

2–17. In the movement of Fig. 2–23, if there are 200 turns each carrying 0.01 ampere and the coil is 2 cm wide and 3 cm high, what is the torque in dyne-centimeters if the flux density in the air gap is 1100 gausses?

2–18. If the flux of 1.8×10^6 lines linking a coil disappears in 0.12 second and generates an average emf of 180 volts, how many turns are there in the coil?

2–19. A coil of 260 turns is linked with 1,250,000 lines. If the flux dies to zero in 0.15 second, what is the average voltage generated during the decay of the current?

2–20. A field coil of a large generator has 1250 turns. The flux in the pole on which the coil is placed is 2.5×10^6 lines. In what time must the flux die to zero, if the average voltage generated in the coil, while the flux is decreasing, is 1100 volts?

2–21. A conductor 8 inches long is moving in a direction perpendicular to a magnetic field of density 30,000 lines per square inch, at a rate of 300 feet per minute. What voltage does it generate in abvolts? In volts?

2–22. A conductor, 12 inches long, is moving in a direction perpendicular to its length and to a magnetic field of density 50,000 lines per square inch. If it is generating 7.2 volts, at what speed, in feet per second, is the conductor moving?

2–23. A conductor carrying a current of 400 amperes is 25 cm long and is being moved in a direction perpendicular to a magnetic field of 6000 gausses with a velocity of 30 cm per second. Calculate the amount of work done in ergs and in foot-pounds in 0.4 second, starting with the force acting on the conductor, and again with the voltage it is generating.

2–24. A conductor carrying a current of 300 amperes is 30 cm long and is being moved perpendicular to a magnetic field of 5000 gausses. If the work done is 11.25 joules per second, at what speed in centimeters per second is the conductor moving? Make the calculation starting with the voltage generated by the conductor and again with the force acting on it.

2–25. A conductor carrying a current of 325 amperes is 30 cm long and is being moved in a direction perpendicular to a magnetic field of 5000 gausses intensity, with a velocity of 1 foot per second. How much work is done, in ergs, and in foot-pounds, in 0.3 second? Make the calculation in both mechanical and electrical units.

2–26. How much voltage does the trailing antenna of an aeroplane, 300 feet long, generate if the plane is moving horizontally on an east-west line at a speed of 120 miles per hour, where the horizontal component of the earth's field, in a north-south direction, is 0.22 gauss? The antenna makes an angle of 45° to the horizontal.

2–27. At Washington, D. C., an aeroplane having metal wings, each of 35 feet spread, is diving in a north-south plane at such an angle to the vertical that it is moving perpendicular to the earth's magnetic field. If it is moving with a velocity of 150 miles per hour, how much voltage is generated between its wing tips?

2–28. A coil having 600 turns is threaded by a flux which is changing at the rate of 100,000 lines per second. The coil has 5.1 ohms resistance and is connected to a storage battery of 2 volts emf and 0.03 ohm resistance. What current flows in the circuit? (Two answers required.)

2–29. A coil of 1500 turns and 4 ohms resistance is connected across a storage battery of 9 volts emf and 0.03 ohm internal resistance. The coil is threaded by a flux which

is changing at a rate of 15×10^4 lines per second. What value of current flows while the flux is changing? (Two answers required.)

2-30. A coil of 1200 turns and 5 ohms resistance is connected across a battery of 0.04 ohm internal resistance. The coil is threaded by a flux which varies at the rate of 18×10^4 lines per second and a current of 1.952 amperes flows. What was the emf of the battery? (Two answers required.)

2-31. A square coil of 50 turns is 25 inches long per side and lies in a field of density 1500 lines per square inch. The coil is attached to a pivot which lies in the plane of the coil and is perpendicular at the center points of two opposite sides. The coil rotates about the pivot at uniform angular speed from a position at right angles to the field, to a position parallel to the field, in 0.125 second. What is the average change of flux interlinkages and what average voltage is induced by the coil during its motion? What value of voltage is generated at the instant the coil is moving parallel to the field? When it is moving at right angles to the field? According to what trigonometric function will the voltage vary as the coil moves?

2-32. What would be the answers to the last problem if the pivot was along one side of the coil?

2-33. If the coil of problem 2-31 turned at a uniform rate of 240 rpm in a field of 3000 lines per square inch, what would be the answers to that problem?

2-34. At what rate is work being done in a circuit in which 37.3 amperes are flowing under a difference of potential of 32 volts? How much work is done in 3.2 seconds?

2-35. An armature with a radius of 12 cm supports 120 conductors each 10 cm long and carrying a current of 20 amperes. The flux density of the field in which the conductors lie is 10,000 gausses. The armature is rotating with a peripheral speed of 1000 cm per second. (*a*) What is the total force exerted by the armature in dynes? (*b*) What is the power generated in ergs per second? In watts? (*c*) What voltage is being generated if all the conductors are in series? (*d*) What is the product of volts and amperes? (*e*) Should the answers to (*b*) and (*d*) be the same? Explain.

2-36. How many foot-pounds of work must be done to pull a coil having a concentrated winding of 220 turns out of a magnetic field, if there are 10^5 lines linking the coil, and the current through the coil is maintained constant at 100 amperes?

2-37. Two conductors on an armature are parallel and ½ inch apart. What is the force, in pounds per inch length, exerted by one of the conductors on the other when the current through them is 250 amperes?

2-38. Two No. 10 copper wires are 2 feet long, parallel to each other, and 2 inches apart. What is the force of attraction between them if they are each carrying 75 amperes of current in the same direction?

2-39. If one wire of problem 2-38 is 4 inches below the other and perfectly free to move vertically, how much current must the conductors be carrying (same in each) if the lower wire is to be lifted by the force between them? (See Table IV in Appendix for weight of wires.)

2-40. If the wires of problem 2-38 are lying parallel to each other, 2 inches apart, on a wooden table, and the coefficient of friction of wire on table is 0.3 and the current in one wire is 500 amperes, how much current must the other wire carry if this is to be dragged on the table by the tractive force between them? (Neglect force for bending wires, etc.)

2-41. A toroid, of average diameter 6 inches, has 400 turns which are carrying 5.6 amperes. How much work would be done in carrying once around the magnetic circuit a pole of 150 units strength? (This procedure is, of course, actually impossible.)

2-42. A cast-steel ring has a mean diameter of 45 cm and a cross-section of 30 sq cm. The flux is 450 kilolines. How many ampere-turns are necessary?

2–43. If there are 1300 turns in the winding of problem 2–42, and the flux increases uniformly from zero to its final value in 0.13 second, what is the average counter voltage against which the magnetizing current has to flow?

2–44. A wooden ring has a circular cross-section of 5 sq cm and a mean radius of 25 cm. A winding of 500 turns is placed upon the ring and carries a current of 20 amperes. What is the flux density within the ring?

2–45. A ring of mean diameter 30 cm and of 40 sq cm cross-section is composed half of cast steel and half of wrought iron (the halves in series), with a hack-saw cut 0.18 cm long. How many turns carrying 1.5 amperes would be needed to set up a flux density of 10,000 gausses in the air gap? (Neglect fringing.)

2–46. A wrought-iron ring, wound with 250 turns, has a circular cross-section of 8 sq cm and a mean radius of 30 cm. The ring has been cut by a saw and has an air gap of 0.1 cm length. Find the ampere-turns required to produce flux densities of 4, 6, 8, 10, and 12 kilolines per sq cm and plot a curve between total flux (ordinate) and amperes excitation.

2–47. What would be the answers to the last problem if the air gap of the ring had been 0.5 cm? Plot the curve as before and compare with that previously obtained.

2–48. A horseshoe-shaped magnet for lifting iron plates is forged of wrought iron and has a magnetic path 40 cm long. The oxide on the iron plates is 0.1 mm thick (note the two gaps) and the path through the plate is 15 cm long. Density in the air gap is 13,000 lines per sq cm, and the cross-section of the plate is twice that of the magnet. How many turns are necessary on the magnet, if the current in the winding is to be 2 amperes?

2–49. A ring core is made of annealed silicon sheet-steel; its cross-section is 7.57 inches by 5.22 inches, and the average length is 22.3 inches. If there are 800 turns in the coil, and a flux density of 13,500 lines per sq cm is required, how much current must flow through the coil?

2-50. In order to raise a certain cast-steel weight the density in a horseshoe-shaped magnet, of cast steel, must be 12,000 gausses. The length of the circuit in the magnet and weight is 50 cm. If there are 1625 turns carrying 2 amperes, how near the weight must the magnet be brought to lift it? If the area of each of the poles is 28 sq cm, how much is the weight?

2–51. A transformer core, made of annealed silicon-steel sheets, has the form and dimensions given in Fig. 2–63. The winding is all on the center leg of the core, and contains 150 turns. How many amperes are required in the winding to produce a flux of 1.4×10^6 lines in the center leg?

2–52. The middle leg of the core of problem 2–51 is cut by a hack-saw to give an air gap of 0.12 cm. How many ampere turns would then be required to set up a flux of 1.4×10^6 lines in the center leg?

2–53. Each outer leg of the core of problem 2–51 is cut by a hack-saw to give an air gap of 0.12 cm. How many ampere-turns would be required to set up a flux of 1.4×10^6 lines in the center leg? Compare with previous problem.

2–54. If the core of Fig. 2–63 were built up of standard annealed sheet-steel, how many amperes would be needed to set up 1.8×10^6 lines in the center leg?

2–55. A closed composite ring as in Fig. 2–36 is built up of wrought iron, annealed silicon sheets and cast iron, and excited by a single winding. If the area of cross-section of each material is 10 sq cm and the mean radius of the ring is 12 cm, how many ampere-turns will be necessary to set up a flux of 60,000 lines in the cast-iron portion? What will be the flux in each of the other two materials?

2–56. In the ring of problem 2–55, what would be the area of cross-section of the wrought iron and silicon-sheet portions of the ring if the flux in each of these portions is also to be 60,000 lines?

2-57. How many ampere-turns per pole would be needed in Fig. 2-46 to set up a flux of 9×10^5 lines in the poles? If the winding per pole has 1800 turns, how much current is necessary?

2-58. How many ampere-turns per pole would be needed in Fig. 2-46 to set up a flux of 14×10^5 lines in the poles, if standard annealed sheets are substituted for the silicon-steel sheets, the other materials being the same?

2-59. The air gap under the pole face of a motor has an area of 18 square inches and is ¼ inch long. The flux density is 40,000 lines per square inch. How many ergs of energy are stored in the air gap? How many foot-pounds?

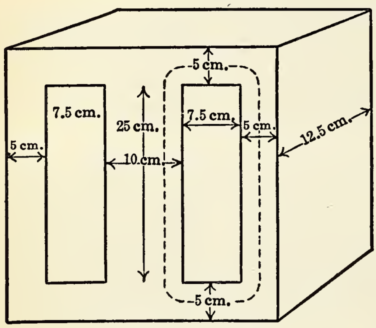


FIG. 2-63.

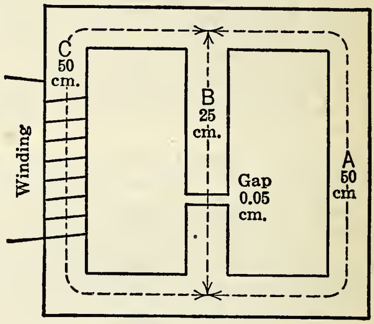


FIG. 2-64.

2-60. How much energy in joules is stored in the core of problem 2-51?

2-61. Calculate the energy in joules stored in the core of problems 2-52 and 2-53.

2-62. In the magnetic circuit shown in Fig. 2-64, the lengths of paths A and C are each 50 cm. The length of path B, including the air gap of 0.05 cm, is 25 cm. The area of cross-section throughout is 20 sq cm. The permeability of the iron (assumed the same throughout) is 2500. If the winding on leg C has 250 ampere-turns, what are the fluxes in A, B, and C?

2-63. A horseshoe-shaped permanent magnet has pole faces each of ½ square inch cross-section. The flux density in the magnet is 3000 gausses. How many pounds can it lift?

2-64. What is the hysteresis loss, in watts, in an iron core being magnetized from a 60-cycle a-c line, if the core weighs 150 pounds and has a maximum flux density of 11,000 gausses, the iron being high-grade annealed electric-steel, weighing 480 pounds per cubic foot? (Let $w_1' = 1.40$.)

2-65. (*a*) If the core of the previous problem were magnetized to the same flux density from a 40-cycle line, what would be the hysteresis loss? (*b*) If with 40-cycle excitation the flux density is raised to 15,000 gausses, what will be the loss?

2-66. If a transformer core of silicon sheet steel weighs 10 pounds and has a hysteresis loss of 3.81 watts when operating on a 25-cycle line, what is the maximum value of flux density in lines per square centimeter? (Let $w_1' = 0.683$.)

2-67. If the core of the previous problem were magnetized to the same maximum flux density by a 60-cycle line, what would be the hysteresis loss?

2-68. If the core of problem 2-49 is being magnetized from a 25-cycle line, the flux density given being the maximum value, what is the hysteresis loss in watts, η being 0.0009?

CHAPTER III

THE ELECTRIC CIRCUIT

1. Ohm's Law. The relation that current and voltage vary in direct proportion in the electric circuit was experimentally determined by the physicist, Ohm, in 1826. Ohm found that if the voltage were doubled in a given circuit the current also would double, etc., and he expressed the relation by writing that

$$I \propto E \quad \text{or} \quad E \propto I$$

where I = current in the circuit,
E = voltage in the circuit.

When a direct proportionality exists between two variables, the relation may be expressed mathematically in an equation as $E = kI$, where k is a constant which generally will be different for every circuit. This constant k is called the *resistance* of the circuit, usually designated by R, and leads to Ohm's law,

$$E = IR \tag{1}$$

Whereas the resistance of a circuit may change due to temperature or other effects, it must be recognized that in the circuits which Ohm used the resistance remained constant.

Ohm's law states further that in the electrical circuit action is equal to reaction, or that effect is equal to cause. If voltage is impressed upon a circuit containing only resistance, the current must increase until the impressed voltage is exactly balanced by the reaction IR.

The factor of proportionality may also be chosen so that

$$EG = I \tag{2}$$

where G is called the *conductance* of the circuit.

The relation between conductance and resistance is evidently given by

$$R = \frac{1}{G} \quad \text{or} \quad G = \frac{1}{R} \tag{3}$$

Ohm's law may also be expressed as

$$I = \frac{E}{R} \tag{4}$$

$$E = \frac{I}{G} \tag{6}$$

$$R = \frac{E}{I} \tag{5}$$

$$G = \frac{I}{E} \tag{7}$$

2. Unit Resistance. The Ohm. If in any form of Ohm's law involving resistance unit current and unit voltage are assumed, the circuit will have unit resistance in *abohms*, or *ohms*, depending upon whether absolute or practical units of voltage and current are used. The practical unit of resistance is the international ohm; it is the resistance offered by a column of pure mercury having a uniform cross-section, 106.300 cm long and weighing 14.4521 grams at 0 C.

The unit of conductance is the *mho* or *reciprocal ohm*; the conductance of a circuit measured in mhos is the reciprocal of its resistance measured in ohms.

The three practical units have now been defined, the volt in terms of standard cells, the ampere in terms of the amount of silver it deposits in a standard solution, and the ohm in terms of a standard mercury column. By international agreement of many years standing, the units of current and resistance have been chosen as bases, since they are easiest to obtain for practical purposes. When measurements were crude, the value of the volt, as fixed by 1/1.434 of the voltage of a standard Clark cell at 15 C, apparently satisfied Ohm's law, as well as the fact that one volt was generated in a circuit if the flux interlinkages changed at a rate of 10^8 per second. But as measurements became more exact it was found that the value of the volt was more nearly the value now accepted, 1/1.4328 of the voltage of a standard Clark cell at 15 C, or 1/1.01830 of the voltage of a standard Weston cell at 20 C.

3. Joule's Law. It has been shown that when a voltage of E volts produces a current of I amperes in a circuit of R ohms resistance, the power, or rate of work, is

$$P = EI \text{ watts} = EI \times 10^7 \text{ ergs per second}$$

This can be seen if we recall the definitions of voltage and current. Voltage is the work per unit charge transferred between two points; current is the charge transferred divided by the time of transfer. Their product is then work divided by time, or power.

If the value of E from Ohm's law, $E = IR$, is substituted, we have

$$P = I^2R \text{ watts} = I^2R \times 10^7 \text{ ergs per second}$$

in which form the equation states that the power converted into heat in the resistance depends upon the resistance as well as the square of the current. The last equation also serves as a definition of the ampere as being that current which produces a dissipation of heat of one watt in a resistance of one ohm.

The electrical energy converted into heat in t seconds by a constant current, I, is

$$W = Pt = EIt \times 10^7 = I^2Rt \times 10^7 \text{ ergs}$$

and, since one joule has been defined as the equivalent of 10^7 ergs,

$$W = EIt = I^2Rt \text{ joules} \tag{8}$$

This relation was first announced by Joule in 1843 and it is now known as Joule's law. By passing known currents through known resistances immersed in a known volume of water, Joule was able to measure the amount of heat generated in the resistance by the rise in temperature of the water.

He found the relation between heat generated in gram-calories * in terms of amperes, volts, ohms, and seconds to be

$$Q = 0.24EIt = 0.24I^2Rt \tag{9}$$

From the relation just stated, Joule was able to connect electrical and heat units by deriving his mechanical equivalent of heat; from Eq. (9) it follows that 1 watt is equal to 0.24 gram-calorie per second or that 1 gram-calorie is equal to 4.18 joules or 4.18×10^7 ergs.

4. Battery Electromotive Forces and Resistances. A cell consists of two dissimilar metals or other conductors, dipping into a conducting solution or electrolyte of some kind. The electromotive force generated by a cell is the result of electrochemical action and, when the cell furnishes current, chemical energy is converted into electrical energy. The value of the electromotive force generated depends upon the conducting materials used as the plates and upon the electrolyte. Thus strips of copper and zinc dipping into a solution of sulphuric acid will show a difference of potential of about 1 volt. With electrodes of carbon and zinc and a solution of sal ammoniac (NH_4Cl) as electrolyte, the emf is about 1.5 volts.

The term battery is generally employed to designate a group of cells in any series-parallel combination. Every cell possesses internal resistance, partly in the plates and terminals, but for the most part in the electrolyte. Therefore when a cell delivers current some of its electromotive force or generated voltage, E, is used up as IR drop within the cell. What is left over is available at the terminals and is known as the terminal voltage, E_T. Therefore

$$E = E_T + IR_b \quad \text{or} \quad E_T = E - IR_b \tag{10}$$

where R_b is the internal resistance of the cell.

When current is forced through a cell *against its generated voltage* by an outside source of voltage, the latter must overcome the generated voltage of the cell and also supply the resistance drop within the cell. Hence,

$$E_T = E + IR_b \tag{11}$$

where E_T is again the voltage at the terminals of the cell.

* One gram-calorie is the amount of heat required to raise the temperature of one gram or cubic centimeter of water through 1 C.

The electromotive force or open-circuit voltage of a cell should be determined when the cell delivers no current. The ordinary voltmeter, which requires some current to deflect, may be used for this purpose if the resistance of the cell is small. Thus a certain lead cell with a generated voltage of 2.100 volts might have an internal resistance of 0.01 ohm. If the 3-volt scale of a voltmeter with a resistance of 300 ohms is used, the voltmeter current would be $2.1/300 = 0.007$ ampere. This would cause an IR drop within the cell of $0.007 \times 0.01 = 0.00007$ volt, which would be negligible and the voltmeter would correctly read the open-circuit voltage of the cell. But if the 1.5-volt range of a voltmeter of 150 ohms resistance were used to measure the voltage of a Weston cell of 300 ohms internal resistance and 1.018 generated volts, the result would be meaningless. The resistance of the entire circuit would be $300 + 150$ or 450 ohms and the current $1.0183/450 = 0.00226$ ampere. The drop within the cell would be $0.00226 \times 300 = 0.678$ volt and the terminal voltage would be 0.340 volt, and this would be the reading of the voltmeter.

5. Reactions in an Electric Circuit. Ohm's law stated that action and reaction were equal in a d-c circuit for a constant current. This law is but a simple case of a more general law which states the same conditions for any electrical circuit, whether with steady or transient conditions, with either direct or alternating currents.

If a brick is pushed along slowly as it rests on a horizontal board, sufficient force must be exerted to overcome the friction between the two surfaces in contact. The resisting force of friction which has to be overcome by the force moving the brick is called the reacting force of friction, or simply frictional reaction. If the brick is pushed up an inclined surface, the impressed force must overcome both the frictional reaction and the component of gravitational force which is tending to make the brick slide down.

In an analogous manner, if electrons are forced to flow steadily through a conductor, a kind of frictional reaction due to the continual collision of the electrons with the atoms of the conductor is set up, which opposes the motion of the electrons; this reaction, called resistance reaction, will be equal and opposite in direction to the impressed force. In the electric circuit we may have other forces which the impressed force has to overcome, such as the opposing force of a storage battery; this corresponds to the force of gravity in the case of the brick pushed up an incline. Each of these forces and reactions is measured by its corresponding voltage, and we shall speak of the resistance reaction as the voltage by which it is measured. In the study of alternating currents still other types of reaction must be considered and each will be measured by a corresponding voltage.

6. Kirchhoff's Laws. Two simple but fundamental laws, first expounded by Kirchhoff, are of great value in the solution of d-c networks.

First Law. *The algebraic sum of the potential drops around every closed circuit is always equal to zero.*

This law is merely another application of Ohm's law and its basis is obvious; if the sum of the reacting forces in the circuit were not equal and opposite to the impressed force, some of this would be left over with no opposing force, and so would serve to increase the current. This increase in current flow would continue until the increased resistance reaction resulting therefrom was just sufficient to build up the total reaction until it became equal and opposite to the impressed force.

Resistance reaction cannot be measured directly; current must be flowing to produce it and current will not flow unless a voltage is impressed on the circuit. In the case shown in Fig. 3–1, the battery E forces current to flow through the two resistances in series, in the direction shown. According to Kirchhoff's first law the sum of the two resistance reactions set up must be equal to the impressed voltage E. If we measure the voltage drop across A-B, and then across B-C, and add these two drops, the resultant must be the same as the impressed voltage, measured across the battery.

It is well at this point to distinguish between "drop" or component of the impressed voltage used across the part of the circuit in question, and the resistance reaction in the same part of the circuit. They are equal in magnitude, but opposite in direction. The drop across A-B for example, is evidently from A to B (A being at higher potential than B) and so causes the current to flow in the direction from A to B. The resistance reaction, however, is opposite in direction, from B to A, being in opposition to the applied voltage, and hence acting in a direction opposite to that in which the current flows. *This difference in direction must be carefully considered* if errors are to be avoided in solving problems.

It should also be noted that when a voltmeter is placed across a resistance, such as A-B, the voltmeter will read the component of the impressed voltage which overcomes the resistance reaction; the resistance reaction may be calculated if current and resistance are known, but cannot be measured directly.

In circuits of the type shown in Fig. 3–2, in which a large voltage, E, and several smaller voltages are shown, it may be assumed that all voltages that act in the same direction around the circuit as the large voltage E are positive, and voltages that act in the opposite direction are negative. We may say that the total impressed voltage acting in the circuit is $E + e_2 - e_1$ and put this impressed voltage equal to the sum of the IR drops in the circuit; then

$$E + e_2 - e_1 = IR_1 + IR_2 \tag{12}$$

Or we might consider the voltage e_1, since it acts to oppose the current,

to be a reaction and place a drop e_1, necessary to overcome this reaction, with the other drops as

$$E + e_2 = IR_1 + IR_2 + e_1 \tag{13}$$

Similarly e_2 might be classed with the drops, but it is then to be noted that, as we go clockwise around the circuit, the direction of potential drop through e_2 is in the opposite direction to that through the resistances and the voltage e_1. We should then write

$$E = IR_1 + IR_2 + e_1 - e_2 \tag{14}$$

It is of course obvious that the three last equations are mathematically really the same and so will yield the same solution.

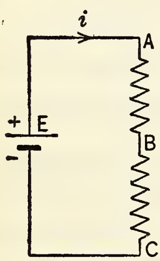


FIG. 3–1.

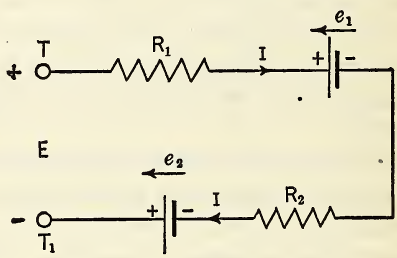


FIG. 3–2.

FIG. 3–1. The sum of the resistance reactions set up in the two parts of this circuit must be equal in magnitude and opposite in direction to the impressed voltage.

FIG. 3–2. Kirchhoff's first law states that the sum of the IR drops in a circuit must be equal to the algebraic sum of the various voltages acting in the circuit.

In some circuits the direction of the current in some branches will not be evident. If an incorrect direction has been assumed the solution of the problem will yield a negative value for that current; in this case the current actually flows in the direction opposite to that assumed. The magnitude of the current will be correct, however, even though its direction has been incorrectly assumed.

For the circuit shown in Fig. 3–3, the direction of current flow is obvious, and the voltage equations required for solution may be written, for the circuit $ABGF$,

$$E = I_1R_1 \quad \text{or} \quad E - I_1R_1 = 0$$

and for the circuit $ACDF$,

$$E = I_2R_2 \quad \text{or} \quad E - I_2R_2 = 0$$

In circuit $BCDG$ there is no voltage acting, and it will be noted that,

either way we go around the circuit, one of the IR drops must be taken as negative. With clockwise direction of traversing the circuit,

$$0 = I_2R_2 - I_1R_1$$

and for counter-clockwise direction

$$0 = I_1R_1 - I_2R_2$$

Either of the last two equations states the obvious fact that

$$I_1R_1 = I_2R_2$$

Kirchhoff's first law is sometimes stated in the form that the difference of electrical potential between any two points in an electrical circuit is the same for all possible paths between the two points. This follows from any of the equations given, but particularly from the last. In Fig. 3–3

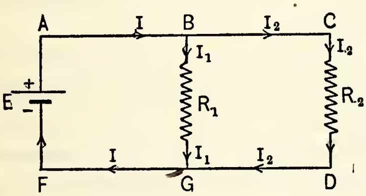


FIG. 3–3.

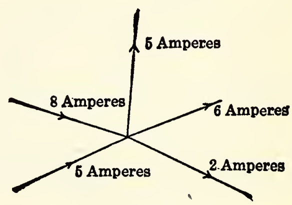


FIG. 3–4.

FIG. 3–3. In writing the equation to utilize Kirchhoff's first law, an IR drop may be reckoned as positive in one circuit and negative in another, depending upon which direction has been assumed for the various currents.

FIG. 3–4. Kirchhoff's second law states that the algebraic sum of all the currents flowing towards any point in the circuit must be zero. From the electron viewpoint the law merely expresses the more or less evident fact that electrons cannot accumulate at a point, without the potential of that point continually changing.

the potential drop from B to G is the same along path $BAFG$ as along path $BCDG$, or through resistance R_1.

In the solution of a network problem, the directions of the currents in the various paths are first assumed. Then in applying Kirchhoff's first law, the voltages in a closed path are added; those that tend to make current flow around the circuit in the direction in which *it is being traversed* are called positive. Those voltages which act oppositely to the direction of traverse are called negative. The total voltage acting in the circuit is then placed equal to the sum of the IR drops, where again the IR drops due to current in the direction in which the circuit is being traversed are called positive, and those due to oppositely flowing currents are called negative. Thus Kirchhoff's first law may be summed up for any closed path by the equation

$$E = \Sigma IR \tag{15}$$

7. Second Law. *The algebraic sum of the currents at any junction of conductors is always zero.*

This is the same as saying that the sum of all currents flowing towards a junction is equal to the sum of all currents flowing away from the junction, as is shown in Fig. 3–4. Were this not so there would be an accumulation (or the reverse) of electrons at the junction, and the potential of the point would continually change.

In the solution of problems involving the second law, we may assume currents flowing in either direction as positive, and those in the opposite direction as negative. When the direction of a current has been incorrectly assumed, the solution will yield a negative value for that current, which merely indicates that the direction is opposite to that assumed; the magnitude of the current is not affected by an incorrect assumption of direction.

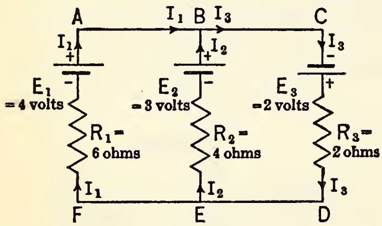


FIG. 3–5. A simple three-branch network, to illustrate the application of Kirchhoff's two laws.

Example 1. In the network of Fig. 3–5, the resistance of the cells is neglected. The direction in which the various currents will flow is reasonably obvious; they are assumed as indicated in the diagram.

Following the procedure outlined in the previous paragraphs, to apply Kirchhoff's first law to circuit *ABCDEF*, we have, starting from *A* in a clockwise direction,

$$2 + 4 = 2I_3 + 6I_1 \quad \text{or} \quad I_1 = 1 - \frac{I_3}{3}$$

For circuit *CDEBC*,

$$2 + 3 = 2I_3 + 4I_2 \quad \text{or} \quad I_2 = \frac{5}{4} - \frac{I_3}{2}$$

Applying the second law to the junction *B*, we have

$$I_1 + I_2 = I_3 \quad \text{or} \quad I_3 - I_2 - I_1 = 0$$

Substituting the values for I_1 and I_2 and solving, we find

$$I_1 = \tfrac{13}{22}, \quad I_2 = \tfrac{14}{22}, \quad I_3 = \tfrac{27}{22} \text{ amperes}$$

Had we used circuit *BEFA*, traversing it clockwise, we should have considered the *IR* drop in the middle branch ($4I_2$) as negative, since the current is flowing in a direction opposite to that with which we are passing around the circuit. We should then have

$$4 - 3 = -4I_2 + 6I_1$$

and, substituting the values obtained above for I_2 and I_1, this extra equation serves as a check:

$$1 + \tfrac{56}{22} - \tfrac{78}{22} = 0$$

which is an identity.

The three IR drops are:

$$I_1R_1 = \tfrac{13}{22}(6) = 3.545 \qquad I_2R_2 = \tfrac{14}{22}(4) = 2.545$$

$$I_3R_3 = \tfrac{27}{22}(2) = 2.455$$

8. Equations Applying Kirchhoff's Laws. When setting up equations for a network by Kirchhoff's laws, it will be evident that there will always be more equations than there are unknown currents. Inspection will show that they are not all independent, but that some equations may be derived from others. To solve for a set of unknown currents a set of independent equations is necessary.

It will be seen by inspection of a circuit that the current law can be applied at each point where three or more wires are joined. Such a point is called a *branch point,* and one equation may be written for the currents at each. It will be found, however, that one of these equations is not independent, but may be derived from the others. In applying the voltage law, an equation may be written for each closed path, and again some of these will not be independent.

The most convenient method is to write equations applying the current law for all branch points *except one,* and then to write equations applying the voltage law, *being sure to traverse every branch at least once,* until a sufficient number of equations have been set up.

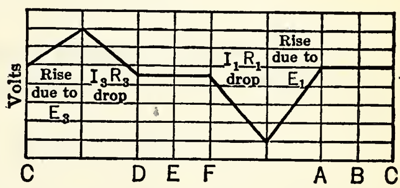


FIG. 3–6. Variation of potential in path $CDEFABC$ of Fig. 3–5, with point C as the reference.

9. Variation of Potential in a Closed Path. In Fig. 3–6 the variation of potential along the path $CDEFABC$ of Fig. 3–5 is plotted with point C as a reference. Cell 3 causes a rise of 2 volts and there is a drop or loss in potential due to the I_3R_3 drop of 2.455 volts, putting the potential of point D below that of point C by 0.455 volt. There is a further loss in potential due to the I_1R_1 drop of 3.545 volts. The rise due to cell 1 of 4 volts brings the potential of point C back to the starting value. Figure 3–7 shows similar conditions for circuit $CDEBC$.

In Fig. 3–8 the variation of potential along the path $BEFAB$ of Fig. 3–5 is plotted with point B as the reference. The current has been assumed as flowing from E to B, the voltage of cell 2 acts in a direction opposite to that taken for the traverse, and this voltage is therefore

considered as a reaction, to cause loss of potential. The same reasoning applied to the I_2R_2 drop requires that this drop be considered negative; potential is gained in traversing R_3 from B to E.

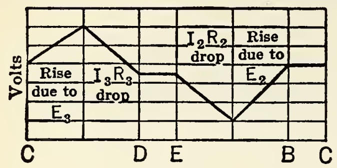


FIG. 3-7.

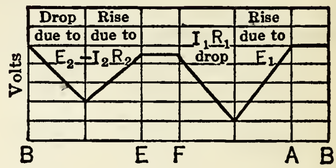


FIG. 3-8.

FIG. 3-7. Variation of potential in path *CDEBC* of Fig. 3-5, with point *C* as reference.
FIG. 3-8. Variation of potential in path *BEFAB* of Fig. 3-5, with point *B* as reference.

Example 2. In the network of Fig. 3-9, the direction of the current I_3 is not obvious. Let it be assumed as flowing down toward the junction E.

At junction B, we have, writing the *one* independent current equation by the second law,

$$I_1 = I_2 + I_3$$

For the circuit *CDFAC*, we have starting at C

$$8 + 10 = 2I_2 + 5I_1 \quad \text{or} \quad I_2 = 9 - \frac{5I_1}{2}$$

For circuit *BEFAB*, we have

$$10 = 2I_3 + 5I_1 \quad \text{or} \quad I_3 = 5 - \frac{5I_1}{2}$$

We now have the required number of independent equations necessary for a solution; we have traversed each branch at least once. Solving, we find

$$I_1 = \tfrac{14}{6}, \quad I_2 = \tfrac{19}{6}, \quad \text{and} \quad I_3 = -\tfrac{5}{6} \text{ ampere}$$

showing that the current in the center path is flowing in the direction opposite to that assumed and is supplied by the cell marked E_2.

Example 3. Wheatstone Bridge. A special network much used for comparing resistances, originated by Wheatstone, is shown in Fig. 3-10. Four resistances A, B, R, and X, are connected in a mesh or square, a battery being connected across two corners $b - b'$, and a galvanometer or other sensitive detector of current across the other two corners, $a - a'$. A and B are usually fixed resistances of appropriate values, R is a resistance which can be varied through a wide range, and X is some unknown resistance.

By varying the resistance R, the desired result, that of no current flowing through the galvanometer G, is achieved. Under these conditions, calling I the current flowing through resistances A and B, and I' the

current through R and X, and applying Kirchhoff's first law to the mesh, we have, passing around in clockwise direction,

$$IR_A + IR_B - I'R_X - I'R_R = 0$$

or

$$IR_A + IR_B = I'R_X + I'R_R$$

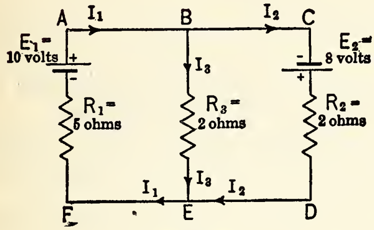


FIG. 3–9.

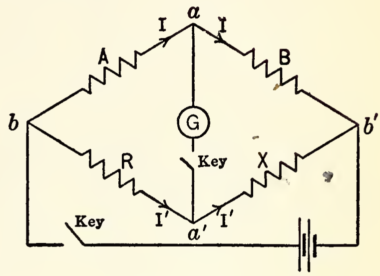


FIG. 3–10.

FIG. 3–9. Sometimes the direction of current flow in a branch of the network is not at all evident, as is the case for the current I_3 in this circuit. A direction is assumed for the current in such a case, and if the solution yields a negative answer it signifies that the direction assumed for the current was incorrect.

FIG. 3–10. The ordinary Wheatstone bridge is a good illustration of the application of Kirchhoff's laws.

As no current is flowing through the galvanometer, G, the points a and a' are at the same potential, and hence

$$IR_A = I'R_R \quad \text{and} \quad IR_B = I'R_X$$

Dividing one equation by the other, we have

$$\frac{R_A}{R_B} = \frac{R_R}{R_X} \quad \text{and} \quad R_X = \frac{R_B}{R_A} \times R_R \tag{16}$$

In commercial and laboratory bridges, all the above members, except the unknown resistance, are suitably arranged in a convenient case with terminals for connecting in the unknown resistance. The variable resistance can be varied in 1-ohm steps from 1 to perhaps 9999 ohms, and the fixed resistances are generally of such values that their ratio may be altered at will, in multiples of 10 from 1/1000 to 1000, permitting of wide ranges in the measurement of resistance.

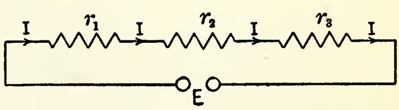


FIG. 3–11. The resistance of a circuit consisting of several resistances in series is obtained by adding the various resistances making up the circuit.

10. The Series Circuit. When a number of resistances, r_1, r_2, r_3, etc., are put in series as in Fig. 3–11, the same value of current must flow

through all of them when a voltage is impressed upon the entire circuit. Applying Ohm's law and Kirchhoff's first law, we have

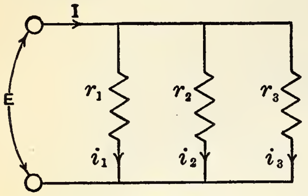


FIG. 3–12. When the resistances are connected in parallel the solution of the problem shows that the circuit resistance is the reciprocal of the sum of the reciprocals of the resistances in the various branches.

$$E = IR = Ir_1 + Ir_2 + Ir_3$$

where E = the impressed voltage,
and R = the total resistance of the circuit.

Dividing by the current I,

$$R = r_1 + r_2 + r_3 \tag{17}$$

which states that resistances in series are to be added to obtain the total resistance.

11. The Parallel Circuit. When a number of resistances, r_1, r_2, r_3, are placed in parallel as in Fig. 3–12, the voltage across each individual resistance must be the same, and the total current supplied must, from Kirchhoff's second law, be equal to the sum of all the branch currents.

$$I = i_1 + i_2 + i_3$$

$$E = IR = i_1r_1 = i_2r_2 = i_3r_3$$

Since
$$I = \frac{E}{R}, \quad i_1 = \frac{E}{r_1}, \quad i_2 = \frac{E}{r_2}, \quad i_3 = \frac{E}{r_3}$$

we have
$$\frac{E}{R} = \frac{E}{r_1} + \frac{E}{r_2} + \frac{E}{r_3}$$

and
$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \tag{18}$$

Since conductance = 1/resistance, Eq. (18) may be written

$$G = g_1 + g_2 + g_3 \tag{19}$$

12. Resistance. It is found experimentally that the resistance of a homogeneous conductor varies directly as its length l, and inversely as its cross-section a, or

$$r = \rho \frac{l}{a} \tag{20}$$

where the factor of proportionality, ρ (rho), is called the *resistivity* or *specific resistance* of the material. The resistivity is dependent upon temperature, as will be discussed later.

Resistivity, as may be seen from Eq. (20), is the resistance of a volume of unit cross-sectional area and unit length. In calculations involving

rectangular conductors, such as bus-bars, etc., resistivity is expressed in microhms (ohms $\times 10^{-6}$) per centimeter cube or per inch cube.

It will be seen from Eq. (20) that the length and area involved determine the resistance of a given volume. A given conductor will have a higher resistance if it is four centimeters long with a cross-sectional area of one square centimeter than if it is one centimeter long with a cross-sectional area of four square centimeters, although the volume in the two cases is the same.

Values of resistivity for the metals and alloys in common use are given in Table II of the Appendix.

In calculations involving round wires, the length of which is usually taken in feet, it is more convenient to express the area in circular measure. The unit used for this purpose is the *circular mil*, which is the area of a

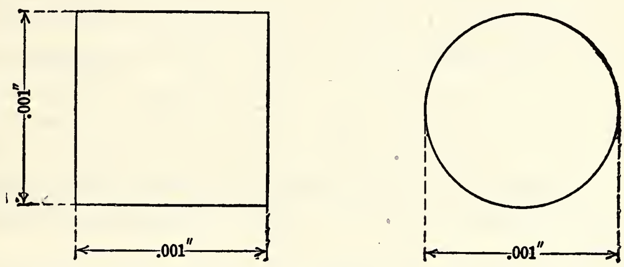


FIG. 3–13. A circular mil is the area of a circle having a diameter of one thousandth of an inch; the square mil is the area of a square one thousandth of an inch on a side, or one millionth of a square inch.

circle one mil, or one thousandth of an inch, in diameter. Thus, to obtain the area of a circle in circular mils, it is necessary merely to square its diameter in mils.

A square mil is the area of a square each side of which is one mil. The area in square mils of a circle one mil in diameter is $1^2 \times \pi/4 = 0.7854$; the area of the same circle is one circular mil. Hence

$$\text{Square mils} = \text{circular mils} \times 0.7854$$

or

$$\text{Circular mils} = \text{square mils} \times 1.2732$$

This relation is indicated in Fig. 3–13.

The circular inch is also used at times, it being the area of a circle 1 inch or 1000 mils in diameter. One circular inch thus equals 1,000,000 circular mils. The exact dimensions of the square and the circle of Fig. 3–13 are one square inch and one circular inch respectively.

When length is in feet and area in circular mils, resistivity may be expressed in ohms per mil-foot, i.e., the resistance of a wire one foot long

and one circular mil in area (Fig. 3–14). Notice that the term circular mil is abbreviated to mil in this expression. Values for resistivity in ohms per mil-foot are also given in Table II of the Appendix.

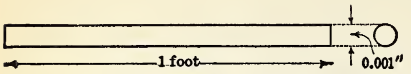


Fig. 3–14. The circular mil-foot (abbreviated to mil-foot) is commonly used to express the resistivity of round wires. It is a wire one foot long, circular in cross-section, with a diameter of one thousandth of an inch.

13. Resistivity of Copper. Up to 1912, Matthiessen's standard for the resistivity of copper, 1.594 microhms per centimeter cube at 0 C, was universally used. This value was the result of measurements made by Matthiessen in 1862 on supposedly pure copper. Later measurements made by the Bureau of Standards upon commercial copper (density 8.89) resulted in the present standard for the resistivity of copper of 1.589 microhms per centimeter cube at 0 C, or 1.7241 microhms per centimeter cube at 20 C. This value is known as the "International Annealed Standard," and is universally accepted. This is equivalent to a resistivity of 10.36 ohms per mil-foot at 20 C.

14. Conductors. Study of Table II in the Appendix will show that silver has the lowest resistivity, followed closely by copper. Copper, because of its reasonable cost, high conductivity, and tensile strength, and because it can be drawn and soldered easily, is the material generally used for conductors.

Aluminum has about 60 per cent of the conductivity and only 30 per cent of the weight of copper. For wires of the same resistance per unit of length, the area of cross-section of aluminum wires must be 63 per cent greater than that of copper wires; round aluminum wires are therefore about 27 per cent greater in diameter. On a basis of equal conductance the weight of aluminum is only about one-half that of copper and in price usually about 10 per cent less. One disadvantage of aluminum is that it is harder to solder than copper.

Because aluminum wires have a greater diameter for the same conductance than copper wires, the use of aluminum in insulated wires is prohibited, at the usual prices of both metals, by the extra cost of insulation. Its use in machine construction is evidently prohibited because of the resulting increase in size, and therefore cost, of the machine. Where it can be used bare, as in high-tension transmission lines and low tension bus-bars, it is employed to some extent. Although the cost of the wire itself is often greatly reduced when aluminum is used in transmission lines, this is more than offset by the need of a greater number of supporting poles because of the lower tensile strength of aluminum. In practice the aluminum wire is reinforced by making up a cable consisting of a number of aluminum wires wrapped about a number of twisted steel wires. Such wire is known as steel-reinforced aluminum cable, the aluminum portion

carrying most of the current and the steel portion supplying the tensile strength. About the only commercial use of iron and steel as conductor is in resistance units and for the third rails of electric railways.

Copper-clad steel wire (steel wire coated with copper) is sometimes used for long spans where higher tensile strength is required. The conductivity of a conductor made of steel and either copper or aluminum will depend upon the relative cross-sections of the two materials. Two commercial grades are common for copper-clad steel wire, 30 per cent and 40 per cent, expressing the conductivity ratio of such wires to solid copper wire of the same cross-section.

A steel-reinforced copper cable is also made and used on long spans. It is similar to the steel-reinforced aluminum cable.

15. Effect of Temperature on the Resistance of Metals. It is shown by experiment that the resistance of metals varies with temperature and that the variation in resistance with temperature is practically linear within ordinary temperatures. Such a relation may be expressed as follows:

$$R_{t_1} = R_t [1 + \alpha_t (t_1 - t)] \tag{21}$$

where R_{t_1} is the resistance at any temperature t_1;
R_t is the resistance at some standard temperature t;
and α_t is the temperature coefficient at the standard temperature t.

If the standard temperature is taken as 0 C, the expression reduces to

$$R_{t_1} = R_0 (1 + \alpha_0 t_1) \tag{22}$$

where α_0 is the temperature coefficient at 0 C.

Copper being used more than any other metal for carrying current, it is important to know the change of resistance with temperature of copper windings in machines and other apparatus.

If the values for resistance of copper at ordinary temperatures, 0 to 100 C, are plotted against temperature, the results lie on practically a straight line which, if extended, reaches zero resistance at -234.5 C. This assumes that, if the rate of decrease in resistance between 100 and 0 C were maintained at all temperatures, the resistance of copper would be zero at -234.5 C. (Fig. 3–15). The same result may be obtained by substituting the temperature coefficient for copper at 0 C, and the value for resistivity from Table II of the Appendix in Eq. (22), equating it to zero and solving for t_1.

$$0 = 1.589 (1 + 0.00427 t_1)$$

from which $t_1 = -234.5$ C.

Similar solutions for other metals indicate that their resistance also becomes zero at about the same temperature, and experimental evidence is available that the resistance of metals is practically zero in the region

of absolute zero, −273 C.* Actually the variation in resistance with temperature is not uniform at very low temperatures.

16. Resistivity and Temperature Coefficient Relations. Since resistivity, either in microhms per cubic centimeter or ohms per mil-foot, is merely the resistance of a body having unit length, and unit cross-sectional area, it follows from Eqs. (21) and (22) that

$$\rho_{t_1} = \rho_t[1 + \alpha_t(t_1 - t)] \tag{23}$$

and

$$\rho_{t_1} = \rho_0(1 + \alpha_0 t) \tag{24}$$

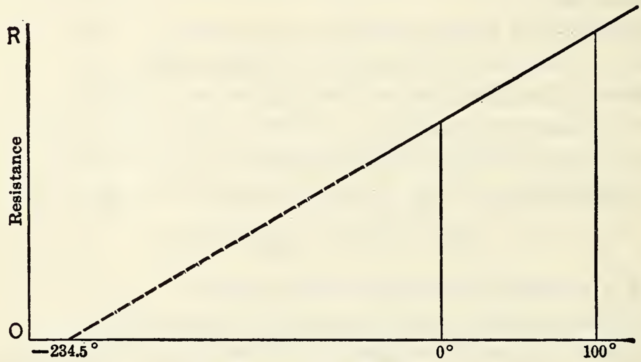


FIG. 3–15. In the ordinary range of temperatures, the curve plotted with resistance of copper and temperature as coordinates is practically a straight line, which if continued backwards cuts the line of zero resistance at −234.5 C.

where ρ_{t_1} is the resistivity at any temperature t_1;
ρ_t is the resistivity at some standard temperature t;
ρ_0 is the resistivity at 0 C;
α_t is the temperature coefficient at standard temperature t;
and α_0 is the temperature coefficient at 0 C.

If we assume again that the temperature coefficient at ordinary temperatures remains constant as the material is cooled to abnormally low temperatures, the temperature at which the resistivity becomes zero may be calculated from Eq. (24).

$$0 = \rho_0(1 + \alpha_0 t_1) = \rho_0 + \rho_0\alpha_0 t_1$$

and

$$t_1 = -\frac{\rho_0}{\rho_0\alpha_0} = -\frac{1}{\alpha_0} \tag{25}$$

* That the resistance of metals is practically zero in the region of absolute zero was shown by an experiment, in which current was induced in a lead ring held at nearly absolute zero temperature by suitable cooling apparatus. Voltage was induced by drawing the pole of a magnet out of the ring and the current detected by a compass needle. The current flowing in the ring remained practically constant for several hours.

For standard copper, the temperature coefficient of which is 0.00427 at 0 C, the apparent critical temperature is $-1/0.00427$ or as before, -234.5 C. If this value for t_1 is substituted in Eq. (25),

$$\alpha_0 = -\frac{1}{-234.5} = \frac{1}{234.5} \tag{26}$$

For tungsten the apparent critical temperature is $-1/0.0047$ or -212.8 C, while for Nichrome V* (a nickel-chromium alloy with a low temperature coefficient) it is $-1/0.00013$ or -7693 C.

The last result follows from the assumption that resistance varies directly with temperature. But it was mentioned that experiment showed

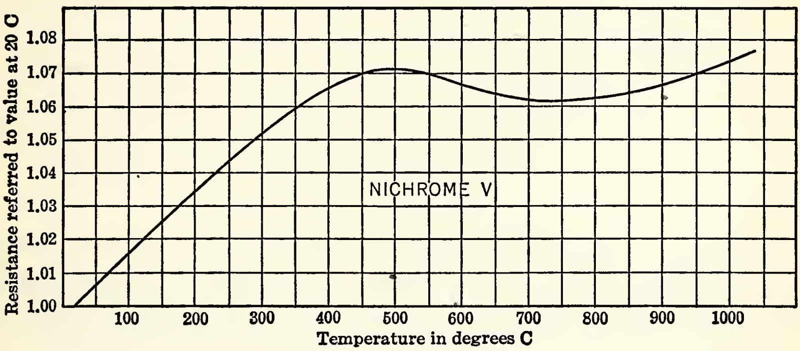


FIG. 3–16. Curve showing the variation of resistance of Nichrome V, a nickel-chromium alloy made by the Driver-Harris Co. The curve is for average material. Nichrome V is suitable for use in electric ranges, radiant heaters, electric furnaces, and all applications where the resistor must operate at high temperatures. Data reproduced by permission of the Driver-Harris Co.

resistance to vary linearly with temperature *within ordinary temperatures*, and it is customary in tables of resistance-temperature coefficients to state the temperature range within which the coefficient may be applied. Many conductors, especially alloys, show very peculiar resistance changes at high and low temperatures, as is shown in the resistance curve of Nichrome V of Fig. 3–16.

In Table III of the Appendix are shown the values of the temperature coefficient of copper at different temperatures. The value shown at 20 C is 0.00393, which means that the resistance of copper increases or decreases by 0.393 of 1 per cent of what it was at 20 C, for each degree centigrade, above or below 20 respectively (at constant mass). Hence, for the resistance to become zero, assuming that the resistance-temperature curve is a

* Trademark name, Driver-Harris Co.

straight line with the same slope as it has at 20 C, the temperature may fall 100/0.393 = 254.5 below 20 or −234.5. Or we may write

$$234.5 + 20 = \frac{1}{0.00393} = \frac{1}{\alpha_{20}}$$

and

$$\alpha_{20} = \frac{1}{234.5 + 20}$$

Hence the temperature coefficient at constant mass (for standard copper) may be determined for any temperature from the expression

$$\alpha_t = \frac{1}{234.5 + t} \tag{27}$$

where t is in degrees C.

If the value $1/\alpha_0$ is substituted for 234.5, Eq. (27) may be rewritten

$$\alpha_t = \frac{1}{\dfrac{1}{\alpha_0} + t} = \frac{\alpha_0}{1 + \alpha_0 t} \tag{28}$$

from which the temperature coefficient at constant mass for any material at any temperature may be determined. This equation is derived on the assumption that the resistance-temperature variation is a linear one, and so may be used only for the temperature ranges in which this relation holds good.

17. Calculation of Resistance at Different Temperatures. It often happens that we know the value of a resistance at one temperature and require it at another temperature. The required result may be obtained by calculating first the value of the resistance at some standard temperature, say 0 C, and from this the value at the desired temperature.

It is frequently desired to know the rise in temperature of a winding from two resistance measurements. A simple formula can be deduced from Eq. (22) to fit this case.

Let R_c = resistance at any initial temperature, t_c,
R_h = resistance at some higher temperature, t_h.

Then

$$R_c = R_0(1 + \alpha_0 t_c)$$

$$R_h = R_0(1 + \alpha_0 t_h)$$

Dividing one equation by the other,

$$\frac{R_c}{R_h} = \frac{1 + \alpha_0 t_c}{1 + \alpha_0 t_h} \tag{29}$$

If copper is the material the temperature rise of which is desired, the value for $\alpha_0 = 1/234.5$ (Eq. 26) may be substituted, so that

$$\frac{R_c}{R_h} = \frac{1 + \dfrac{t_c}{234.5}}{1 + \dfrac{t_h}{234.5}} = \frac{234.5 + t_c}{234.5 + t_h} \tag{30}$$

Equation (30) is evidently the equation for the straight-line curve of Fig. 3–15, *but may be applied only to copper.* A similar equation may easily be derived for any other metal or alloy for the region in which there is a linear relation between resistance and temperature.

18. Low Temperature Coefficient of Certain Alloys. It will be noted in Table II of the Appendix that several of the alloys listed have tempera-

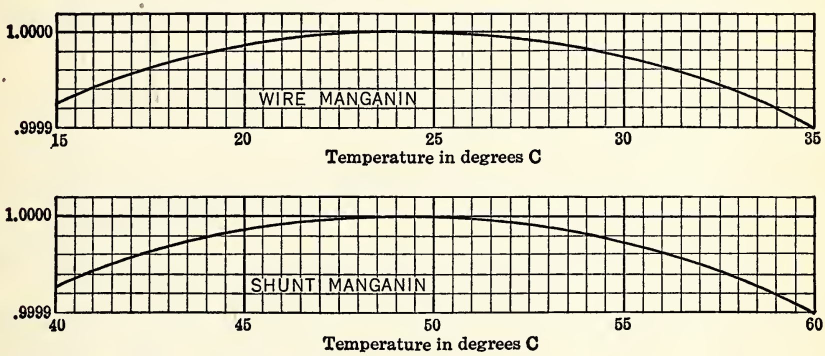


Fig. 3–17. Change in resistance of a 1-ohm wire or strip of manganin with increasing temperature. Wire manganin, on spools and in coils, is used in precision bridges, potentiometers, etc. Shunt manganin is used for ammeter shunts and precision resistance units which are heated by the passage of current. The curves are reproduced, by permission, from data published by the Driver-Harris Co.

ture coefficients which are nearly zero. Such alloys are particularly valuable for use as standard resistances and in instruments where a variation of resistance with temperature is likely to cause appreciable error.

The curves of Fig. 3–17 show the variation in resistance of two grades of manganin which were developed to have maximum resistance at the temperatures at which they are likely to be used. Wire manganin is used for resistance spools in precision instruments; shunt manganin is used for ammeter shunts.

19. Effect of Temperature on the Resistance of Non-metals. The variation of resistance with temperature of most non-metals which are conductors is found to be different from that in metals, in that their resist-

ance decreases with increase of temperature; they therefore have a negative coefficient.

This is true for carbon, most solutions of acids and salts, and many materials like glass which are conductors only at high temperatures. The variation in resistance of the latter class of materials is, however, irregular and is best expressed by curves. The change of resistance with temperature of materials which are much used for insulation is of importance. In Fig. 3–18 is shown the variation of resistance with temperature of rubber and of varnished cambric, the latter an impregnated insulating material much used for machine and cable insulation. It will be seen that their insulating properties decrease very rapidly as the temperature increases; it is because of the effect of heat on such insulating materials that a definite

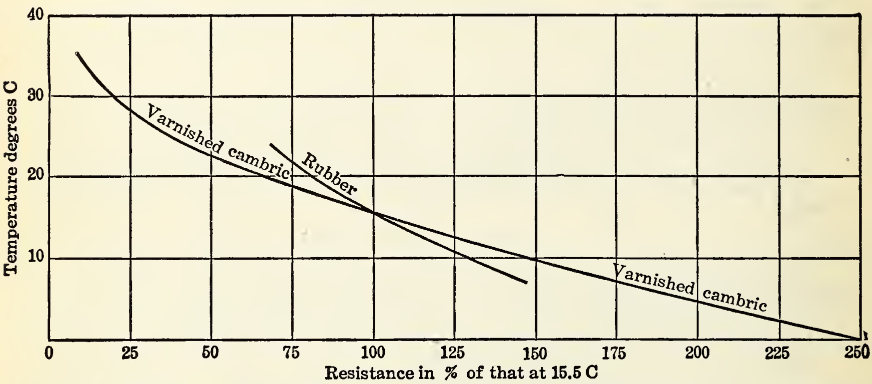


Fig. 3–18. The resistance of most insulating materials decreases rapidly with temperature increase, a fact which must be considered in machine and cable design. The above curves are for varnished cambric and rubber.

temperature is specified, above which it is not safe to operate an electric machine or cable.

20. Wire Table. The manufacture of wire to be used for electrical purposes has been standardized by gauge numbers, according to a table originally known as the Brown and Sharpe, but now properly known as the American Wire Gauge (AWG). The wire numbers are retrogressive, a larger number denoting a smaller size.

The wire sizes are related according to a simple mathematical law, which may be expressed by saying that the ratio of any diameter to the next smaller is a constant number, 1.123. As the sixth power of this ratio is approximately 2, it means that the diameter approximately doubles or halves every six numbers, as may be seen from the table for solid copper wire, Table IV, in the Appendix.

The resistance, mass, and cross-section of a wire vary as the square of the diameter; hence the ratio of the cross-section of any size to the next

smaller is the square of 1.123, or 1.261. The cube of 1.261 is so very nearly 2 that it follows that the cross-section is doubled or halved every three numbers, and the resistance and mass correspondingly changed.

Inspection of the table for solid copper wire shows that No. 10 wire has practically

a resistance of 1 ohm per 1000 feet at 20 C,
a diameter of 0.1 inch,
a cross-section of 10,000 cir mils,
a weight of 31.4 or 10π pounds per 1000 feet.

With these characteristics in mind, the application of the "three numbers" rule enables one to determine the approximate resistance and weight of any size of wire.

If the resistance, weight, and area of, say, No. 3 solid copper wire is desired, it is found that the resistance of No. 7 is one-half and of No. 4 is one-fourth that of No. 10. Hence the resistance of No. 3 is

$$\frac{1}{4 \times 1.26} = \frac{1}{5} = 0.2 \text{ ohm per 1000 feet}$$

Similarly the area of No. 3 is $10{,}000 \times 4 \times 1.26$ or 50,000 cir mils and its weight $31.4 \times 4 \times 1.26 = 157$ pounds per 1000 feet. These figures are sufficiently close for all ordinary problems, being within 2 per cent at 20 C.

It will also be seen from Table IV that for every ten numbers the area and weight increase practically ten times or become one-tenth as much. This follows from what was said above. If we start with No. 10 wire and count over nine larger sizes the area of No. 1 doubles three times or is $2 \times 2 \times 2$ that of No. 10. Then the area of No. 0 becomes $2 \times 2 \times 2 \times 1.26 = 10.09$ times that of No. 10. The actual areas of No. 10 and No. 0 are, from Table IV, 10,380 and 105,500 cir mils respectively. Similarly the resistance of No. 0 wire will be one-tenth that of No. 10, etc.

21. Stranded Cables. In Table V of the Appendix, data for stranded cables are given. As it is impracticable to handle solid wires in sizes greater than No. 0000 and often advantageous to have smaller sizes in a flexible form, cables built up of a number of strands of smaller wires are commonly available in sizes beginning with No. 8. As flexibility is a function of the number of strands and of the size of wire used, four classes of stranding, known as A, B, C, and D, are available. Class A is for bare, weatherproof, and slow-burning cables and has the least number of strands. Class B is for rubber, varnished cambric, and paper cables and is much used; it was formerly known as "standard." Classes C and D are for use where still greater degrees of flexibility are desired. Class C has more strands of smaller-sized wire than Class B, and Class D more than Class C.

Since it is desired to keep the circumference of a stranded cable which is to be insulated at a minimum, the strands are concentrically laid. Six wires will just fit about a center wire and twelve wires will just fit about the second layer of six, etc.; hence the standard numbers of strands will be 1, 7, 19, 37, 61, etc. Wires larger than No. 0000 are designated by their circular mil area, the standard sizes beginning with 250,000 cir mils and increasing successively by 50,000 up to 1,000,000 cir mils. After 1,000,000 cir mils each successive size increases by 100,000 cir mils.

22. Electrical Factors Which Determine the Size of Wire in Practice. In selecting the size of wire for transmitting and distributing electric current, the electrical factors to be considered are: (*a*) the allowable voltage variation at the load or the voltage lost in the line as IR drop; (*b*) the power lost in the lines, or I^2R loss; (*c*) the allowable current-carrying capacity of the wire.

23. Voltage Regulation. If the voltage at the generator end of a d-c transmission system is maintained constant, the voltage at the receiving end will vary with the load on account of the IR drop in the line. At no load the voltage at both ends will be the same, $E_g = E_r$, but at full load the voltage at the receiving end will be less, according to the relation

$$E_r = E_g - IR \tag{31}$$

where E_r = voltage at receiving end;
E_g = voltage at generator end;
I = current;
R = resistance of outgoing and return wires.

The regulation of a line is the ratio of the change in voltage, from no load to full load, to the value at full load. The regulation of a line is an important matter, as it affects the salability of the energy. It is found that the amount of light given out by modern tungsten lamps varies directly as the 3.6th power of the ratio of the applied voltage to the rated voltage and their life varies inversely approximately as the 13th power of the ratio of the applied voltage to the rated voltage. If the load over the line varies rapidly through wide limits, unpleasant flickering of the lamps results. Voltage change also affects the speed of d-c motors.

Circuits designed to transmit relatively large amounts of power, usually at high voltages with few or no branches, are generally called transmission lines; circuits used for delivering power in smaller amounts to numerous points, by many branches usually at lower voltages, are termed distribution circuits. A transmission line usually has a distribution system at its receiving end. Because of the low voltages employed, d-c systems generally serve only as distribution circuits.

The usual loss in voltage in d-c systems supplying light and power

varies from 2 to 5 per cent of the delivered voltage. In some cases where provision is made for raising the voltage at the supply end, these values may be exceeded, but in any case the voltage variation at incandescent lamps should never exceed 2.5 per cent above or below normal. For systems supplying power only, the allowable voltage variation may be greater, because the voltage variation is here not so important; and by allowing a greater voltage variation, smaller and correspondingly cheaper wires may be used to deliver the power.

24. Voltage and Weight of Conductor. The weight of conductor necessary to transmit a given amount of power a certain distance with the same loss varies inversely as the square of the voltage.

Let V_1 and V_2 be two voltages at which an amount of power, P, is to be transmitted. The currents will then be

$$I_1 = \frac{P}{V_1} \quad \text{and} \quad I_2 = \frac{P}{V_2}$$

and the power loss

$$P_1 = I_1^2 R_1 \quad \text{and} \quad P_2 = I_2^2 R_2$$

where R_1 and R_2 are the respective resistances of conductor used.

If the losses in the two cases are to be the same,

$$I_1^2 R_1 = I_2^2 R_2$$

so that

$$\frac{R_1}{R_2} = \frac{I_2^2}{I_1^2} = \frac{\left(\dfrac{P}{V_2}\right)^2}{\left(\dfrac{P}{V_1}\right)^2} = \frac{V_1^2}{V_2^2} \tag{32}$$

Weight of conductor for the same length varies inversely as resistance; hence, if weight of conductor is designated by W,

$$\frac{W_1}{W_2} = \frac{V_2^2}{V_1^2} \tag{33}$$

Further, since resistance of conductor of constant cross-section varies directly as length, the relative distances L for equal power loss are given by

$$\frac{L_1}{L_2} = \frac{V_1^2}{V_2^2} \tag{34}$$

It appears from these relations that the higher the voltage, the less the weight of copper necessary to transmit a given amount of power, or the greater is the distance a certain amount of power can be transmitted with a given loss.

When alternating current is used to transmit power, voltages as high as 275,000 volts are used, being made feasible by the transformer. This piece of stationary apparatus makes it possible to transform economically from one voltage to another. Thus the power might be generated at 15,000 volts, stepped up through transformers in one or two steps to 275,000 volts. After being transmitted at this voltage for many miles the power might be stepped down through other transformers, in several steps, to 120 volts or any voltage at which the power is to be used.

In the case of direct currents, to transform from one voltage to another it is customary to use rotating machinery, usually a motor-generator set. If d-c power at 500 volts were available and it was required at 110 volts, a 500-volt motor would be used to drive a 110-volt generator. Such transformation is relatively inefficient and is limited by the fact that the maximum voltage for which large d-c power generators can successfully be built is about 2000 volts.

The present development of mercury-arc rectifiers and inverters provides an efficient means of changing from a-c power to d-c power, or vice versa. This holds the possibility of the use of high-voltage direct currents for the transmission of power with its many advantages. The power would be generated in alternating form, stepped up to high voltage, rectified to d-c form, and then transmitted. At the distribution point it would be inverted back into alternating form and the voltage then stepped down to any desired value by transformers.

Whereas voltages of 500 to 3600 volts are used for d-c railway service (obtained sometimes from two generators in series), the voltage for incandescent lamps and general household service is limited to 125 volts. This is done because shocks from high voltages are dangerous and because small lamps of higher voltage are less efficient and require longer and thinner filaments to obtain the required resistance.

25. Power Loss in Line. Kelvin's Law. The energy converted into heat in a line, or I^2Rt loss, besides heating the conductors, also represents an actual money loss, and so has bearing on the size of conductor selected.

Any installation of machinery, wire, etc., represents an investment of money, and in connection with such installations there will be many items of expense called fixed charges. There is the interest which must be paid for the loan of the money invested. In practically every installation some money must be spent in upkeep to care for depreciation. As the installation grows older it may eventually become obsolete and have to be replaced; to care for this a certain sum of money must be laid aside each year, so that when replacement becomes necessary the original loan may be repaid or the original cost written off. The problem is a complicated one, and a complete analysis is far beyond the scope of this book.

In the case of a transmission line, the problem is a little simpler, in that upkeep, depreciation, and obsolescence are small and at times even may be neglected. The most important economic question is to balance the cost of the energy lost in the line with the interest on the investment.

The cost of the energy lost in the line varies directly with the cost per kilowatthour and inversely as the area of cross-section of the wire used. If the cost of the energy is high, or copper is cheap, it usually pays to put in more copper, that is, use larger sizes of wires. But this involves a greater first cost for copper with higher interest charges and a greater expense for poles or underground conduits.

If only interest charges and cost of energy are considered, a law, known as Kelvin's law, applies. Kelvin's law may be stated in the following form: That system of electrical transmission or distribution is most economical in which the annual interest on the first cost of the copper is equal to the value of the energy lost in the line in one year.

Thus, depending upon the market price for copper and the cost of energy, Kelvin's law would dictate larger or smaller conductors. However, there is another factor which is frequently more important than this economic consideration, that is, satisfaction of the customer buying the energy. A line of small cross-section, possibly dictated by Kelvin's law, will furnish to the customer a voltage which fluctuates widely with the current he draws. Such a fluctuating voltage will result in poor motor action, poor performance of electric lamps, etc., and hence dissatisfaction of the customer.

It will be appreciated further that the size of a feeder installed must be determined largely by the load which it will probably have to carry during the next five or ten years, rather than the present load. Thus the real value of Kelvin's law appears only when it dictates *larger* feeders than are called for by the other considerations, particularly that of regulation.

26. Current-carrying Capacity of Conductors. Inasmuch as the passage of current through a conductor generates heat, and as the temperature of a conductor must rise until the rate of dissipation of heat is equal to the rate of generation, it becomes necessary to limit the current so as not to allow the temperature of the conductor to reach such values as might cause deterioration of the insulation. The limiting current capacity for cables and wires is dependent upon the type of insulation used, the temperature of the surrounding air, and other factors. For interior wiring in buildings, the limits are set by the National Board of Fire Underwriters, in a set of rules and instructions known as the National Electric Code. In general, municipal authorities require that installations be carried out in accordance with this set of rules. The carrying capacity of wires, according to the NEC code, is given in Tables VI and VII of the Appendix.

27. Wire and Cable Insulation. The principal materials used for insulating electric conductors intended for transmission, distribution, building wiring, etc., are compounds containing rubber, varnished cambric, paper, flexible plastics, and impregnated cotton braid.

Rubber insulation is a compound of rubber with various other substances, the amount of new rubber varying from 20 to 60 per cent by weight. The various properties of rubber insulation are controlled by the selection and amount of the added ingredients, some of the more important being sulphur, litharge, zinc oxide, inert mineral matter such as talc, clay, whiting, etc., carbon black, mineral waxes, resins, reclaimed rubber, vulcanized oil, and other organic compounds.

Rubber insulation is affected by heat, air, and sunlight, the relative effects produced varying with the composition of the compound. Each compound when exposed to a critical temperature loses strength and becomes brittle, and as the temperature is further increased (100 C or higher) the insulation may melt and burn. Subsequent cooling does not restore any of the properties lost in heating. The effect of air and sunlight is to produce checking and loss of elasticity. By vulcanization, a heat process in which the insulation is subjected to temperatures between 248 and 304 F, to produce a chemical union between rubber and sulphur, the rubber compound is rendered more or less immune to deterioration by air and sunlight.

As generally used, the rubber compound is pressed around the bare or stranded wire, the thickness of the insulation varying with the voltage at which the wire is to be used. Rubber-covered wire is often covered with one or two thicknesses of cotton braid, the braid being subsequently saturated with pitch or an asphaltic material. The cost of rubber-covered wire is generally governed by the rubber market, as well as by the cost of manufacture. In smaller sizes of wire, say up to about No. 8, it is the cheapest for interior wiring.

Three grades of rubber insulation are at present generally accepted, known as Code, Performance, and Heat-Resistant, for operation respectively at maximum conductor temperatures of 50, 60, and 75 C. Code grade insulation is the cheapest and should not be used above 5000 volts; the other grades may be used at higher voltages, the thickness of insulation increasing with voltage.

Varnished cambric is a cotton fabric, coated with multiple layers of insulating varnish. For use on wire, the material is made up in the form of tape and wound spirally over the copper. Overlapping joints are staggered in successive layers, and every two or three layers the wrap is reversed. Between layers a film of oil of petroleum is applied, so that the layers may slip while the wire is being bent.

Varnished cambric does not absorb moisture as paper does, and is

generally more flexible; it is not injured by contact with lubricating or transformer oil, as is rubber. In general varnished cambric cable is more expensive than rubber-insulated cable in the smaller sizes, but cheaper in the larger sizes, the point of equal cost depending upon market conditions of the materials. It is not so good a heat conductor as rubber, but as it may be permitted to attain a maximum temperature of 85 C, the current-carrying capacity of a cable of given size insulated with varnished cambric may be greater than one insulated with rubber. It is thus often possible to use a smaller varnished cambric cable and thereby cause a reduction in the cost of conduit, hangers, etc.

The paper used for cable insulation is made of special wood pulp, and is wound spirally in a sufficient number of layers to the required thickness. The cable is then thoroughly dried, evacuated, and impregnated with some oily insulating compound and the whole enclosed in a lead sheath. Paper-insulated cable is much used above 5000 volts; its maximum operating temperature is 85 C.

Some plastic materials are used; they exhibit many of the general properties of rubber. In many cases there is a tendency for these materials to stiffen and crack at low temperatures and care must be exercised in their use. A wide variety of colors may be used with this material, making the tracing of wires very easy.

To protect cables from mechanical injury or water, they are given coverings of lead, steel wire or tape, treated cotton, asphalted jute or asbestos, rubberized cotton tape, braided fiber glass, hardened or reinforced rubber, vulcanized oil, treated paper, etc. Cotton braid saturated with an asphaltic material is the most usual covering for rubber-insulated conductors. For underground cables, either a sheath of lead or of other water-resistant material, such as vulcanized oil, asphalt-saturated jute or asbestos, is used. A lead sheath, either pure or alloyed with small amounts of tin, calcium, or antimony, is formed by pressing hot lead around the conductor, forming a continuous, close-fitting tube.

A list of the different insulations with their general characteristics as recognized by the National Electric Code is given in Table VIII of the Appendix.

When several layers of cotton braid, thoroughly impregnated with a black, weatherproofing compound, are put upon bare, hard-drawn copper wire, so-called "weatherproof wire" results. This wire is much used for outdoor overhead work, where great insulating strength is not required. It is a very cheap form of insulation.

In American practice, overhead lines carrying power at voltages greater than 2300 are not insulated at all; for the higher voltages, insulation which has been exposed to the weather is not reliable as a safeguard against shock, and so gives apparent security where there really is none. Hence

the wires of overhead power lines for voltages above 2300 are always bare.

28. Calculation of Simple Distribution Systems. Of the various types of circuits used in distribution systems, we shall consider only the more common forms.

(*a*) *Series System.* Find the size of wire necessary to supply 100 series incandescent lamps over a 5-mile series circuit. Each lamp requires 6.6 amperes at 30 volts. Allowable loss in voltage 3 per cent. (Fig. 3–19.)

FIG. 3–19.

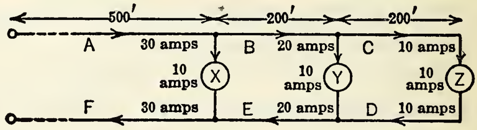


FIG. 3–20.

FIG. 3–19. In the series distribution system, used principally in street-lighting systems, but one wire is used, all the lamps being in series. The middle of the system is often connected to ground so that the potential varies progressively, above and below with respect to ground, as the source is approached. High voltages (possibly several thousand volts) may therefore result.

FIG. 3–20. In the more commonly used parallel distribution system the voltage to ground is practically the same at all points on one side of the circuit, but the current in the circuit varies widely at different points; in the series circuit of Fig. 3–19 the current is the same throughout the entire circuit, but the voltage to ground varies.

Let R = resistance of wire per 1000 feet.

Then Total voltage consumed = 3000;
Length of wire in thousands of feet $= 5 \times 5.28 = 26.4$;
Allowable voltage loss $= 3000 \times 0.03 = 90$;
Allowable IR drop $= 6.6 \times 26.4 \times R = 90$.

$$R = \frac{90}{6.6 \times 26.4} = 0.517 \text{ ohm}$$

According to the wire table, the nearest size of wire is No. 7.

(*b*) *Parallel or Multiple System.* Three loads, each of 10 amperes, are 500, 700, and 900 feet from the distribution center. Find the size of wire necessary to keep the drop within 5 volts, using wire of the same size throughout. (Fig. 3–20.)

Let R = resistance of wire per 1000 feet, in ohms.

Then
$$5 = I_AR_A + I_BR_B + I_CR_C + I_DR_D + I_ER_E + I_FR_F$$
$$5 = 2 \times 30 \times 0.5R + 2 \times 20 \times 0.2R + 2 \times 10 \times 0.2R$$
$$5 = 30R + 8R + 4R = 42R$$
$$R = \tfrac{5}{42} = 0.119 \text{ ohm}$$

Choose No. 0 wire ($R' = 0.09827$ ohm).

No. 1 wire would almost serve, its resistance being 0.1239, and would cause a maximum drop of 5.20 volts. Using No. 0 wire the actual drops to loads X, Y, and Z are:

Drop to $X = 2 \times 30 \times 0.5 \times 0.09827 = 2.948$ volts.
Drop to $Y = 2.95 + 2 \times 20 \times 0.2R' = 2.948 + 0.786 = 3.734$ volts
Drop to $Z = 3.734 + 2 \times 10 \times 0.2R' = 3.734 + 0.393 = 4.127$ volts

(c) *Anti-parallel System.* With the same three loads as in (b) connected now as in Fig. 3–21, determine size of wire necessary when allowing a maximum voltage drop of 5 volts at the middle load.

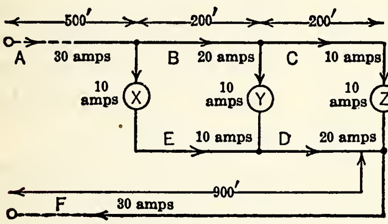


Fig. 3–21.

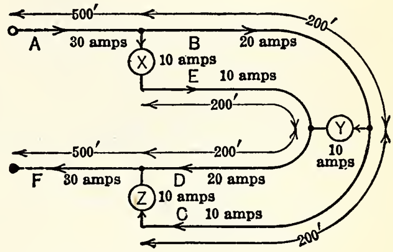


Fig. 3–22.

Fig. 3–21. In the so-called "anti-parallel" system of distribution there is less voltage variation throughout the circuit than if the straight parallel circuit of Fig. 3–20 is used, although more wire is generally used.

Fig. 3–22. If the loads of an "anti-parallel" circuit can be arranged so as to make the outgoing and return wires of equal length, wire is saved and less voltage variation among the loads obtained.

Let R = resistance of wire per 1000 feet, in ohms.

Then
$$5 = I_A R_A + I_B R_B + I_D R_D + I_F R_F$$
$$5 = 30 \times 0.5R + 20 \times 0.2R + 20 \times 0.2R + 30 \times 0.9R$$
$$5 = 15R + 4R + 4R + 27R = 50R$$
$$R = \tfrac{5}{50} = 0.100 \text{ ohm.}$$

Choose No. 0 wire ($R' = 0.09827$ ohm).
With No. 0 wire the actual drops to loads X, Y, and Z are:

$$\text{Drop to } X \text{ and } Z = 30 \times 0.5R' + 10 \times 0.2R' + 20 \times 0.2R' + 30 \times 0.9R'$$
$$= 15R' + 2R' + 4R' + 27R' = 48R'$$
$$= 48(0.09827) = 4.717 \text{ volts}$$

Drop to Y, from the solution, $= 50\,R' = 50(0.09827) = 4.914$ volts
Although the drops are greater, in the anti-parallel system they differ by only 0.20 volt, while in the parallel system, with copper of the same size, they differ by nearly 1.18 volts.

If the loads just considered could be rearranged as in Fig. 3–22 so as to

make the out-going and return wires equal in length with 5 volts drop allowed at the middle load, a smaller-sized wire is possible.

$$5 = 30 \times 0.5R + 20 \times 0.2R + 20 \times 0.2R + 30 \times 0.5R$$
$$5 = 15R + 4R + 4R + 15R \doteq 38R$$
$$R = \tfrac{5}{38} = 0.1316 \text{ ohm} \qquad \text{Choose No. 1 wire.}$$

(*d*) *Tapered Conductor.* Given three loads, as in Fig. 3–23. This figure shows the distribution by means of a *one-line diagram*, but it will be understood that actually two wires are required wherever one is shown; thus the length of wire causing IR drop between A and B is actually 400 feet. The allowed drop to each load is 5 volts. Find sizes of wires.

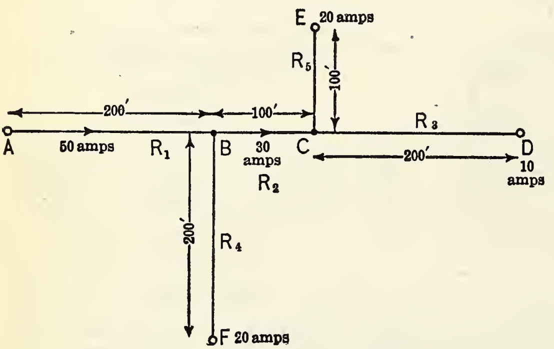


FIG. 3–23. In parallel networks of any appreciable magnitude, conductors of decreasing sizes are used at points more distant from the power supply. This sketch illustrates the ordinary scheme of showing circuit layouts, one wire only being shown where a pair is actually installed.

Assume the "drop per foot" to the most distant load to be uniform; this assumption, which gives uniform current density in the copper from the generator to the most distant load, results in a minimum amount of copper for a specified IR drop to this load. On this assumption, the drop in AB is equal to 2 volts, in BC it is 1, and in CD, 2. Then if the total drop to loads F and E is to be 5 volts, that in $BF = 3$ and in $CE = 2$.

Let R_1 = resistance of wire per 1000 feet between A and B;
R_2 = resistance of wire per 1000 feet between B and C, etc.

Then	$2 = 50 \times 2 \times 0.2R_1$	$R_1 = 0.1$	Choose No. 0 wire.
	$1 = 30 \times 2 \times 0.1R_2$	$R_2 = 0.167$	Choose No. 2 wire.
	$2 = 10 \times 2 \times 0.2R_3$	$R_3 = 0.500$	Choose No. 7 wire.
	$3 = 20 \times 2 \times 0.2R_4$	$R_4 = 0.375$	Choose No. 5 wire.
	$2 = 20 \times 2 \times 0.1R_5$	$R_5 = 0.500$	Choose No. 7 wire.

(*e*) *Feeder and Main.* Given five loads, each of 10 amperes, as in Fig. 3–24, the maximum drop to be 10 volts. Find size of feeder and mains.

Assume drop over feeder as 8 volts, leaving 2 volts drop for the mains. This is about the proportion used in practice.

Let R_1 = resistance of feeder per 1000 feet;
R_2 = resistance of main per 1000 feet.

Then $8 = 50\times 2R_1$ $R_1 = 0.080$ Choose No. 00.
$2 = 20\times 2\times 0.1R_2 + 10\times 2\times 0.1R_2$ $R_2 = 0.333$ Choose No. 5.

(*f*) *Double-end Feed.* In electric-railway systems the double-end feed is much used, the power in any section of the system being furnished from stations located along the right of way. Such stations are usually 6 to 12 miles apart.

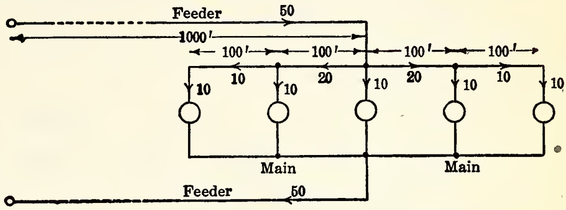


Fig. 3–24. A long feeder supplying a concentrated load; the connections between the individual load units are known as the mains.

In the system of Fig. 3–25, the trolley wire is No. 000 of 0.326 ohm per mile, and the track consists of two 80-pound rails well bonded. The resistance of such track is 0.0375 ohm per mile. With car currents and distances as indicated, find the current furnished by each generator and the voltage at each car.

Let I be the current furnished by generator X; the current furnished by generator Y is then $400 - I$. Assume all currents in trolley to be

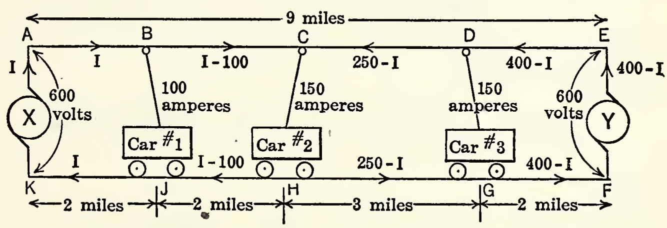


Fig. 3–25. In electric railway systems, stations generating direct current or converting alternating current into direct current are situated every 6 to 12 miles along the right of way.

flowing towards car No. 2. The current in sections BC and HJ is then $I - 100$ amperes, and that in sections CD and GH will be $400 - I - 150$, or $250 - I$.

Since a section of trolley, such as AB, and a section of track, as JK, are always in series, the drop in AB plus that in JK is

$$I \times 2(0.326 + 0.0375) = 2I(0.3635).$$

Letting $R = 0.3635$, the section drops are as follows:

$$AB + JK = 2IR \qquad CD + GH = (250 - I)3R$$
$$BC + HJ = (I - 100)2R \qquad DE + FG = (400 - I)2R$$

The voltage at car No. 2 may be calculated starting at either generator. Thus, from generator X,

$$\begin{aligned} E_2 &= 600 - I_{AB}R_{AB} - I_{JK}R_{JK} - I_{BC}R_{BC} - I_{HJ}R_{HJ} \\ &= 600 - 2IR - (I - 100)2R = 600 - 4IR + 200R \\ &= 672.70 - 1.454I \end{aligned}$$

Starting from generator Y,

$$\begin{aligned} E_2 &= 600 - I_{DE}R_{DE} - I_{FG}R_{FG} - I_{CD}R_{CD} - I_{GH}R_{GH} \\ &= 600 - (400 - I)2R - (250 - I)3R = 600 - 1550R + 5IR \\ &= 36.575 + 1.8175\,I \end{aligned}$$

Equating the two values for E_2,

$$672.70 - 1.454I = 36.575 + 1.8175I$$

from which

$$\begin{aligned} I = I_X &= 194.44 \text{ amperes} \\ I_Y &= 400 - 194.44 = 205.56 \text{ amperes} \\ I_{BC} &= 94.44 \text{ amperes} \\ I_{CD} &= 55.56 \text{ amperes} \end{aligned}$$

Voltage at car No. 1 is

$$E_1 = 600 - 194.44[2(0.326 + 0.0375)] = 458.64 \text{ volts}$$

Voltage at car No. 2 is

$$E_2 = 458.64 - 94.44 \times 2 \times 0.3635 = 389.98 \text{ volts}$$

Voltage at car No. 3 is

$$E_3 = 600 - 205.56 \times 2 \times 0.3635 = 450.56 \text{ volts}$$

From the last value the voltage at car No. 2 may be checked,

$$E_2 = 450.56 - 55.56 \times 3 \times 0.3635 = 389.97$$

29. The Three-wire System. In order to take advantage of the saving in copper by the use of higher voltages, and yet use low-voltage incandescent lamps, Edison devised the three-wire system of distribution. Originally, two 110-volt generators were put in series at the station and three wires led out, one from each outside terminal of the two generators and one from their common connection, as shown in Fig. 3–26.

By this arrangement, we have 110 volts between the middle wire, or *neutral,* and either outside wire, and 220 volts across the two outer conductors.

If there were no neutral, to transmit a given amount of power at 220 volts would require one-fourth the copper necessary for transmission at 110 volts. If the cross-section of the neutral wire is taken the same as that of the outside wires, then we should require 37.5 per cent of the copper required by a 110-volt system. In some three-wire distribution systems, the cross-section of the neutral is less than that of the outer wires, so that the saving in copper is even greater than this amount.

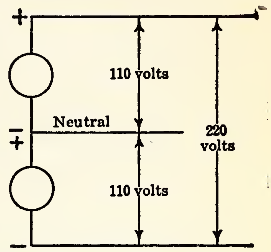


FIG. 3–26. Nearly all power for lighting loads is delivered at the customer's premises by the three-wire system; the voltage is twice as great between the two outside wires as between either outside wire and the middle, or neutral, wire. This neutral wire is generally grounded as a protection against shock.

The scheme of operation of the three-wire system is simple. If the load is exactly balanced, the neutral carries no current, as shown in Fig. 3–27. If the load on the lower leg be reduced to 2 lamps, the system is said to be unbalanced, as in Fig. 3–28, and the neutral will carry current. The neutral wire is maintained by the generators at a difference of potential of 110 volts from each of the outer wires, being thus negative with respect to the upper, and positive to the lower, wire. Since the voltage across the lamps is thus maintained at 110 volts (neglecting the effect of the IR drop in the wires), each lamp must still continue to take 5 amperes. Hence, the upper bank of lamps will require 20 amperes, and the lower bank 10 amperes. The difference must be taken care of by the neutral, the current in which will therefore flow toward the generators.

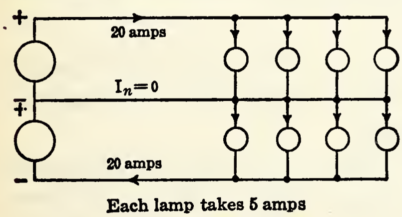


FIG. 3–27.

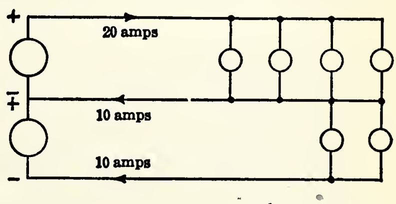


FIG. 3–28.

FIG. 3–27. In a well-maintained three-wire system the neutral wire carries practically no current, as the two loads are kept balanced by shifting various loads from one side to the other as required.

FIG. 3–28. In case of unbalanced loads, as illustrated here, the neutral may carry current; in this example the current in the neutral is towards the power supply.

If the unbalancing is reversed, as in Fig. 3–29, the current distribution must be as shown. The neutral again carries the difference between the two outer currents, but, as the lower leg requires more current than the

upper, its current is now out from the generators. The current in the neutral may thus flow in either direction, and is always the difference between those in the two outer conductors. Every effort is made to keep the amount of unbalancing as small as possible, by proper connection of the load. In house wiring, complete unbalancing is likely to occur, and for this reason the neutral is usually of full size. In distribution systems, however, by careful connection of the customers to the system, the amount of unbalance is very small.

Modern methods for maintaining a constant voltage in d-c three-wire systems will be discussed later.

The accidental opening of the neutral of a three-wire system, with balanced loads as in Fig. 3–27, would not be felt, but with unbalanced loads trouble is likely to occur. If the system in Fig. 3–29 is considered as having 110 volts across each side, and the feeders as having no voltage

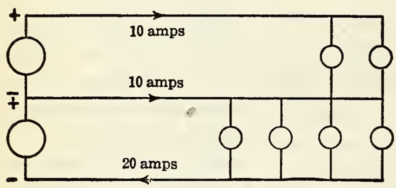


FIG. 3–29.

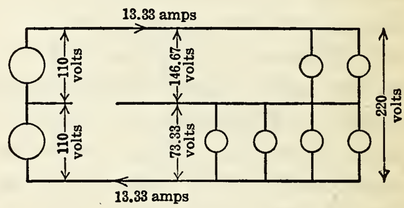


FIG. 3–30.

FIG. 3–29. Here the unbalance is the opposite of that shown in Fig. 3–28 and the direction of the neutral current is also reversed.

FIG. 3–30. In case the neutral wire opens on an unbalanced system a big difference in voltage on the two sides of the system may result, probably burning out the lamps on the lightly loaded side.

drops, the resistance of the two lamps together on the upper side taking 10 amperes would be $110/10 = 11$ ohms; the resistance of the four lamps on the lower side would be $110/20 = 5.5$ ohms.

If the neutral were opened somewhere between the supply and the loads, as in Fig. 3–30, 220 volts would be impressed on $5.5 + 11.0 = 16.5$ ohms, and a current of $220/16.5 = 13.33$ amperes would flow through the two loads in series. The drop across the upper lamps would be $13.33 \times 11 = 146.7$ volts, and across the lower ones, $13.33 \times 5.5 = 73.33$.

This calculation assumed that the resistance of the lamps would remain constant with other than their rated voltages, which, of course, is not the case. It is evident, however, that the upper bank of lamps would have impressed on it a higher voltage than normal, resulting in increased candlepower and shorter life, the reverse being true for the lower bank. The amount of voltage unbalance in such a case is dependent upon the extent to which the load is unbalanced.

To obviate the results of an open neutral, the neutral wire is not fused, and it is the custom to ground this wire both at the distribution centers and on each customer's premises. This practice also limits the voltage of a shock to which a person might be subjected in simultaneously touching any live parts of the wiring system and a ground connection, such as water pipes, radiators, or a bath-tub.

30. Calculation of Voltage Drop in Three-wire System. With loads connected as in Fig. 3–31, we shall calculate the voltage at each load. Outside wires are No. 4 (R = 0.25 ohm per 1000 feet) and neutral is No. 7 (R = 0.5 ohm per 1000 feet).

By inspection of the distribution of the various loads, it will be evident that the various lines must carry currents as indicated in the diagram. It

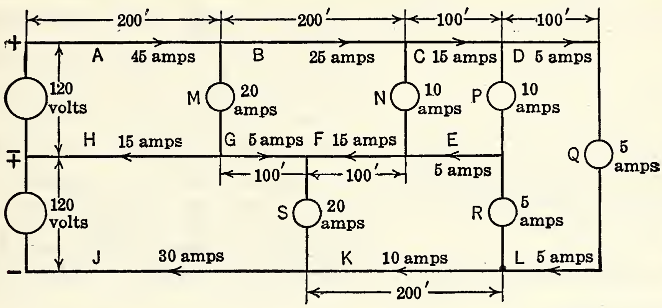


FIG. 3–31. A problem circuit, for the calculation of voltages at various points in the three-wire system.

is to be noted that in section G the current flows in the opposite direction from that in the other sections of the neutral.

Using Kirchhoff's first law, the section drops are determined as follows:

$$
\begin{aligned}
A &= 45 \times 0.2 \times 0.25 = 2.250 & G &= 5 \times 0.1 \times 0.50 = 0.250 \\
B &= 25 \times 0.2 \times 0.25 = 1.250 & H &= 15 \times 0.2 \times 0.50 = 1.500 \\
C &= 15 \times 0.1 \times 0.25 = 0.375 & J &= 30 \times 0.3 \times 0.25 = 2.250 \\
D &= 5 \times 0.1 \times 0.25 = 0.125 & K &= 10 \times 0.2 \times 0.25 = 0.500 \\
E &= 5 \times 0.1 \times 0.50 = 0.250 & L &= 5 \times 0.1 \times 0.25 = 0.125 \\
F &= 15 \times 0.1 \times 0.50 = 0.750
\end{aligned}
$$

$$
\begin{aligned}
\text{Voltage at load } M &= 120 - I_A R_A - I_H R_H \\
&= 120 - 2.250 - 1.500 = 116.250 \\
\text{Voltage at load } N &= 116.250 - I_B R_B - I_F R_F + I_G R_G \\
&= 116.250 - 1.250 - 0.750 + 0.250 = 114.500 \\
\text{Voltage at load } P &= 114.500 - I_C R_C - I_E R_E \\
&= 114.500 - 0.375 - 0.250 = 113.875 \\
\text{Voltage at load } S &= 120 + I_H R_H - I_G R_G - I_J R_J \\
&= 120 + 1.500 - 0.250 - 2.250 = 119.000
\end{aligned}
$$

$$
\begin{aligned}
\text{Voltage at load } R &= 119.000 + I_F R_F + I_E R_E - I_K R_K \\
&= 119.000 + 0.750 + 0.250 - 0.500 = 119.500 \\
\text{Voltage at load } Q &= 240 - I_A R_A - I_B R_B - I_C R_C - I_D R_D - I_L R_L \\
&\qquad - I_K R_K - I_J R_J \\
&= 240 - 2.250 - 1.250 - 0.375 - 0.125 - 0.125 \\
&\qquad - 0.500 - 2.250 = 233.125.
\end{aligned}
$$

The problem indicates one of the disadvantages of the three-wire system; the voltage on the loaded side falls, while that on the unloaded side may even rise. This is accounted for by the fact that if the upper side is more heavily loaded, so that the unbalanced current flows from right to left over the neutral, there must be a drop in voltage along the neutral from right to left to cause the unbalanced current to flow. This drop of potential in the neutral tends to offset the drop occurring in the lower wire JK; in case the drop in $EFGH$ is greater than the drop in JK, the voltage at load R will be greater than either the generator voltage or that across load S. This occurs, however, only with badly unbalanced loads.

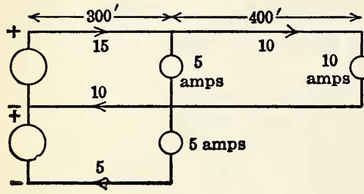


FIG. 3–32. A problem circuit, for calculating the proper sizes of conductors, the allowable drop to the various loads being specified.

As a check on the figures obtained in the above problem, we can find the voltage across P and R in series

$$
\begin{aligned}
E_{PR} &= 240 - I_A R_A - I_B R_B - I_C R_C - I_K R_K - I_J R_J \\
&= 240 - 2.250 - 1.250 - 0.375 - 0.500 - 2.250 = 233.375
\end{aligned}
$$

From our first calculations

$$E_{PR} = E_P + E_R = 113.875 + 119.500 = 233.375$$

As a further check

$$E_Q = E_{PR} - I_D R_D - I_L R_L = 233.375 - 0.125 - 0.125 = 233.125$$

Calculation of Size of Wire. Given three loads, as shown in Fig. 3–32, assume maximum drop to be 5 volts; neutral wire of same size as others.

Let R = resistance of wire per 1000 feet.

Then
$$
\begin{aligned}
5 &= 15 \times 0.3R + 10 \times 0.4R + 10 \times 0.7R \\
5 &= 4.5R + 4R + 7R = 15.5R \\
R &= \frac{5}{15.5} = 0.323. \quad \text{Choose No. 5 wire.}
\end{aligned}
$$

PROBLEMS

3–1. A tungsten lamp has resistance of 440 ohms when hot. How much current does it draw from a 110-volt line, and how much power is used? What is the conductance of the lamp?

3–2. If we neglect losses due to radiation and convection, how long will it take to heat 5 gallons of water from 60 F to boiling, if the power being supplied to heat the water is 4.75 kw? How many joules are supplied? How many calories are supplied?

3–3. If, in the above problem, losses due to radiation and convection are equal to the energy used in raising the water temperature, how much does the boiling of the water cost, if power is worth 4 cents per kwhr?

3–4. How much does it cost to heat 2300 gallons of water (in a tank having good insulation), from 50 F to 190 F, the rate for electric energy being 5 cents per kwhr for the first 30 kwhr and 2 cents per kwhr thereafter?

3–5. An electric automobile weighing 2600 pounds is moving up a 10 per cent grade at a speed of 20 miles per hour. The force required to overcome friction due to tires, gears, etc., at this speed, is 120 pounds. What power is being developed by the motor? At what rate is energy being wasted in heat, and at what rate stored?

3–6. A carbon-filament incandescent lamp has a cold resistance of 392 ohms. When burning on a 120-volt line, the hot resistance is 288 ohms. How many amperes and watts are drawn by the lamp at the instant it is connected to a 120-volt source? What are the final values of current and power?

3–7. The field rheostat of a generator has a maximum resistance of 42 ohms, and the field with which it is placed in series across a 115-volt line is 200 ohms. Through what range will the field current change as the arm of the rheostat is moved from the position of maximum resistance to that of zero resistance? How much power is lost in the rheostat at each current?

3–8. How much resistance must be placed in series with the 10-ohm incandescent lamp of a projector so that, when the circuit is connected to a 115-volt source, the voltage across the lamp shall be 50 volts?

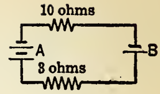
3–9. What will it cost to operate the circuit of problem 3–8 for four hours, if power costs 4 cents per kwhr?

3–10. A charging current of 15 amperes is being sent through a storage battery which has a resistance of 0.16 ohm and 12 volts open-circuit voltage. At what rate is energy being wasted, and at what rate stored?

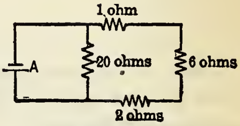
3–11. If 70 per cent of the ampere-hours used in charging the above battery is available for discharge, how long must the charging current be kept on if the battery is to furnish 8 amperes for 12 hours? How many calories of heat are wasted during discharge?

3–12. A storage battery, being charged at the rate of 12 amperes, requires an impressed voltage of 13.6 volts. When 12 amperes discharge current are drawn, the terminal voltage is 12.85. What is the internal resistance of the battery?

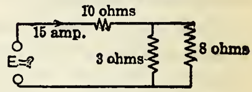
3–13. The generator of an automobile is furnishing 10 amperes to head and tail lights and is sending 8 amperes into the storage battery. The internal resistance of the generator itself is 0.1 ohm and its terminal voltage is 6.8 volts. Resistance of the storage battery is 0.042 ohm. What is the resistance of the lamp load? Open-circuit voltage of the battery? Power used in the generator as I^2R loss?



3-14. Emf of $B = 2.1$ volts. Resistance of $B = 0.16$ ohm. Emf of $A = 6.2$ volts. Resistance of $A = 0.22$ ohm. How much heat is generated in B in 1 hour and 20 minutes? What is the voltage across B?

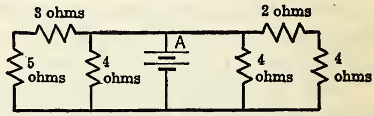


3-15. Emf of $A = 60$ volts. Resistance of $A = 0.82$ ohm. How much current flows through it, and what is the voltage across its terminals?



3-16. How much voltage is required to force 15 amperes through this circuit?

3-17. Fifteen 500-ohm lamps and twelve 350-ohm lamps are in parallel. Resistance of circuit between lamps and generator is 0.36 ohm; voltage at the generator is 118. Find the voltage across the lamps.



3-18. Emf of A is 50 volts. Resistance of A is 0.65 ohm. How much current does the battery supply, and what is the voltage across its terminals? (Simplify diagram.)

3-19. A parallel bank of 50 lamps is supplied by a generator over conductors, the resistance of which is 0.6 ohm together. If the total current is 15.5 amperes and the voltage at the generator is 133.3 volts, what is the resistance of each lamp?

3-20. There are three resistances, A, B, and C. When A and B are in series across a 120-volt source, the current is 6.67 amperes; when B and C are in series, the current is 8.57 amperes; and when C and A are in series, the current is 7.50 amperes. What are the values of the three resistances?

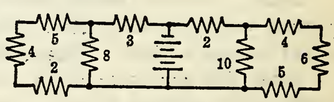
3-21. Show that a battery is delivering maximum power to a load when the resistance of the load equals the internal resistance of the battery. What will be the terminal voltage of the battery at that time?

3-22. Resistance A is 12 ohms. When a resistance B is placed in parallel with A, across a 120-volt source, the combination draws 30 amperes. What is the resistance of B? How much power is consumed in each resistance?

3-23. When resistances X and Y are in series across a 120-volt source, a current of 15 amperes flows. When resistance Z is added to the combination, in parallel with Y, the current is 30 amperes. What is the value of resistances Y and Z, if X is 2 ohms?

3-24. A current of 10 amperes flows through a combination of resistances R, S, and T, in parallel. The resistances of R and S are 30 and 7.5 ohms, respectively. What must be the resistance of T if its current is 5 amperes? What value of voltage was impressed?

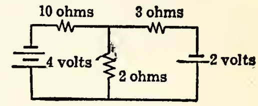
3-25. The resistance of a track of two well-bonded 80-pound rails is 0.375 ohm per mile. What will be the voltage drop in millivolts over a distance of 20 feet, if the track is carrying 400 amperes? (The two rails of a track are electrically in parallel. In well-bonded track the resistance of the rail joints is neglected.)



3-26. If the battery supplies 24.06 amperes, how much current flows in the 3-ohm resistance? In the 10-ohm resistance? What is the voltage across the battery? Resistances are in ohms.

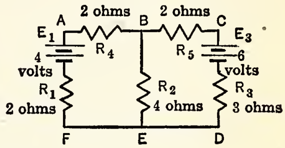
3-27. A resistance of 1.7 ohms is connected across two batteries, A and B, which are themselves connected in parallel. The emf of A is 2.1 volts, and its resistance is 0.08 ohm; the emf of B is 1.9 volts, and its resistance is 0.016 ohm. How much current is supplied by each battery?

3–28. Batteries A and B and a 10-ohm resistance are all in parallel. The open-circuit voltages are 50 and 60 volts respectively. How much resistance must be placed in series with B, so that battery A will have zero current? What current will B then deliver?



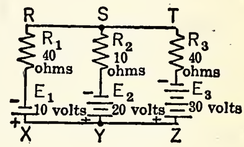
3–29. What is the current in the 2-ohm branch, and in what direction does it flow?

3–30. What will be the answers to problem 3–29 if the right-hand battery has its polarity reversed?



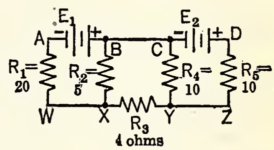
3–31. What will be the current flowing in R_2 and in what direction will it flow?

3–32. What will be the answers to problem 3–31 if the polarity of the right-hand battery is reversed?



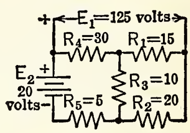
3–33. Solve for the three currents and state the direction in which they flow.

3–34. What will be the answers for problem 3–33 if the battery E_1 had its polarity reversed?



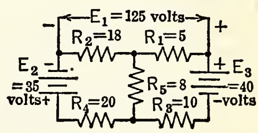
3–35. The voltage of battery E_1 is 125 volts and that of battery E_2 is 100 volts. What are the currents and in which direction do they flow? Resistances are in ohms.

3–36. If battery E_2 has its polarity reversed, what will be the answers to problem 3–35?



3–37. Solve for all currents and state their direction. What is the voltage across R_3? Resistances are in ohms.

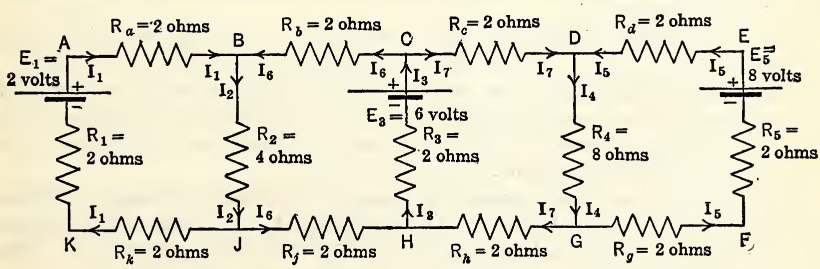
3–38. What will be the answers to problem 3–37 if the polarity of E_2 is reversed?



3–39. What is the current in each resistance and in what direction does each flow? Resistances are in ohms.

3–40. If battery E_2 had its polarity reversed, what would be the answers to problem 3–39?

3–41. Solve for all currents in the following complicated network.



3–42. How many circular mils in a bus-bar 1 inch by ⅜ inch? In a copper bar ¼ inch square?

3–43. What must be the diameter of an aluminum wire which has a resistance of 0.24 ohm per mile at 0 C?

3-44. A hard-drawn copper wire, of 0.125 inch diameter, has a resistance of 2.26 ohms at 30 C. What is its length?

3-45. It is desired to measure the current in a rectangular copper bus-bar, 2 inches by ¼ inch, using a millivoltmeter which draws 0.15 ampere at 0.050 volt to give full-scale deflection. How far apart must the millivoltmeter contacts be placed on the bar to have full-scale deflection when 500 amperes are flowing in the bar? Temperature of the bar is 27 C.

3-46. Calculate the resistivity of standard copper in ohms per circular mil-foot at 47.5 C.

3-47. The resistance of a certain tungsten lamp, when lighted, is 220 ohms, the temperature being 2200 C. What is the resistance of the lamp at 20 C? Assume that the temperature-resistance coefficient for tungsten is constant up to 2200 C.

3-48. When the lamp of problem 3-47 is first connected to a 110-volt line, how many watts of power are taken by it? How many when it is burning?

3-49. Twenty-five feet of Nichrome V wire are used in making a toaster which draws 7.3 amperes from a 110-volt line, its temperature being 1000 F. What is the diameter of the wire? Use curve of Fig. 3-16.

3-50. How much No. 20 Nichrome V wire must be used to make the heating element of a flatiron which is to consume 550 watts from a 110-volt line, the operating temperature of the heating element being 500 F? How much power does it draw from the line when first connected? Use curve of Fig. 3-16.

3-51. The resistance of the field coils of a generator is 22.3 ohms at 19.6 C. After the machine has operated for several hours, the resistance of the coils is found to be 29.2 ohms. What is the temperature of the coils?

3-52. What is the resistance per 1000 feet at 20 C of a No. 0 copper-clad steel wire, the area of the copper being 31.8 per cent and that of the soft-steel core 68.2 per cent of the total? What per cent conductivity does the wire have compared to a solid copper wire of the same size?

3-53. Solve problem 3-52 for 0 C and determine the temperature-resistance coefficient (referred to 0 C) for the copper-clad wire of the proportions given, between 0 and 20 C.

3-54. How many feet of No. 18 Nichrome V wire must be used to make the heating element of a certain device which is to consume 600 watts from a 115-volt line? Temperature of the wire is to be 1200 F. Use curve of Fig. 3-16.

3-55. At 20 C the field coils of a small 110-volt motor take 0.186 ampere. In use, with the surrounding air at a temperature of 110 F, the motor field current was 0.142 ampere, when operating on a 110-volt line. How many degrees C are the coils above the surrounding air?

3-56. If an armature has a resistance of 0.0162 ohm at 18.5 C, what will its resistance be at 80 C?

3-57. When measured at a temperature of 19.2 C the field coils of a small 115-volt motor required a current of 0.18 ampere. After several hours operation on 115 volts, in a room temperature of 40 C, the motor field current was 0.14 ampere. How many degrees above room temperature are the coils?

3-58. A cable consists of 61 strands of wire, each 0.112 inch in diameter. Of these 19 strands are of bronze and the rest of standard copper. If bronze has 23 per cent the conductivity of copper, find the resistance of 1000 feet of this cable at 20 C. What percentage of the total current carried by the cable will flow in the bronze?

3-59. A bare steel-cored aluminum cable consists of 7 strands of No. 12 soft-steel wire, surrounded by 42 strands of No. 12 aluminum wire. The cable is to be operated at 120 F. What is the resistance per 1000 feet and the current division between aluminum and steel?

3–60. If copper were substituted for aluminum in problem 3-59, what would be the answers to that problem? How much more current could the steel-cored copper cable carry for the same drop?

3–61. A cable using rubber insulation of the grade described in Fig. 3-18 has an insulation resistance of 400 megohms at 60 F. What value would be expected at 75 F?

3–62. The insulation resistance of a certain machine using varnished cambric insulation is 55 megohms at 15.5 C. How much will it probably be at 35 C? See Fig. 3–18.

3–63. One hundred kilowatts of power is to be transmitted 10 miles, the voltage at the generator being 240 volts, and 10 per cent line drop being allowed. What size wire must be used, and how many watts of power are used in the line? Would the line be feasible?

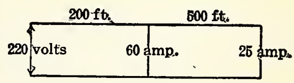
3–64. How many kilowatts of power can be transmitted over an aluminum cable, 750,000 cir mils area, 60 miles long, if the voltage at the station is 45,000 volts and 10 per cent line drop is allowed? How much could be transmitted if copper cable of the same cross-section were used?

3–65. A factory motor, requiring 100 amperes, is 50 feet away from a 120-volt generator. What size wire should be used if the drop to the motor is not to exceed 5 per cent of the generator voltage?

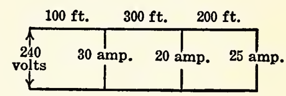
3–66. A motor is connected to a generator 7500 feet away, by copper conductors. The voltage at the generator is 250 volts and, when the motor is drawing 19.76 amperes, the voltage at the motor is 220 volts. The temperature of the conductor is 27.5 C. Calculate the circular mil area of the conductor. What size is the conductor?

3–67. If copper costs 8 cents per pound, What is the cost of the copper for the conductors of problem 3-66? Does it seem economical?

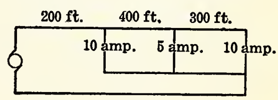
3–68. What size wire must be used if the voltage at the 25-ampere load is to be 200 volts?



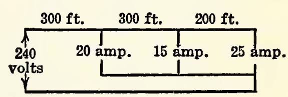
3–69. What size wire must be used if the voltage at the 25-ampere load is to be 220 volts?



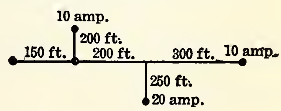
3–70. Drop to the middle load is 8 volts. What size wire is used?



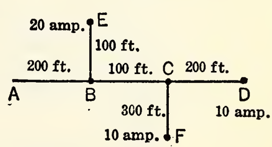
3–71. What size wire must be used if the voltage at any load is not to be less than 220 volts?



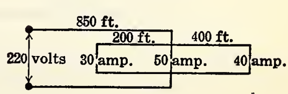
3–72. The allowed drop to each load is 12 volts. Find sizes of wire, using proper taper.



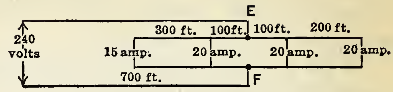
3–73. Find sizes of wire, using proper taper, if the allowed drop to each load is 10 volts.



3–74. Maximum drop allowed is 10 per cent, of which 7 per cent is in the feeder. Find proper size of feeder and main, and calculate voltage at each load.

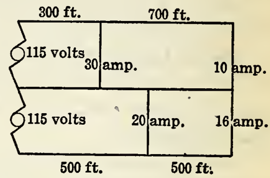


3–75. If no one load is to operate at less than 220 volts, and 15 volts is the maximum allowed drop in the feeder, find proper sizes of feeder and mains, and calculate voltage at each load.



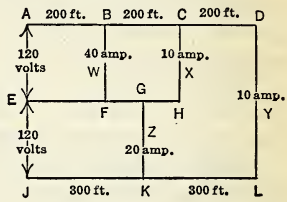
3–76. Outside wires No. 1 and neutral No. 4. Find voltage at each load.

3–77. If the current in the 10-ampere load of problem 3–76 were increased to 36 amperes, what would be the voltage at each load?

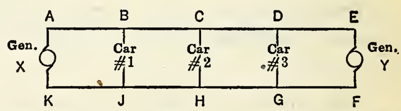


3–78. Outside wires No. 0 and neutral No. 3. Find voltages at each load.

3–79. If the current in load Z of problem 3-78 increased to 50 amperes, what would be the voltage at each load?



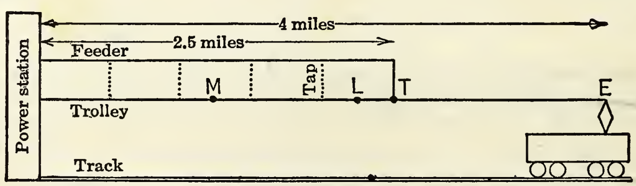
3–80. In diagram of a double-end feeder system, distances in miles are as follows: $AB = 2$, $BC = 3$, $CD = 3$, $DE = 2$. Car 1 draws 75 amperes, car 2 draws 150 amperes, and car 3 requires 100 amperes. Trolley wire is No. 000 of 0.326 ohm per mile and track consists of two well-bonded 80-pound rails. Resistance of track is 0.0375 ohm per mile. Each generator furnishes 600 volts. Find current supplied by each generator and voltage at each car.



3–81. In diagram, distances in miles are: $AB = 3$, $BC = 2$, $CD = 3$, and $DE = 3$. Car 1 takes 100 amperes, car 2 draws 125 amperes, and car 3 requires 150 amperes. Each generator supplies 600 volts, and trolley and track are as in problem 3–80. Find voltages at each car and the current supplied by each generator.

3–82. Solve problem 3–81 if the voltage of generator X is 600 volts and that of generator Y is 550 volts.

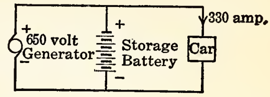
3–83. As shown in following figure, an electric railway is fed by a 4-mile, No. 0000 trolley wire of hard-drawn copper, resistance of which is 0.269 ohm per mile. A 250,000 cir mil feeder, 2.5 miles long, parallels the trolley. The resistance of the track is 0.038 ohm per mile. Considering that the feeder is connected to the trolley only at its end (disregard dotted taps), calculate the voltage at a car, drawing 150 amperes, when it is (*a*) at the end of the line, E, (*b*) at the tapping point T of the feeder, and (*c*) at the point M, midway between the power station and point T. Power station voltage is 600 volts.



3–84. Consider that the feeder of problem 3–83 is tapped into the trolley every half mile (shown by dotted lines). Calculate the voltage at the car when the latter is

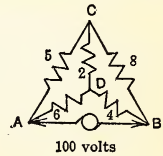
respectively at points E, T, L, and M. Point L is midway between point T and the next tap.

3–85. Distance from generator to battery is 7 miles, and from battery to car is 3 miles. Resistance of battery is 0.63 ohm, and its emf is 600 volts. Resistance of feeders, including rail return, is 0.12 ohm per mile. How much current is the storage battery delivering, and what is the voltage at the car?



3–86. In problem 3–85 to what value would the voltage of the generator have to be raised in order that the battery supply 100 amperes, the car still taking 330 amperes? To what value would the voltage of the generator have to be raised in order that the battery supply no current to the car? What would be the value of the voltage at the car in each case?

3–87. Find the current supplied by the generator. What is the resistance between A and B? The figures indicate resistances in ohms.



CHAPTER IV

SELF-INDUCTION AND MUTUAL INDUCTION

1. Coil Moving in a Magnetic Field. When the case of a coil moving in a magnetic field was considered (page 39), the conclusion was reached that when the flux threading a coil is changing voltage is induced, in amount proportional to the rate of change of interlinkages between flux and turns. If the coil is part of a closed circuit the induced voltage causes a current to flow, and the action of this resultant current is always such that it tends to prevent any change in the existing number of interlinkages. Thus the act of moving a coil into a field establishes a certain number of flux interlinkages; conversely, the act of moving a coil out of a field destroys a certain number of flux interlinkages. In either case voltage is induced in such a direction that the current set up by it develops electrical and mechanical reactions which oppose the change taking place in the number of interlinkages. The electrical reaction comes about in that the current develops a magnetomotive force which opposes or assists that setting up the original field into which the coil is respectively introduced or from which it is being removed.

The act of establishing or destroying flux interlinkages in a coil or circuit is called *induction.* It is logical to reason that if flux interlinkages are established or destroyed in a circuit *by any method other than that of moving the coil into or out of a field, voltage of the same value and direction must be induced.*

2. Self-induction. When the current flowing in an electric circuit varies with time, any magnetic field set up by the current will correspondingly vary with time. Consider a turn of wire; if current is passed through the turn, as in Fig. 4–1*a*, being made to increase uniformly to a final value I, by the application of a voltage E, a uniformly increasing magnetic field, of final value Φ, will be set up. The current has been chosen so as to set up a field through the turn in a direction from left to right, as indicated by dotted lines. The increasing current thus establishes a definite number of flux interlinkages just as motion of the turn does in Fig. 4–1*b*, or in Fig. 2–26, page 40.

From Fig. 4–1*b*, it follows that a voltage must be induced, which, by application of the rule given in section 21, page 37, is seen to be out of

the paper in the lower turn-side. Since flux interlinkages are being established in the same direction in the turn of Fig. 4–1*a*, voltage must be induced, which, in its lower turn-side, must be in the same direction as that shown in Fig. 4–1*b*. This induced voltage, E_L, is called the *voltage of self-induction*, and is obviously acting in a direction opposite to that of the impressed voltage, E, thus resulting in a decrease in the net voltage of the circuit, E–E_L. The current at any instant is proportional to the net voltage acting in the circuit, and it is therefore evident that the counter voltage which is due to the rate of change of current tends to limit the increase of current.

The whole phenomenon is analogous to a body having inertia being moved at a changing velocity. As the body is accelerated a reaction ap-

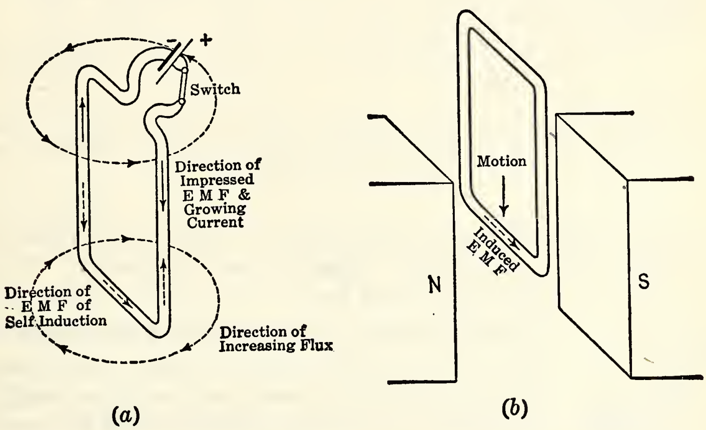


Fig. 4–1. As the current in a coil increases from zero, flux is set up in the circuit, establishing interlinkages; the voltage produced thereby is the same as though the interlinkages had been established by moving the coil into the field of a permanent magnet.

pears opposing the force causing the acceleration. This reaction is present only as long as the body is being accelerated, i.e., its velocity changed, and disappears when its velocity becomes constant.

The whole effect in the electrical circuit is in accord with the idea expressed by Lenz's law; the original increase in current strength which produces the increase in flux interlinkages immediately brings into existence an opposing or counter voltage, which tends to retard the increase in current, thereby opposing the increase in flux interlinkages.

Conversely, when the current dies down within the turn, the number of flux interlinkages decreases, just as if the turn were moved out of the field. From Fig. 4–2*a*, it will be seen that the decrease in current strength, which causes a reduction in interlinkages, at once induces a voltage. This, being in the same direction as the impressed voltage, tends to retard the

decrease in current, thereby again opposing the change taking place in the flux interlinkages.

When the turn of Fig. 4–1*b*, is moved into the field in the direction

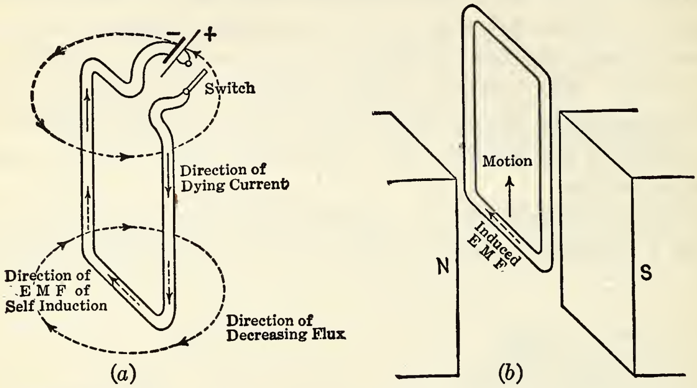


Fig. 4–2. When the current in a coil decreases, the voltage of self-induction acts to aid the impressed emf, thus tending to prevent the current decrease, to which the voltage of self-induction is due.

shown, its lower turn-side cuts the flux, or, in other words, flux interlinkages are being established only along the lower turn-side. Voltage is therefore being induced only in the lower turn-side. However, when an increasing current is passed around the turn, as in Fig. 4–1*a*, the flux interlinkages set up are due to the current; and, as the strength of the current at any instant is uniform over the entire turn, it follows that the flux interlinkages are being set up uniformly about the entire length of the turn (Fig. 4–3). Hence the generation of voltage of self-induction must be uniform along the entire length of the turn; in other words, each element of the turn is assisting in the generation of voltage of self-induction. If the turn is not circular, the above statement is not quite true; the voltage generated per unit of length is greatest where the curvature of the turn is greatest.

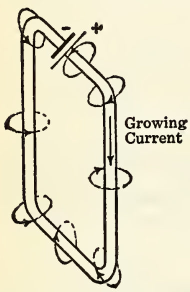


Fig. 4–3. The voltage of self-induction is set up nearly uniformly throughout the length of the turn; each centimeter of length develops nearly the same amount of voltage of self-induction.

Any circuit capable of setting up a magnetic field when current flows in it is said to be *inductive*, or to possess inductance. Whenever the current in an inductive circuit changes, a counter voltage of self-induction will be generated, which lasts as long as the current (and flux) varies, and always has such direction as to oppose the change in current (and flux) taking place. This property of a circuit, by virtue of which

it tends to prevent a change of current (and flux) *in itself*, is called its *self-induction*.

3. Voltage of Self-induction. Consider again the toroid of Fig. 2–35, page 49, of N turns wound on an iron core of permeability μ, of mean radius r centimeters, and of cross-section A square centimeters. When a steady current of I amperes flows through the toroid the value of the flux set up is, from Eq. (37), page 52,

$$\Phi = \frac{0.4\pi NI\mu A}{l} \tag{1}$$

If the current changes uniformly at a rate of I/t amperes per second, and the permeability of the iron is constant for the flux change considered, the flux will correspondingly change at a uniform rate of Φ/t lines per second. Since the flux is changing at this rate through N turns in series, the average value of the voltage of self-induction will be

$$E_{\text{av}} = \frac{\Phi N}{t\, 10^8} \text{ volts} \tag{2}$$

Substituting for Φ its value above, we have

$$E_{\text{av}} = \frac{0.4\pi N^2 \mu A}{l\, 10^8} \cdot \frac{I}{t} \text{ volts} \tag{3}$$

That the counter voltage of self-induction varies as the square of the number of turns follows because the flux set up by a given current depends directly upon the number of turns, and so the voltage induced *per turn* (for a given rate of change of current) is directly proportional to the number of turns in the coil. There being N turns connected in series, and thus adding their respective voltages, the total voltage of self-induction will vary as the square of the number of turns.

As noted in section 42, page 64, the part $0.4\pi N^2\mu A/l\, 10^8$ is a constant (if μ is constant) and is called the *coefficient of self-induction*, or merely the *self-inductance* of the circuit, and is represented by the symbol L. Thus

$$L = \frac{0.4\pi N^2 \mu A}{l\, 10^8} = \frac{0.4\pi N^2}{\mathcal{R}\, 10^8} \tag{4}$$

where $\mathcal{R} = l/\mu A$ is the reluctance of the magnetic circuit. The coefficient L obviously depends upon the physical dimensions of the circuit and the value of μ. If no iron is used in the construction of the coil, the flux set up will vary directly with the current. When iron is used, permeability and therefore self-inductance will change with current.

By reference to the magnetization and permeability curves of Fig. 2–37, page 54, it is evident that for the very low values of current the value of

L will increase with increase of current. For higher values of current, when saturation begins, the permeability will decrease and L as well.

In place of Eq. (3) the shorter form may be written,

$$E_{av} = L\frac{I}{t} \text{ volts} \tag{5}$$

where I/t is the average rate of change of current.

If the rate of change of current in an inductive circuit is not uniform, but varies from instant to instant, the voltage of self-induction will likewise vary from instant to instant. Accordingly we must write, in terms of instantaneous current and voltage, that

$$e = -L\frac{di}{dt} \text{ volts} \tag{6}$$

Equation (5), as it has been written, expresses only the magnitude of the voltage. Equation (6) has been written with a negative sign to indicate that the voltage is a reaction or counter voltage; that is, if the current i is increasing, making di/dt positive, the voltage e acts in the circuit so as to *oppose the increase of current.* This means that e is *opposite* in direction to the voltage which is causing i to flow in the circuit. If the current is decreasing, making di/dt negative, the voltage e will be in the same direction as that causing i to flow.

4. Unit of Self-inductance. The practical unit of self-inductance is called the *henry;* it is obtained by making all the terms in Eq. (5) equal to unity. The henry is thus the self-inductance of a circuit of such form that a counter voltage of one volt is induced by a rate of change of current of one ampere per second.

Another definition, which perhaps gives a better concept of self-induction, is derived through the use of Eqs. (2) and (5). Since

$$E_{av} = L\frac{I}{t} \quad \text{or} \quad L = \frac{E_{av}\,t}{I}$$

substituting for E_{av} its value $\Phi N/t\ 10^8$,

$$L = \frac{\frac{\Phi N}{10^8}}{I} = \frac{\Phi N}{I} \times 10^{-8} \tag{7}$$

From this equation the henry may be defined as the amount of self-inductance possessed by a circuit in which one ampere is capable of setting up 10^8 flux interlinkages. If the current changes at a rate of one ampere per second, the flux interlinkages will change at a rate of 10^8 per second, and so generate a counter voltage of one volt.

It was shown on page 64 that one henry is equal to 10^9 abhenrys.

5. Iron-core Coils. For problems involving air-core coils, the coefficient of self-induction of which is constant, any of the above equations involving L may be used. But in the case of coils containing iron, these equations do not always serve.

From the definition of unit self-inductance as the amount of self-inductance possessed by a circuit in which one ampere is capable of setting up 10^8 interlinkages, it will be realized that the self-inductance of a coil with iron in its magnetic circuit will vary with the current.

Consider again the rectangular core of Fig. 2–40, page 57. To set up 700,000 lines or a flux density of 14,000 lines per sq cm required 10.8 ampere-turns per cm of length. With a total length of path of 116 cm, 1252.8 ampere-turns were needed. As there were 110 turns in the coil a current of 11.39 amperes was required.

The self-inductance of the coil from Eq. (7) would be

$$L = \frac{\phi N}{10^8 I} = \frac{7 \times 10^5 \times 110}{10^8 \times 11.39} = 0.0676 \text{ henry}$$

Suppose that the total flux was raised to be 750,000 lines or the flux density 15,000 lines per sq cm. This would require 22 ampere-turns per cm or a total of $22 \times 116 = 2552$ ampere-turns. The required current would be $2552/110 = 23.20$ amperes.

Now the self-inductance of the coil is

$$L = \frac{75 \times 10^4 \times 110}{10^8 \times 23.20} = 0.0356 \text{ henry}$$

Thus, as the iron of the magnetic circuit becomes more highly saturated, the self-induction of the coil decreases; the flux per ampere becomes less.

If it was required to calculate what average voltage of self-induction would be generated within the coil just considered, if the current decreased from 11.39 to 0 ampere in 0.03 second, either Eq. (2) or Eq. (5) could be used.

$$\text{From Eq. (2)} \quad E_{\text{av}} = \frac{7 \times 10^5 \times 110}{0.03 \times 10^8} = 25.67 \text{ volts}$$

$$\text{From Eq. (5)} \quad E_{\text{av}} = \frac{0.0676 \times 11.39}{0.03} = 25.66 \text{ volts}$$

But if we want to know what average voltage of self-induction would be generated within the coil if the current changed from 11.39 to 23.20 amperes in 0.03 second, we must calculate it on the basis of the flux change, using Eq. (2). With 11.39 amperes the total flux is 70×10^4 lines; with 23.20 amperes it is 75×10^4 lines.

Thus the average voltage is

$$E_{av} = \frac{(75 - 70)10^4 \times 110}{0.03 \times 10^8} = 1.83 \text{ volts}$$

It may be remarked that the voltage here generated is low, but it must be noted that, although the current is doubled, the flux change is small; the iron is nearly saturated to begin with.

In this calculation, Eq. (5) cannot be used. The value of L is also a variable in this problem, and we do not know how L changes with I. If we had some mathematical relationship which told us how L changed with I, it might be possible to solve the problem, but it would be a complicated calculation.

It might be suggested that we use the mean value of L in Eq. (5), as follows:

$$E_{av} = \frac{0.0676 + 0.0356}{2} \times \frac{23.20 - 11.39}{0.03} = 20.31 \text{ volts}$$

but this answer is obviously incorrect.

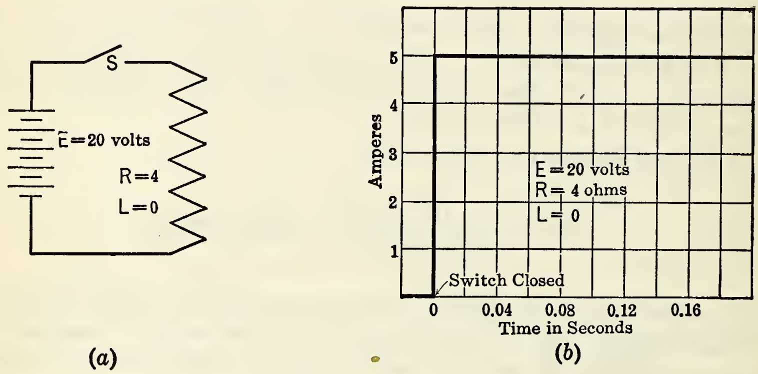


FIG. 4–4. In a circuit containing no self-induction the current rises to its final value the instant the switch is closed.

In conclusion it may be stated that, for solving problems involving iron-core coils, the use of Eq. (2) is always correct, but that the use of equations involving L depends upon whether or not L has a variable value.

6. Growth of Current in an Inductive Circuit. If a voltage is impressed upon a circuit possessing no inductance, the current rises instantaneously to a value as determined by Ohm's law. In Fig. 4–4 is shown the rise in current when 20 volts is impressed on a circuit of 4 ohms resistance, by closing of the switch S; the current rises at once to a value of 5 amperes.

When voltage is impressed on a circuit possessing inductance, and the current starts to increase, the change of flux interlinkages induces a voltage of self-induction which opposes the increase of the current.

Consider a circuit possessing inductance and resistance as in Fig. 4–5*a*, in which closing of the switch S impresses a voltage E on the circuit. Just before the closing of the switch, the current is necessarily zero and is not changing, and no voltage of self-induction exists; but as soon as the switch is closed the current immediately starts to rise. Now the rate with which the current will increase is obviously fixed by the amount of voltage available for the purpose; at the instant of closing the switch there is no appreciable IR drop in the circuit, and so the entire impressed voltage is available for producing current increase; the current, therefore, begins rising at a high average rate, as shown by the line *oa* in Fig. 4–5*b*.

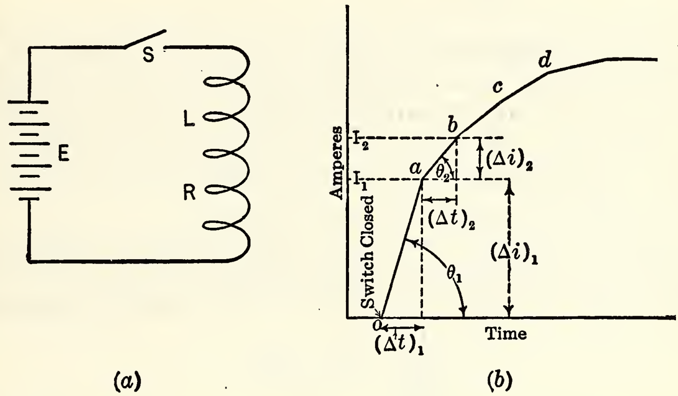


FIG. 4–5. When a circuit contains appreciable self-induction the current cannot at once assume its final value; it starts to increase, from zero value, at a rate determined by the impressed voltage and the self-induction of the circuit.

During the first short interval of time considered $(\Delta t)_1$, the resistance drop, although actually present, may be neglected, because the current is so small. Hence the counter voltage of self-induction must balance the impressed voltage E, and so we have, using Eq. (6),

$$E = L\frac{(\Delta i)_1}{(\Delta t)_1}$$

and from this we get

$$\frac{(\Delta i)_1}{(\Delta t)_1} = \frac{E}{L} \tag{8}$$

It therefore appears that the rate of increase in the current, $\dfrac{(\Delta i)_1}{(\Delta t)_1}$, when the switch is first closed, is given by the value of E/L. In Fig. 4–5*b* this rate of current increase is given by $\tan\theta_1$, so we have $\tan\theta_1 = E/L$.

After the current has risen to an appreciable value (say at the end of

time interval $(\Delta t)_1$), the resistance drop can no longer be neglected; it is equal to I_1R. At this instant of time we may write

$$E = I_1R + L\frac{(\Delta i)_2}{(\Delta t)_2} \tag{9}$$

in which $\frac{(\Delta i)_2}{(\Delta t)_2}$ represents the rate of increase in current at the beginning of interval of time $(\Delta t)_2$. This relation may evidently be written

$$\frac{(\Delta i)_2}{(\Delta t)_2} = \frac{E - I_1R}{L} = \tan \theta_2 \tag{10}$$

Since part of the impressed voltage is now used up as an I_1R drop, less voltage is available to act across the inductance. The rate of rise of the current will then fall off, until the voltage of self-induction just balances the remainder. Hence the rate of change of the current cannot be so high. This decreased rate of current increase is shown in Fig. 4–5*b* by the lower value of $\tan \theta_2$ as compared to $\tan \theta_1$.

At the end of the second increment of time $(\Delta t)_2$ the current has risen to the value I_2 and there is, therefore, a greater IR drop. For a third increment of time $(\Delta t)_3$ the rate of current increase would be given by

$$\frac{(\Delta i)_3}{(\Delta t)_3} = \frac{E - I_2R}{L} \tag{11}$$

which is evidently less than the value given in Eq. (10). So, with increasing time, the increase in current becomes correspondingly less, until finally (actually in the ordinary circuit after a small fraction of a second) the IR drop is essentially equal to E; for this condition the rate of current increase is zero.

The form of current growth in an inductive circuit, considered from the standpoint of current increments, is shown by the broken line *oabcd* of Fig. 4–5*b*; it will be evident that this solution is approximate only, because, to make our analysis accurate, the increments of time and current should be taken extremely small. The result of so taking the time and current increments is given in the following section.

The case of current rise in an inductive circuit is analogous to that of the motion of a body being accelerated in a liquid. When the body is at rest and a constant force is applied to it, the body accelerates rapidly, the entire force applied being available to overcome its inertia. However, as the body starts to move, the friction reaction, which we shall assume is proportional to the velocity, comes into play and some of the impressed force is used to overcome it, and only the remainder of the impressed force

is available to overcome inertia for further acceleration. As more and more of the impressed force is required to overcome the increasing friction reaction, and less and less is available for acceleration, the velocity of the body increases more and more slowly, until finally the entire impressed force is required to overcome friction alone, and the velocity becomes constant.

7. Exact Form of Current Growth in an Inductive Circuit. The equation for the growth of current in an inductive circuit of constant inductance, L, can be derived by the use of the calculus, and is found to be as follows:

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right) \tag{12}$$

where i is the current at any elapsed time, t, after closing switch;
E is the constant impressed voltage;
R is the resistance of the circuit;
L is the inductance;
ϵ is the Napierian logarithmic base = 2.71828.

The derivation of Eq. (12) is obtained if we remember that, at any instant during the growth of the current, the impressed voltage is equal to the sum of the reactions, i.e., the iR reaction plus the counter voltage of self-induction, or

$$E = iR + L\frac{di}{dt} \tag{13}$$

which may be rewritten

$$(E - iR)dt = Ldi$$

Dividing by $-R$ and transposing

$$\frac{di}{i - \frac{E}{R}} = -\frac{R}{L}dt$$

Integrating, there results

$$\log_\epsilon\left(i - \frac{E}{R}\right) = -\frac{Rt}{L} + C$$

where C is a constant of integration.

To determine the value of C, we utilize the relation that, when $t = 0$, i also is zero, so that

$$\log_\epsilon\left(-\frac{E}{R}\right) = C$$

Using this relation we find that

$$\log_\epsilon\left(i - \frac{E}{R}\right) = -\frac{Rt}{L} + \log_\epsilon\left(-\frac{E}{R}\right)$$

or

$$-\frac{Rt}{L} = \log_\epsilon\left(i - \frac{E}{R}\right) - \log_\epsilon\left(-\frac{E}{R}\right) = \log_\epsilon\left(\frac{i - \frac{E}{R}}{-\frac{E}{R}}\right)$$

$$\epsilon^{-\frac{R}{L}t} = \frac{i - \frac{E}{R}}{-\frac{E}{R}}$$

Then

$$i = \frac{E}{R} - \frac{E}{R}\epsilon^{-\frac{R}{L}t} = \frac{E}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right) \tag{12}$$

In Fig. 4–6 is shown the rise of current in a circuit possessing 0.16 henry inductance and 4 ohms resistance, when 20 volts is impressed, the final

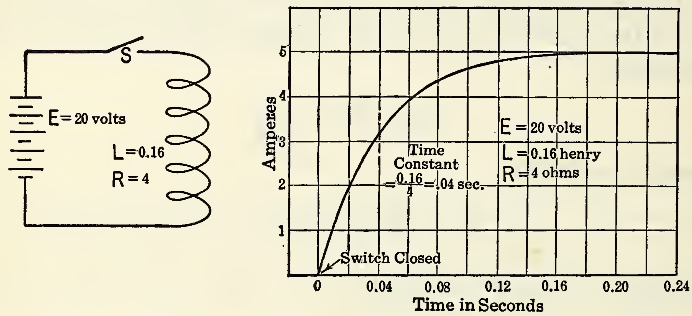


FIG. 4–6. Exact analysis of the growth of current in an inductive circuit shows it to be an exponential curve, as given here.

value of the current being 5 amperes. Figure 4–7 is an oscillograph record of the growth of current in an air-core inductance, the permeability and inductance, L, of which remain constant. Figure 4–8 is a similar record for a coil wound on an iron core; the permeability and L of this coil varied with the current, accounting for the difference in the form of this curve as compared with that of the air-core coil.

8. Decay of Current in an Inductive Circuit. If an inductive circuit is short-circuited as in Fig. 4–9a, the current in the coil does not die down immediately, as it would in a non-inductive circuit, but continues to flow for an appreciable time after the short circuit is applied; this is due again to the counter voltage of self-induction.

As soon as the short circuit is applied and the current starts to decrease, i.e., change, the counter voltage of self-induction makes its appearance and

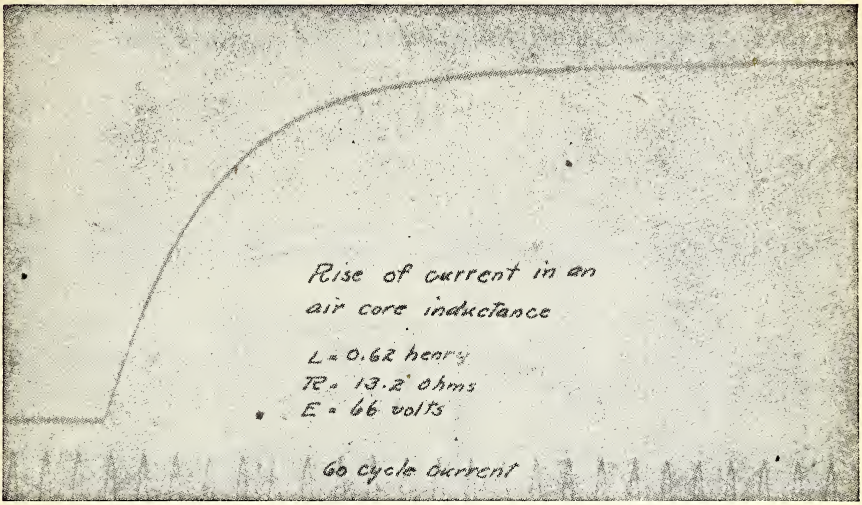


Fig. 4–7. An oscillogram of the rise of current in an air-core inductor; it is seen to be of exactly the same form as the theoretical curve of Fig. 4–6.

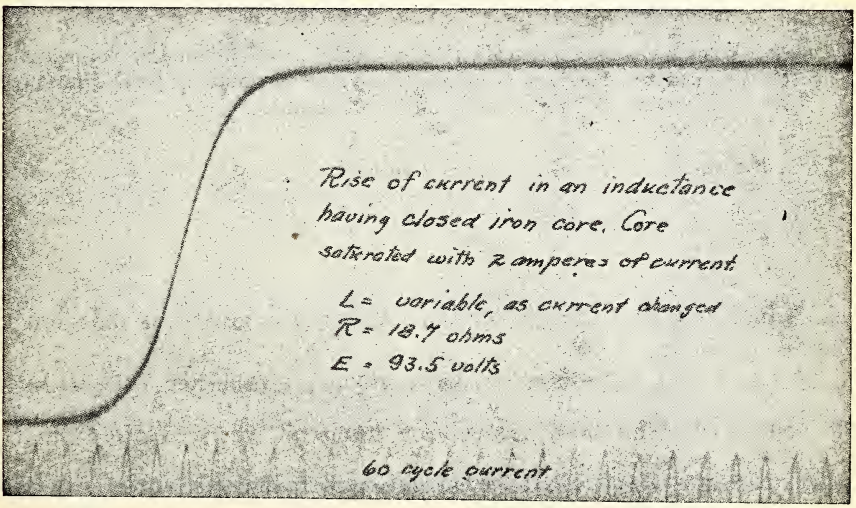


Fig. 4–8. If the inductor has a closed iron core, the permeability is likely to change as the current rises, thus giving a variable L. The form of current rise is therefore considably different from that of Fig. 4–7.

acts to maintain the current. As there is now no impressed voltage on the circuit, this counter voltage of self-induction is the only agent causing cur-

rent to flow, and it must therefore be overcoming the IR drop, which exists so long as current flows.

If we assume the current to decrease to zero along the line oa in Fig. 4–9b, the time required to reach zero current would be t_1, and the rate of change of current would evidently be I/t_1. We may then write

$$0 = IR + L\frac{I}{t_1}$$

or

$$-L\frac{I}{t_1} = IR \tag{14}$$

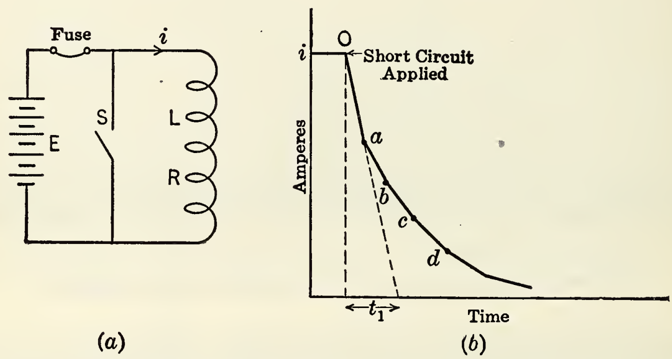


FIG. 4–9. When an inductive circuit is short-circuited, as will be done by closing switch S of the above circuit, the form of current decay will resemble, in general form, the broken line shown in diagram b.

The last equation may be transposed to give

$$\frac{I}{t_1} = -\frac{IR}{L} \tag{15}$$

which indicates that the greater the IR drop the greater is the slope of the line oa, in Fig. 4–9b.

But as IR, at the instant the short circuit is established, is equal to E, the voltage which has been impressed on the circuit, we may write $\frac{I}{t_1} = -\frac{E}{L}$, which expresses the idea that the current starts to decrease at the same rate as that at which it started to rise when the voltage was impressed on the circuit. This is seen by reference to Eq. (8).

With the high rate of decrease of the current, as shown by the line oa, the value of the counter voltage is large, as is necessary, inasmuch as the IR drop is still considerable. However, as the current falls, the IR

drop decreases in proportion, and the rate of change of the current need not be as large to generate the necessary counter voltage to balance the IR drop. From the standpoint of time increments, the current, therefore, decreases its rate of decrease, first along the line ab for a while, and then along bc, and so on.

The exact equation for the decay of current in a circuit having constant inductance is found to be

$$i = I\epsilon^{-\frac{R}{L}t} \tag{16}$$

where I is the value of the steady current through the inductive circuit, an instant before it is short-circuited; the other terms have the same significance as in Eq. (12).

To derive Eq. (16) we may rewrite Eq. (14) in the form

$$-L\frac{di}{dt} = iR \quad \text{or} \quad \frac{di}{i} = -\frac{R}{L}dt$$

Integrating, we have

$$\log_\epsilon i = -\frac{Rt}{L} + C$$

The value of C is obtained from the relation that, when $t = 0$, $i = I$, so that

$$\log_\epsilon I = C$$

and

$$\log_\epsilon i = -\frac{Rt}{L} + \log_\epsilon I$$

Then

$$-\frac{Rt}{L} = \log_\epsilon i - \log_\epsilon I = \log_\epsilon \left(\frac{i}{I}\right)$$

$$\epsilon^{-\frac{R}{L}t} = \frac{i}{I}$$

and

$$i = I\epsilon^{-\frac{R}{L}t} \tag{16}$$

Figures 4–10, 4–11, and 4–12 show the decrease in current in inductive circuits, Fig. 4–10 showing it for the circuit possessing constant inductance of 0.16 henry and 4 ohms resistance, Fig. 4–11 being an oscillograph record of current decay in an air-core coil, and Fig. 4–12 a similar record for an iron-core coil. Changing permeability accounts for the shape of the curve in the last figure.

9. Time Constant. Study of Eq. (12) will indicate that the value of the fraction R/L is the factor which actually fixes the rate of increase of the current. In circuits of equal resistance, the greater the inductance, the longer it will take for a given impressed voltage to set up the same final value of current in the circuit.

In the case of a body being accelerated in a liquid, bodies of the same size, shape, and surface will encounter the same frictional force at the same velocity; but a mass having high inertia, as lead, will accelerate much more slowly than a mass of low inertia, as aluminum, when the same force is applied. In the end, however, each will have the same velocity.

The greater the resistance of a circuit, the lower the final value of the current becomes with a given impressed voltage. The initial rate of increase of the current being fixed by the inductance, it follows that, in circuits of equal inductance, the current will reach its final value very

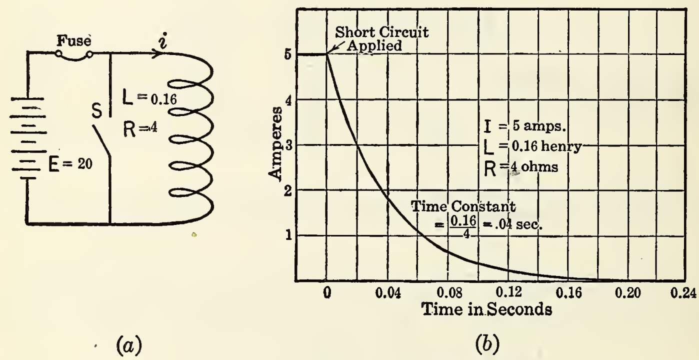


FIG. 4–10. The exact form of current decay in a circuit of constant inductance is exponential, as shown in this curve, which was calculated from Eq. (16).

much sooner in a circuit of high resistance than in one of low resistance; that is, the effect of inductance is less evident in a high-resistance circuit than in one of low resistance.

The ratio of the inductance to the resistance of a circuit, L/R, is called the time constant of the circuit. If the numerical value of L/R, in seconds, is substituted for t in Eq. (12), it will be found that it is the time it takes for the current to reach 63.2 per cent of its final value; if substituted in Eq. (16), it represents the time it takes the current to fall 63.2 per cent of its initial steady value.

The time constant is therefore useful in determining the rapidity with which current rises or falls in one inductive circuit in comparison with others. The time constant of the circuit used in Figs. 4–6 and 4–10 is $0.16/4 = 0.04$ second.

In order to get a large time constant, it is evidently necessary to have

a high ratio of L/R; this requires the use of a large amount of copper or iron. This is illustrated by the fact that an air-core coil using even 100 pounds of copper wire will have a time constant of less than one-tenth of a

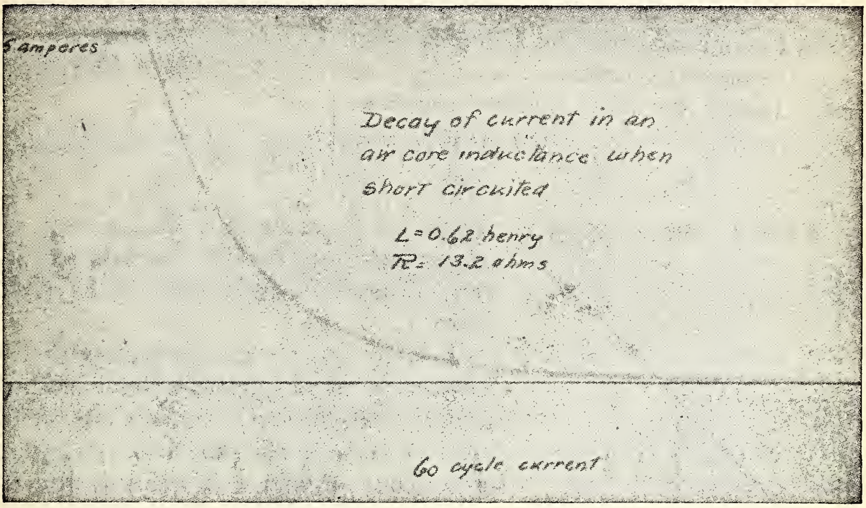


FIG. 4–11. Oscillogram of current decay in a circuit of constant inductance which has been short-circuited.

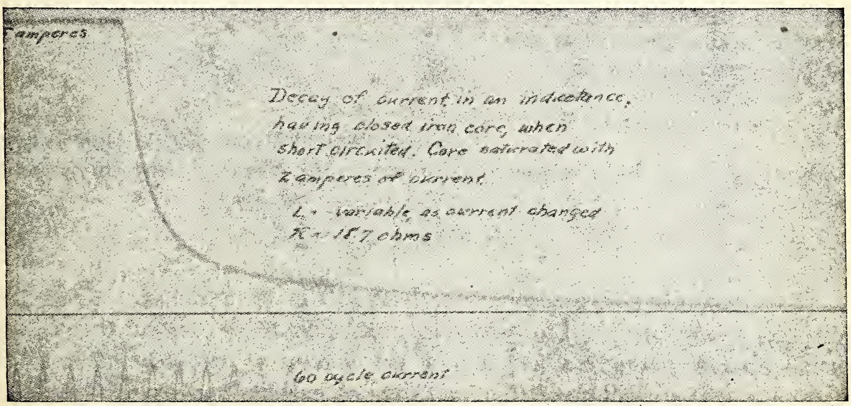


FIG. 4–12. In case the inductor has a closed iron core, the decay of current may be very rapid at first, and then the decrease takes place more slowly as the self-induction of the circuit increases.

second; the field circuits of large generators, having iron magnetic circuits of large dimensions, with many pounds of wire in the field coils, may have a time constant of several seconds.

10. Energy Stored in a Magnetic Field. It was shown in section 42, page 64, that the energy stored in a magnetic field was

$$W = \frac{4\pi N^2 \bar{I}^2 \mu A}{2l} = \frac{\bar{L}\bar{I}^2}{2} \text{ ergs}$$

where $\bar{I}$ is in abamperes and $\bar{L}$ in abhenrys.

If the current is expressed in amperes and the coefficient of self-induction in henrys, the above equation becomes

$$W = \frac{0.4\pi N^2 I^2 \mu A}{2l\, 10^8} = \frac{LI^2}{2} \text{ joules} \tag{17}$$

When a magnetic field is established by a current, this amount of energy must be stored in the field as the current increases from zero to its final value, I.

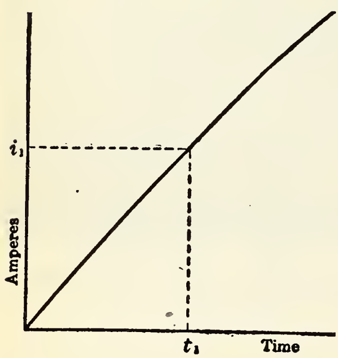


FIG. 4–13. During the first short time interval after the switch of an inductive circuit is closed, the current rise is essentially a straight line, the slope being equal to E/L.

Let us consider the first part of the current rise in an inductive circuit during the time this curve is essentially a straight line, as, for example, the first hundredth of a second in Fig. 4–6, which is shown enlarged in Fig. 4–13. During this time the current is so small compared with its final value that the resistance drop is negligible, so that Eq. (5) may be used, that is

$$E = L\frac{I_1}{t_1}$$

The average current during this time is evidently $I_1/2$, so that the energy supplied to the circuit during this time is

$$E \cdot \frac{I_1}{2} \cdot t_1 = L\frac{I_1^2}{2t_1} \cdot t_1 = L\frac{I_1^2}{2} \tag{18}$$

If we had taken into account the heat lost in the resistance of the circuit, the energy supplied *to the circuit* would have been found, by an accurate analysis, equal to $LI_1^2/2$ *plus the amount of heat generated in the resistance.* The amount of this heat cannot be obtained by this analysis because the current is not truly a straight line during the time considered.

In any case, the amount of energy stored in the magnetic field, for any value of current I, is found to be just equal to

$$W = \frac{LI^2}{2} \tag{19}$$

This is true no matter what the form of the current curve, as it rises from

zero to I. This expression for the energy stored in a magnetic field checks with that derived in section 42, page 64, by an entirely different method.

The exact mathematical derivation of Eq. (19) is based upon Eq. (12). At any instant while the current is rising in an inductive circuit according to Eq. (12), the total power input is Ei, and of this a part, i^2R, is used up in heating the winding. The portion of input which is stored in the magnetic field is then

$$p = Ei - i^2R \tag{20}$$

Substituting for i its value from Eq. (12),

$$p = \frac{E^2}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right) - \frac{E^2}{R}\left(1 - 2\epsilon^{-\frac{R}{L}t} + \epsilon^{-\frac{2R}{L}t}\right)$$

$$p = \frac{E^2}{R}\left(\epsilon^{-\frac{R}{L}t} - \epsilon^{-\frac{2R}{L}t}\right) \tag{21}$$

The total energy stored in the magnetic field is the integral of the power from $t = 0$ to $t = \infty$; that is,

$$W = \int_0^\infty p\,dt = \frac{E^2}{R}\int_0^\infty \left(\epsilon^{-\frac{R}{L}t} - \epsilon^{-\frac{2R}{L}t}\right) dt$$

$$= \frac{E^2}{R}\left[-\frac{L}{R}\epsilon^{-\frac{R}{L}t} + \frac{L}{2R}\epsilon^{-\frac{2R}{L}t}\right]_0^\infty$$

Inserting the limits,

$$W = \frac{E^2L}{2R^2} = \frac{LI^2}{2} \tag{22}$$

11. Danger of Opening Circuits Possessing Much Inductance. In discussing the decay of current in inductive circuits, it was considered that the coil was short-circuited, and the current was found to decrease to zero; all the energy stored in the magnetic field must therefore have been dissipated in the form of heat generated within the coil.

If the switch of a circuit possessing inductance, as in Fig. 4–6, is opened, it will be noticed that an appreciable arc occurs across the switch contacts, much greater in magnitude than would be the case if the circuit contained no inductance, as in Fig. 4–4. This arc is due to the voltage of self-induction.

The value of the voltage of self-induction is always given by the expression $L\dfrac{\Delta i}{\Delta t}$, where $\Delta i/\Delta t$ is the rate of change of the current. If the switch is opened very rapidly, so that the current is quickly ruptured, the rate of change of the current is high, as is also the value of the voltage of self-induction, the action of which is to maintain the flow of current even across the open switch contacts, in the form of an arc.

When an inductive circuit is opened, the energy stored in the magnetic field must be dissipated in some way by the time the current reaches zero, and this energy generally appears as heat in the arc at the switch contacts, and in the circuit.

If the energy stored in the field is $\frac{1}{2}LI^2$, and opening the switch reduces the current to zero in time t, then the average power must be $LI^2/2t$, which, if t is very small, will be large.

(It is to be pointed out that the circuit is not actually opened as quickly as the switch is opened, because the arc at the switch contacts maintains the circuit closed as long as it lasts, even though the switch itself is actually open.)

The average power must be equal to the average of the products of the instantaneous induced voltage and current, averaged over the interval t. Since the magnitude of the current in field circuits is not high the induced voltage must be large if the rate of energy dissipation is high.

Suppose that the field circuit of a large generator has 5 henrys inductance, and that its current of 10 amperes is opened completely in 0.05 second. The average induced voltage of self-induction will then be $5 \times 10/0.05 = 1000$ volts. The energy originally stored in the field is, from Eq. (19), $5 \times 10^2/2 = 250$ joules, which, if dissipated in 0.05 second, represents power to the amount of $250/0.05 = 5000$ watts. The average current, assumed to decrease uniformly, is $10/2 = 5$ amperes, and the product of 1000 volts and 5 amperes is again 5000 watts.

The case is analogous to the stopping of moving bodies; a rifle bullet, striking a steel wall, develops great force and great power for a fraction of a second.

The opening of circuits containing much inductance, as the fields of generators, motors, etc., is likely to cause severe burning at the switch contacts, but the real danger lies in the possibility that the voltage of self-induction may break down the insulation between the coil and the frame of the machine. If the opening of the switch induces a high voltage across the ends of a field coil, as diagrammatically represented in Fig. 4–14, this high voltage is also impressed on the path $WXYZ$, that is, across the insulation at WX, through the steel pole of the machine, XY, and the insulation at YZ. By puncturing the insulation at both WX and YZ and thus "grounding" the coil at two places, current may flow over the ground circuit, instead of through the coil as desired, and the energy stored in the field may be thus dissipated. Such grounding of the field circuit may be dangerous for the operator, resulting in severe shocks, or may even make the machine inoperative in that only a small current will flow through the coil (as will appear later in this book).

Incidentally, if a voltmeter is connected across the field side of the switch, to measure the voltage originally impressed on the coil, say 110

volts, and the switch is opened, the voltage of self-induction will be impressed on the voltmeter. If this voltage reaches high values, even only momentarily, the instrument will be injured and may even be burnt out.

12. Field Discharge Switches. In order to guard the fields of machines against puncture of their insulation when the field circuit is opened, special switches, called discharge switches, are used. The action of these switches is diagrammatically indicated in Fig. 4–15; in the position shown, current flows only through the field *F*, but before the blades of the switch, *bb*, break contact with the main jaws, *aa*, one blade makes contact with an auxiliary jaw, *c*. This action places a discharge resistance, *R*, in parallel with the field, and, in this position of the switch, current from the supply continues to flow through the field, in addition to flowing through the resistance. Further motion of the switch opens the contacts between the blades and the main jaws, opening the line but leaving the discharge resistance in parallel with the field. The energy stored in the field is thereby dissipated slowly and without the induction of high voltages. A common form of field discharge switch is illustrated in Fig. 4–16.

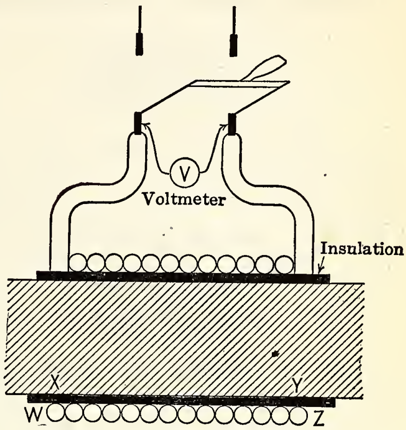


Fig. 4–14. If a field circuit is opened quickly, the high voltage of self-induction of the winding is likely to cause rupture of the insulation of the field winding; a breakdown is likely to occur through the insulation to the frame of the machine at two points, such as across *W–X* and *Y–Z*.

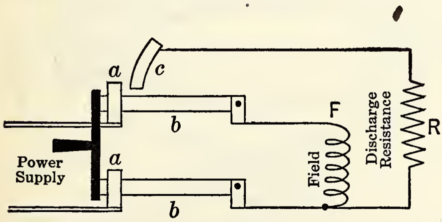


Fig. 4–15.

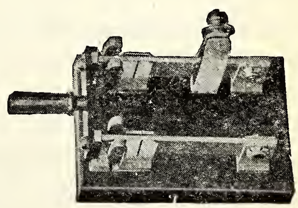


Fig. 4–16.

Fig. 4–15. A common method of lessening the chances of the rupture of the field-winding insulation is to open the field circuit by a specially designed switch, such as that indicated here; before the field is disconnected from the power supply, a resistance *R* is connected across the field circuit, and this serves to keep the rate of decay of the current to reasonable values.

Fig. 4–16. A common type of the switch indicated in Fig. 4–15; the extra clip, for connecting the field discharge resistance across the field winding is seen on the upper blade.

13. Mutual Induction. Consider two turns of wire in close proximity, as in Fig. 4–17. If an increasing current is made to flow in turn 1, some of the flux set up will thread turn 2 and establish flux interlinkages with it. Voltage must therefore be induced in turn 2, as long as its flux interlinkages are increasing; and this voltage, from previous reasoning, will have a direction as shown, or opposite to the direction of the impressed voltage in turn 1. The induced voltage in turn 2 will set up current in the same direction as itself, or in a direction opposite to that of the current in turn 1. Hence, the resultant current in turn 2 sets up a mmf in the opposite direction to that due to the current in turn 1; but inasmuch as it owes its existence to the growing field due to the current in turn 1, it cannot actually overcome the flux set up by turn 1 and set up a flux of its own in the opposite direction; the action of this mmf in turn 2 is merely to decrease the amount of flux from turn 1, which links turn 2. In other words, the effect of the current in the second circuit is to limit the rapidity of the increase of flux through itself, the flux being due to the current in circuit 1.

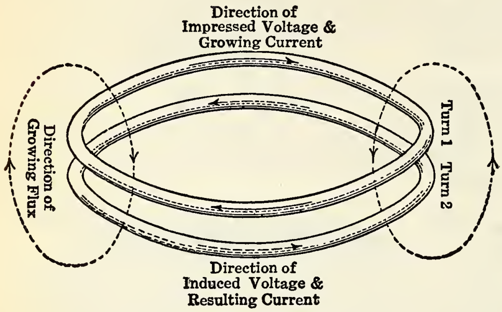


Fig. 4–17. If current rises in one circuit in proximity to another, a voltage is induced in the second, by the change of flux linking the second, this flux being set up by the first circuit. If the second circuit is closed, a current will flow in it, due to this induced voltage.

If the current in turn 1 is allowed to die down, the decreasing flux linking turn 2 induces a voltage in turn 2, the direction of which is obviously as shown in Fig. 4–18. The induced current in turn 2 will now tend to set up a field in the same direction as the decreasing field of turn 1, thereby again opposing the change taking place in the latter.

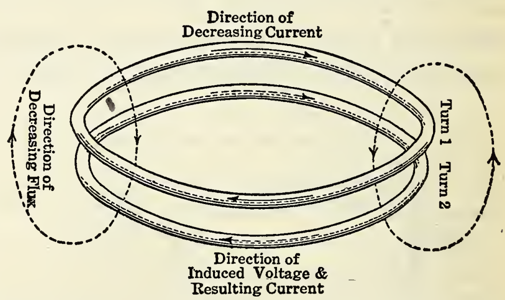


Fig. 4–18. If the current in the first circuit decreases, the voltage induced in the second will be in the opposite direction to that it had when the current in the first was rising.

Therefore, if two circuits are so placed with respect to each other that the magnetic field due to current in one circuit links wholly or partly with the other, any change in the strength of the current in the first circuit or

primary will induce in the second circuit or secondary a voltage of *mutual induction.* If the secondary is closed, this voltage of mutual induction sets up a current in such direction in the secondary as to oppose the change of the flux linking this circuit. This, again, is in accord with Lenz's law. If the secondary is open, it offers no reaction to the establishment of a field by the primary, because no current can flow in this open circuit even though a voltage is induced in the secondary; it is only by current flow that a back mmf is set up.

It is also evident that while primary current is increasing, and secondary current is flowing, a force of repulsion exists between the coils. If the secondary could move away, the reaction is such as to prevent an increase in secondary flux interlinkages. Similarly, when primary current decreases, a force of attraction exists; if the coils can move closer together, the reaction tends to maintain the secondary flux interlinkages.

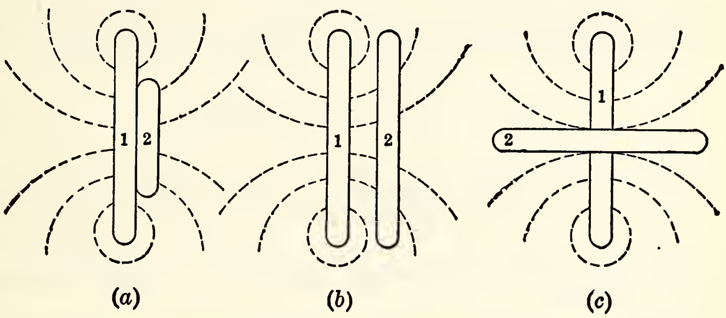


FIG. 4–19. The value of the mutual induction between two circuits depends upon their relative shapes and positions, being about the same for cases *a* and *b*, and being zero for case *c* shown above.

The magnitude of the voltage of mutual induction depends only upon the rate of change of the flux interlinkages of the second circuit; for a uniform decrease in flux,

$$E_{2\,\text{av}} = \frac{\Phi_m N_2}{10^8 t} \tag{23}$$

where $E_{2\,\text{av}}$ is the average voltage of mutual induction in the secondary;
Φ_m is the *mutual* flux that is set up, linking both circuits;
N_2 is the number of turns in the secondary;
and t is the time, in seconds, taken for the flux Φ_m to be set up.

The amount of flux from the primary actually linking with the secondary will depend upon the relative shapes and relative positions of the two circuits. If the first circuit is larger than the second (Fig. 4–19*a*), or at a distance from the second (Fig. 4–19*b*), only a small portion of the flux set up by the first circuit will thread the second.

If the two circuits have their axes at right angles (Fig. 4–19*c*), no flux

from either will thread the other; hence there is no mutual induction between them.

14. Unit of Mutual Inductance. Even though the two circuits considered before are brought as close together as possible, only a fraction of the total flux set up by the first will link the second; some of the primary flux will leak past the secondary. We may write then, if Φ_1 is the total flux set up in the first circuit,

$$\Phi_m = K_1\Phi_1 \tag{24}$$

where K_1 is a constant for any particular circuit and position of the two turns; it is called the leakage coefficient of circuit 1 with respect to circuit 2.

Substituting the above value for Φ_m in Eq. (23), we have

$$E_{2\,\text{av}} = \frac{K_1\Phi_1 N_2}{10^8 t} \tag{25}$$

Since K_1 and N_2 are constants, and multiplying numerator and denominator by the current I_1, we may further write

$$E_{2\,\text{av}} = \frac{K_1\Phi_1 N_2}{10^8 I_1} \cdot \frac{I_1}{t} = M\frac{I_1}{t} \tag{26}$$

where

$$M = \frac{K_1\Phi_1 N_2}{10^8 I_1} \tag{27}$$

and is called the coefficient of mutual induction, or mutual inductance between the two coils.

In this analysis also, if the change in current or flux is not uniform, the equations must be expressed in the form of instantaneous values; thus Eq. (26) becomes

$$e_2 = M\frac{di_1}{dt} \tag{28}$$

The henry is also the unit of mutual inductance; it may be defined from either of the last two equations. From Eq. (26), we find that two circuits have mutual inductance of one henry if a rate of change of one ampere per second of the current in the first, or inducing, circuit generates one volt in the second circuit. From Eq. (27), we may define the henry as the amount of mutual inductance possessed by two circuits when one ampere in the first circuit sets up 10^8 interlinkages in the second.

15. Coefficient of Mutual Induction. The coefficient of mutual induction, M, has the same value whichever coil is treated as the primary. If current in the first circuit, I_1, is varied at constant rate, the average voltage in the second circuit is

$$E_{2\,\text{av}} = \frac{K_1\Phi_1 N_2}{10^8 t} = \frac{K_1\Phi_1 N_2}{10^8 I_1} \cdot \frac{I_1}{t} = M_{12}\frac{I_1}{t} \tag{29}$$

where $K_1\Phi_1 = \Phi_2$ represents the portion of the flux set up by the first coil which links the second, and M_{12} is the mutual inductance when coil 1 is the primary: $M_{12} = K_1\Phi_1N_2/10^8I_1$.

If current in the second circuit is varied at constant rate, the average voltage induced in the first circuit is

$$E_{1\text{ av}} = \frac{K_2\Phi_2N_1}{10^8t} = \frac{K_2\Phi_2N_1}{10^8I_2} \cdot \frac{I_2}{t} = M_{21}\frac{I_2}{t} \tag{30}$$

where $K_2\Phi_2 = \Phi_1$ is again the mutual flux and M_{21} is the mutual inductance when coil 2 is the primary: $M_{21} = K_2\Phi_2N_1/10^8\,I_2$.

It was shown on page 45 that, when a coil of N turns, carrying a current $\bar{I}$, was moved from infinity into a magnetic field Φ, set up by another agency, the work done was

$$W = \Phi N\bar{I}$$

If I is expressed in amperes this expression becomes

$$W = \frac{\Phi NI}{10}$$

If coil 1, carrying current I_1 and setting up flux Φ_1, is fixed and coil 2 carrying current I_2 is moved up to it so that a flux $K_1\Phi_1$ links it, the work done is

$$W = \frac{K_1\Phi_1N_2I_2}{10} \tag{31}$$

but if coil 2 is fixed and coil 1 is moved into the same relative position so that it links a flux $K_2\Phi_2$, the work done is

$$W = \frac{K_2\Phi_2N_1I_1}{10} \tag{32}$$

The work done in either case must be the same, so that

$$K_1\Phi_1N_2I_2 = K_2\Phi_2N_1I_1$$

From Eqs. (29) and (30) the ratio of M_{12} and M_{21} is

$$\frac{M_{12}}{M_{21}} = \frac{\dfrac{K_1\Phi_1N_2}{10^8I_1}}{\dfrac{K_2\Phi_2N_1}{10^8I_2}} = \frac{K_1\Phi_1N_2I_2}{K_2\Phi_2N_1I_1} = 1$$

Therefore $M_{12} = M_{21} = M$ and

$$E_{1\text{ av}} = M\frac{I_2}{t} \quad \text{and} \quad E_{2\text{ av}} = M\frac{I_1}{t} \tag{33}$$

If in the equations

$$M_{12} = \frac{K_1\Phi_1N_2}{10^8I_1} \quad \text{and} \quad M_{21} = \frac{K_2\Phi_2N_1}{10^8I_2}$$

the values of Φ_1 and Φ_2 in terms of magnetomotive force and reluctance are substituted,

$$M_{12} = \frac{0.4\pi K_1N_1N_2}{10^8 \dfrac{l_1}{\mu_1A_1}} = M_{21} = \frac{0.4\pi K_2N_1N_2}{10^8 \dfrac{l_2}{\mu_2A_2}} \tag{34}$$

where l_1, μ_1, A_1 and l_2, μ_2, A_2 are the length, permeability, and area respectively of the separate magnetic circuits of the two coils.

Whereas the coefficient of mutual induction, M, has the same value whichever coil is treated as primary, the values of K_1 and K_2 will not be equal unless the reluctances of the individual magnetic paths are identical. An example of coils with unequal leakage coefficients is that of a long air-core solenoid with a shorter one wound over the middle of the long coil. All the flux set up by the long coil will link the shorter one, but only a portion of the flux set up in the short coil will link the long one.

If two air-core coils are of exactly the same shape and size and are symmetrically placed, the magnetic circuits will be identical and K_1 will equal K_2; the coefficients will also be equal when iron is used provided that the core is symmetrically placed with respect to both coils. But if the system is unsymmetrical in any way, K_1 and K_2 will have different values.

Since the permeability of a magnetic circuit containing iron will vary with the current, the values of K_1 and K_2 will be constant only if there is no iron in the magnetic paths.

If the self-inductance of circuits 1 and 2, alone, are

$$L_1 = \frac{\Phi_1N_1}{I_110^8} \quad \text{and} \quad L_2 = \frac{\Phi_2N_2}{I_210^8}$$

then

$$L_1L_2 = \frac{\Phi_1N_1}{I_110^8} \cdot \frac{\Phi_2N_2}{I_210^8} = \frac{K_1\Phi_1N_1}{K_1I_110^8} \cdot \frac{K_2\Phi_2N_2}{K_2I_210^8}$$

$$= \frac{M_{12}}{K_1} \cdot \frac{M_{21}}{K_2} = \frac{M^2}{K_1K_2}$$

and

$$M = \sqrt{K_1K_2} \cdot \sqrt{L_1L_2} \tag{35}$$

where $\sqrt{K_1K_2}$ is sometimes called the *coefficient of coupling;* it is evident that it will always be less than one.

For two circuits of the same size and shape and symmetrical position, $K_1 = K_2 = K$ so that

$$M = K\sqrt{L_1 L_2} \tag{36}$$

If iron is present in the magnetic circuit, the equation is true, but the values of L_1, L_2, and M will vary with the currents.

Equation (35) may be written in the form

$$M = \sqrt{K_1 L_1} \cdot \sqrt{K_2 L_2}$$

which shows that the individual inductances enter into the mutual inductance in the same ratio as the individual fluxes enter into the total flux. This is apparent if L is defined in terms of flux interlinkages.

Coupling between two circuits is greatest when they are interwound, with the turns of one circuit in closest proximity to the turns of the other. Perfect coupling could be attained only if the two circuits were to occupy exactly the same position, which is obviously impossible.

An equation to express the value of M in terms of the mutual magnetic circuit may be derived from Eq. (34).

Since $M_{12} = M_{21} = M = \sqrt{M_{12}M_{21}}$

$$M = \sqrt{\frac{0.4\pi K_1 N_1 N_2}{10^8 \dfrac{l_1}{\mu_1 A_1}} \cdot \frac{0.4\pi K_2 N_2 N_1}{10^8 \dfrac{l_2}{\mu_2 A_2}}}$$

$$= \frac{0.4\pi N_1 N_2}{10^8} \sqrt{\frac{K_1 K_2}{\dfrac{l_1 l_2}{\mu_1 \mu_2 A_1 A_2}}}$$

If

$$\frac{1}{\dfrac{l}{\mu A}} = \sqrt{\frac{K_1 K_2}{\dfrac{l_1 l_2}{\mu_1 \mu_2 A_1 A_2}}}$$

then

$$M = \frac{0.4\pi N_1 N_2}{10^8 \dfrac{l}{\mu A}} \tag{37}$$

The derivation shows that even with a perfectly symmetrical system the reluctance of the common path is different from the individual magnetic paths unless the coupling is perfect and there is no magnetic material in the path.

16. Currents Due to Mutual Induction. The exact form of currents set up in circuits, due to mutual induction with other circuits in which the current is varying, may be deduced by the use of differential equations.

Figure 4–20 is an oscillogram showing an experimentally determined time variation of the currents in two closely coupled air-core circuits.

The rise of current in the primary circuit, when it is connected to a voltage source, is seen to be similar to the rise of current in a circuit possessing only self-inductance. There is, however, a reaction of the secondary on the primary and it is not identical to the rise in such a circuit. The current in the secondary rises from zero to a maximum in the opposite direction, since it flows as a result of a voltage having the same relative direction as the counter voltage in the primary. We might say that this secondary current with its magnetic field opposes the rise of the magnetic field that accompanies the rising current in the primary. As the primary current approaches its final, i.e., steady or unchanging, value, the sec-

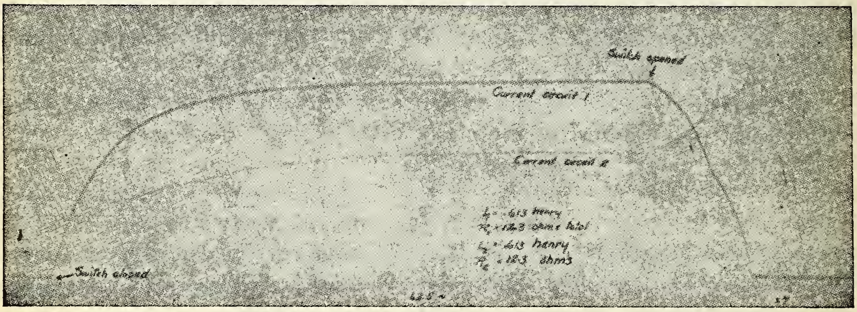


Fig. 4–20. An oscillogram showing the variation of primary and secondary currents as the primary circuit is first connected to, and later disconnected from, a voltage source. The peculiar form of the decay of primary current is due to arcing at the switch blades when the primary circuit is opened, being caused by the self-inductance and mutual inductance of the two circuits.

ondary current falls to zero since there is no longer a changing flux produced by the primary to induce voltage in the secondary.

The oscillogram of Fig. 4–20 also shows the current variation in both primary and secondary when the primary circuit is quickly broken by opening the switch. The peculiar form of decay of current in the primary, accompanied by a corresponding peculiarity in the form of the secondary current, was due to arcing at the switch blades when the primary circuit was opened, this arcing being caused by the self- and mutual inductances of the two circuits. It will be seen that the secondary current has its highest value when the rate of change of the primary current is the greatest, that is, when the arc is extinguished, and persists for some time thereafter. It would reach zero when all the magnetic energy stored in the system has been dissipated.

Had the secondary been open, only the effect of self-induction would have been present when the primary was opened and all the stored mag-

netic energy would have been removed by the time the primary current reached zero, there being no secondary current. With the secondary closed, some energy may still be stored in the system after the primary has been ruptured, to be dissipated as a secondary copper loss.

If the primary circuit had been disconnected from the source by short-circuiting, as was done for a self-inductive circuit in Fig. 4–9, both primary and secondary current would persist after voltage was removed. The form of primary current decay would be similar to that for a self-inductive circuit; the form of the secondary current would be the same as for an increasing value of primary current except that it would have the same relative direction as it had with the decreasing primary current. The stored magnetic energy is dissipated as a copper loss in both primary and secondary and both currents approach zero together.

17. Coils in Series. If two coils of self-inductance L_1 and L_2 are so placed as to have a coefficient of mutual inductance M, and are electrically connected so that the same current flows through both, each coil will induce a voltage of mutual induction in the other when the current varies, in addition to the voltages of self-induction induced in each coil separately. The coils may however be so placed with respect to each other that the voltages of mutual induction will either aid or oppose the voltages of self-induction. Assuming that the current is changing at a constant rate, so that average values of the voltages of self- and mutual induction may be used, the impressed voltage will then be balanced by a number of reactions. For the case where the voltages of mutual induction augment the voltages of self-induction, we may write that

$$E_{av} = \frac{L_1 I}{t} + \frac{L_2 I}{t} + \frac{M_{12} I}{t} + \frac{M_{21} I}{t}$$

$$= (L_1 + L_2 + 2M)\frac{I}{t} = L'\frac{I}{t} \tag{38}$$

where the term $M_{12}I/t$ expresses the voltage induced in the second coil due to variation of the flux set up by the first coil, and the term $M_{21}I/t$ the voltage induced in the first coil due to variation of the flux set up by the second coil.

If the coils are placed so that the voltages of mutual induction oppose the voltages of self-induction, the voltage equation becomes

$$E_{av} = \frac{L_1 I}{t} + \frac{L_2 I}{t} - \frac{M_{12} I}{t} - \frac{M_{21} I}{t}$$

$$= (L_1 + L_2 - 2M)\frac{I}{t} = L''\frac{I}{t} \tag{39}$$

The two coils carrying the same current therefore act as though they were a single coil having self-induction. When the coils are placed so that they augment, the value of the coefficient of self-induction is

$$L' = L_1 + L_2 + 2M \tag{40}$$

and, when the connection is such that the coils oppose, then

$$L'' = L_1 + L_2 - 2M \tag{41}$$

Equations (40) and (41) afford convenient methods of determining the mutual inductance of two coils. If the self-induction of the coils is measured, first augmenting and then opposing, then

$$L' - L'' = 4M$$

and

$$M = \frac{L' - L''}{4} \tag{42}$$

Another method is to measure L_1 and L_2 separately and then L' with the coils assisting. The value of M may then be found from Eq. (40).

The case of the two coils aiding each other is also interesting as shedding further light on the nature of self- and mutual induction. Substituting for L_1 and L_2 their values from Eq. (4) and for M_{12} and M_{21} their values from Eq. (34),

$$E_{\text{av}} = \frac{0.4\pi N_1^2}{\mathcal{R}_1 10^8}\cdot\frac{I}{t} + \frac{0.4\pi N_2^2}{\mathcal{R}_2 10^8}\cdot\frac{I}{t} + \frac{0.4\pi K_1 N_1 N_2}{\mathcal{R}_1 10^8}\cdot\frac{I}{t} + \frac{0.4\pi K_2 N_1 N_2}{\mathcal{R}_2 10^8}\cdot\frac{I}{t} \tag{43}$$

If the coils are now brought very close together with the same axis so that the individual magnetic paths as well as the mutual magnetic path are all the same, $K_1 = K_2 = 1$ and all reluctances become equal. Equation (43) may then be written

$$E_{\text{av}} = (N_1^2 + 2N_1N_2 + N_2^2)\,\frac{0.4\pi}{\mathcal{R}10^8}\cdot\frac{I}{t}$$

$$= (N_1 + N_2)^2\,\frac{0.4\pi}{\mathcal{R}10^8}\cdot\frac{I}{t} = L\,\frac{I}{t}$$

The two coils when moved together thus become in effect a single coil and the nature of self-induction is thus revealed as a case of mutual induction in which each individual turn induces voltage of mutual induction in every other turn.

PROBLEMS

4–1. A loop of one turn is linked with 10^6 lines of flux. If the flux is reduced to zero in 0.0123 second, what average value of voltage is generated?

4–2. What is the self-inductance of a solenoid of 5000 turns wound on a wooden ring, if the area of cross-section of the ring is 4 square inches and the length of the ring is 50 inches?

4–3. What is the inductance of an air-core coil with 1000 turns, if a 3-ampere change in current changes the flux by 3.6×10^5 lines?

4–4. When the current through an air-core coil changes from 2.4 to 4.8 amperes, the flux interlinkage change is 192×10^5. What is the inductance of the coil?

4–5. The current through the field of a certain motor increases from zero to 1.92 amperes in 0.011 second. If the inductance of the coils is 20 henrys, what average voltage is induced?

4–6. The inductance of the field of a certain motor is 25 henrys. If the current through the field increases from zero to 1.88 amperes in 0.012 second, what average voltage is generated?

4–7. A coil has an inductance of 1.5 henrys. What was the initial value of the current, if by decreasing it to zero in 0.9 second, an average voltage of 70 volts was induced?

4–8. An air-core coil is wound with 800 turns. With a current of 24 amperes the total flux set up is 6.2×10^4 lines. What is the inductance of the coil? If the current is broken in 0.11 second, what average value of voltage is generated?

4–9. What is the inductance of an air-core coil of 600 turns, carrying a current of 2.8 amperes, if the total flux set up is 5.8×10^4 lines? If the current is opened in 0.12 second, what is the average voltage induced?

4–10. A coil is wound with 200 turns, and, when a current of 1.8 amperes is flowing, the total flux set up is 4×10^4 lines. What is the inductance of the coil? If the exciting current dies to zero in 0.12 second, what average voltage is generated?

4–11. The core of a certain transformer is 15 cm by 20 cm in cross-section; the winding has 500 turns. The density with a current of 0.2 ampere is 5000 lines per sq cm. What is the coefficient of self-induction, and what is the average induced voltage, if the current is reduced to zero in 0.001 second?

4–12. What average voltage is induced in an air-core coil having 0.003 henry inductance, when the current is changing at a rate of 5×10^4 amperes per second? What value of flux will 30 amperes set up in the coil, if the coil has 200 turns?

4–13. A voltage of 361 volts is induced at a certain instant in an air-core coil of 2400 turns. If $L = 0.30$ henry, at what rate is the current changing? How much flux would a current of 2 amperes set up through the coil?

4–14. When the current through a coil is changing at a rate of 1000 amperes per second, the generated voltage within a coil of 2000 turns is 400 volts. How much flux would a current of 5 amperes set up in the coil?

4–15. At what rate is the current changing in a coil of 2000 turns and 0.32 henry inductance, if the voltage at a certain instant is 359 volts? How much flux would a current of 2.2 amperes set up in the coil?

4–16. An air-core coil of 200 turns of wire is threaded by 100,000 lines of force. The exciting current is halved in 0.01 second. What is the average induced voltage? If the original exciting current was 5 amperes, what is the inductance of the circuit?

4–17. A coil of 1000 turns has a magnetic circuit of 18 sq cm cross-section. With 9 amperes, the density is 14,500 lines per sq cm; with 4 amperes, it is 10,000 lines per sq cm. What is the inductance for each current? If the above current change took place in 0.00146 second, what average voltage is induced?

4–18. An iron-core coil of 1600 turns has a magnetic circuit of 20 sq cm. With 1.5 amperes flowing, the total flux set up is 258,000 lines; with 0.75 ampere, the total flux is 226,000 lines. (*a*) What is the inductance for each current? (*b*) If the current increases from 0.75 to 1.5 amperes in 0.0125 second, what average voltage is generated?

4–19. When 8 amperes flow through an iron-core coil of 1200 turns, the total flux set up within the core is 2.88×10^5 lines; with 4 amperes flowing it is 2.4×10^5 lines. (*a*) What is the inductance for each current, if the area of cross-section of the core is 22 sq cm? (*b*) How long did it take the current to change from 4 to 8 amperes, if the average induced voltage was 523.6 volts?

4–20. A wrought-iron ring of mean radius 40 cm has a cross-section of 25 sq cm and carries a winding of 1500 turns. What is the value of the necessary exciting current (Fig. 2–38) to set up a total flux of 3×10^5 lines? If the exciting current changes from the value just determined to 2.912 amperes in 0.012 second, what average value of voltage would be induced?

4–21. A wrought-iron ring has a mean radius of 30 cm and a cross-section of 20 sq cm. How many amperes must flow through a winding of 1200 turns on the ring to set up a total flux of 26×10^4 lines (Fig. 2–38)? If the exciting current decreased to a new value in 0.011 second and generated an average value of voltage of 21.8 volts, to what value did the current decrease?

4–22. A wrought-iron ring (Fig. 2–38) has a circular cross-section of 8 sq cm, a mean radius of 12 cm, and a winding of 580 turns. If the current through the winding changes from 1.3 to 2.6 amperes in 0.025 second, what average voltage is induced?

4–23. The inductance of a certain magnetic circuit of 1000 turns is 40 henrys when 0.87 ampere flows and 31.1 henrys when 1.25 amperes flow. If the current changed from 1.25 amperes to 0.87 ampere in 0.0136 second, what is the average induced voltage?

4–24. In a certain magnetic circuit with 700 turns, when a current of 10 amperes flows, the reluctance of the circuit is 0.0733 cgs units. When the current is increased 10 per cent, the reluctance increases 6.63 per cent. If the current increase takes place at the rate of 12,000 amperes per second, (*a*) what is the average voltage induced? (*b*) what is the inductance in henrys for each current value?

4–25. With a current of 7.24 amperes through 860 turns, the reluctance of a magnetic circuit is 0.130 cgs units. When the current increases 3 per cent, the reluctance increases 1 per cent. The current increase takes place at the rate of 18,000 amperes per second. What is the average induced voltage, and what is the inductance in abhenrys and in henrys for each of the current values?

4–26. The inductance of a certain magnetic circuit with 600 turns is 0.220 henry with a current I_1, and the reluctance is 0.02055 cgs unit. If the current increases 10 per cent (I_2), the reluctance of the magnetic circuit increases by 7.94 per cent, and the inductance is 0.204 henry. If the current increase takes place in 3×10^{-5} second, the average voltage generated is 410.4 volts. What was the initial current, I_1?

4–27. A coil has an inductance of 0.6 henry and a resistance of 10 ohms. How long does it take for the current to reach 75 per cent of its final value, when connected to a 110-volt source? To reach 99 per cent?

4–28. A coil with $L = 0.58$ and $R = 12$ is connected to a 110-volt line. How fast does the current start to rise? After the switch has been closed 0.01 second, how much will the current be?

4–29. A circuit of 0.216 henry and 4.63 ohms resistance is connected to a battery of 102 volts and 1.63 ohms resistance. At what rate does the current start to rise immediately after the switch is closed? What is the time constant of the circuit? What

is the current this number of seconds after closing the switch?

4–30. What must be the resistance and inductance of a coil if the current is to be 0.5 ampere, 0.07 second after it is connected to a 110-volt source, if the time given corresponds to the time constant of the coil?

4–31. An air-core coil of 0.628 henry inductance and 10 ohms resistance is connected suddenly across a 125-volt source. What is the time constant of the coil? What current flows at the time represented by the time constant? What current flows through the coil 0.1 second after the coil was connected to the source?

4–32. How much energy is stored in the coil of the last problem when the current is 5 amperes? 10 amperes? When the current has reached its final value?

4–33. A coil has 40 ohms resistance and 3 henrys inductance. When the coil is connected to a 240-volt source, determine (*a*) the rate of increase of the current at the instant the coil was connected to the source, and (*b*) the final amount of energy stored.

4–34. After the coil of the last problem has been connected to its source for an hour, it is to be disconnected through a discharge resistance of 30 ohms. At what initial rate will the current begin to decrease and what will be the voltage across the coil and across the resistance at the instant the current starts to fall?

4–35. An air-core coil has a resistance of 20 ohms and an inductance of 0.3 henry and is connected to a 120-volt source. If the coil is short-circuited upon itself, calculate the rate at which current starts to decrease. What is the current 0.01 second after the short circuit was applied, and what energy is stored in the coil at that time?

4–36. If the coil of the last problem were short-circuited through a discharge resistance of 20 ohms, what would be the answers?

4–37. An air-core coil has a resistance of 15 ohms and an inductance of 0.5 henry. Calculate the amount of energy stored in the coil, 0.03 second after the coil is connected to a 120-volt source, in joules and in percentage of the energy stored when the current reaches its final value. At what rate does the current start to increase at the instant the switch is closed?

4–38. If the coil of the last problem is suddenly short-circuited after its current has built up to its final value, calculate the amount of energy left in the coil 0.038 second after the short circuit was applied.

4–39. How much energy is stored in the field of a coil of 1000 turns carrying 10 amperes if there are 1.5×10^9 interlinkages?

4–40. A coil carrying a current of 0.5 ampere stores 0.004 joule of energy. What is the energy in foot-pounds, and what is the inductance in henrys?

4–41. A coil of 1500 turns carries a current of 6 amperes which sets up 1.4×10^9 interlinkages. How much energy is stored in joules and in foot-pounds?

4–42. How much energy in joules and in foot-pounds is stored in the field of a coil of 2000 turns carrying 10 amperes, if there are 1.6×10^9 flux interlinkages?

4–43. The inductance of the field of a certain generator is 40 henrys. If the field current is 1.6 amperes, how much energy is stored in ergs, joules, and foot-pounds? If the current is reduced to zero in 0.08 second, at what average rate in kilowatts is the energy expended?

4–44. How much energy in ergs, joules, and foot-pounds is stored in the field of a generator of 32 henrys when a current of 1.25 amperes is flowing? In what time was the current ruptured if the energy was expended at an average rate of 300 watts?

4–45. The inductance of the field of a 110-volt generator is 40 henrys, and the exciting current is 120 amperes. If the induced voltage across the field terminals is not to exceed 1000 volts, what is the minimum time in which this field may be opened? How much energy is liberated in opening the field and what is the average power during the minimum time as just calculated?

4–46. The inductance of the field of a certain generator is 40 henrys. When the current is 12.5 amperes, how much energy is stored in ergs, joules, and foot-pounds? If the current is reduced to zero (by opening the switch) in 1/60 second, what is the average power expended?

4–47. An air-core coil weighing 45 pounds and having $L = 0.30$ henry carries a current of 11 amperes. How fast must the coil be moving to store kinetic energy of motion equal to the magnetic energy stored? How high must it be lifted to store the same amount of potential energy?

4–48. A certain air-core coil having $L = 0.62$ and $R = 12$ will safely carry 8 amperes and weighs 75 pounds. How high must it be lifted to store gravitational potential energy equal to the maximum energy of self-induction the coil may store? How fast must it be moving to store kinetic energy of motion equal to the maximum energy of self-induction?

4–49. An air-core coil weighing 100 pounds and having a resistance of 15 ohms carries a current of 9.1 amperes. What is the value of the self-induction of the coil if the magnetic energy stored is the same as the potential energy when the coil is lifted 3.3 inches. How fast must the coil be moving to store the same amount of kinetic energy?

4–50. The four field coils of a 4-pole shunt generator are connected in series electrically, and each coil has 1000 turns. When a current of 3.5 amperes is flowing, there are 3×10^5 lines linking each coil. Calculate the inductance of the entire winding and the total energy stored. If the current is ruptured in 0.4 second, what average value of voltage will be generated across the ends of the winding?

4–51. The six field coils of a 6-pole generator are electrically connected in series, and each coil has 1200 turns. When a current of 8.8 amperes is flowing, the flux linking each coil is 72×10^5 lines. What is the inductance of the entire field winding and the total energy stored? If the current is ruptured in 0.5 second, at what rate is the stored energy expended?

4–52. What is the total inductance of the field of a 10-pole generator with all 10 field coils electrically connected in series, if each coil has 800 turns, and there are 87×10^5 lines linking each coil when a current of 12.5 amperes is flowing? In what time was the circuit opened if the energy stored was dissipated at an average rate of 19 kilowatts?

4–53. Two circuits are magnetically coupled by a mutual inductance of 3.26 henrys. If the current in one circuit changes from 2.63 amperes to 0.97 ampere in 0.0142 second, what average voltage is induced in the second circuit?

4–54. Two coils, R and S, have a common magnetic circuit. When placed in series augmenting, the inductance of the two coils is 1.286 henrys; when placed in series opposing, the inductance is 0.314 henry. What is the coefficient of mutual induction? What is the inductance of each coil, if the inductance of coil S is 0.6 that of coil R?

4–55. Two coils, X and Y, have a common magnetic circuit. The inductance of coil X alone is 0.34 henry and that of coil Y alone is 0.19 henry. With the coils in series the inductance of both coils was found to be 1.00 henry. What is the mutual inductance and the coefficient of coupling?

4–56. Two air-core coils 1 and 2, of the same dimensions, are placed with their axes coincident, as in Fig. 4–19*a*. Coil 1 has 300 turns, coil 2 has 175 turns, and 75 per cent of the flux set up by either coil links the other. When 2 amperes flow in coil 1, the flux linking it is 180,000 lines. If a current of 2 amperes in coil 1 is interrupted in 0.5 second, what average voltage will be induced in coil 2? What is the self-induction of coil 1 with coil 2 open-circuited? What is the mutual inductance of the two coils?

4–57. Two air-core coils, P of 1500 turns and Q of 1225 turns, have the same magnetic circuit. The self-inductance of coil P is 0.6 henry and the coefficient of coupling is 0.70. How much flux will be set up in coil Q, if a current of 2 amperes is passed through coil P? How much flux through coil P will a current of 1 ampere in coil Q set up? What is the self-inductance of coil Q? What is the mutual inductance? The self-inductance of the combination when in series augmenting? In series opposing?

CHAPTER V

PARTS OF A DYNAMO-ELECTRIC MACHINE—FUNCTION, MATERIAL, CONSTRUCTION

1. Dynamo. A dynamo-electric machine, or a *dynamo,* is a machine for converting mechanical energy into electrical energy, or vice versa. It may be either a direct-current (d-c) machine or an alternating-current (a-c) machine. When used to convert mechanical energy into electrical energy, a dynamo is called a *generator*; when used to convert electrical energy into mechanical form, it is called a *motor.* The generic term of

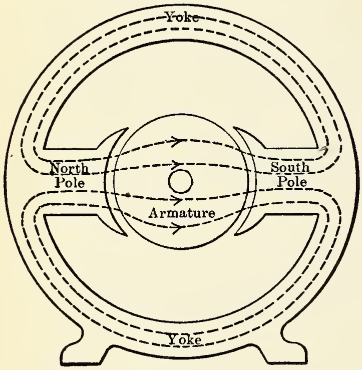


Fig. 5–1.

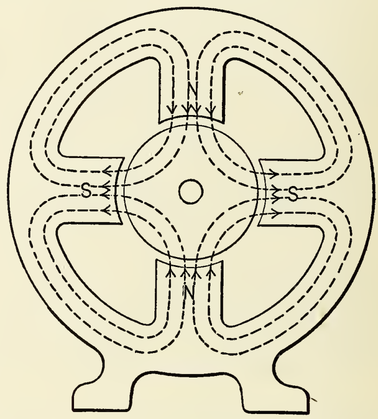


Fig. 5–2.

Fig. 5–1. The general form of the magnetic circuit of a modern bipolar machine; the yoke is double, each part carrying half as much flux as the pole pieces and the armature.

Fig. 5–2. For a multipolar machine the ring-shaped frame is used as in the bipolar; the length of magnetic path in a multipolar machine is less than it would be in a bipolar machine of the same capacity.

dynamo designates a reversible machine; any dynamo may be used either as a generator or as a motor.

2. Parts of a D-C Dynamo. A classification of the parts of a modern d-c generator or motor, together with their functions, may be made as follows:

1. *Field frame or yoke*; forms part of the magnetic circuit, supports the field poles and end frames.

2. *Field poles*; project from the yoke and carry flux to and from the armature.

3. *Armature core*; completes the magnetic circuit and carries the conductors which serve to generate voltage (if the machine is a generator), or to carry current by which torque is developed (if the machine is a motor).

4. *Commutator*; rectifies the alternating voltages induced in the armature conductors and, together with the brushes, forms the electrical contact between the revolving conductors and the external circuit.

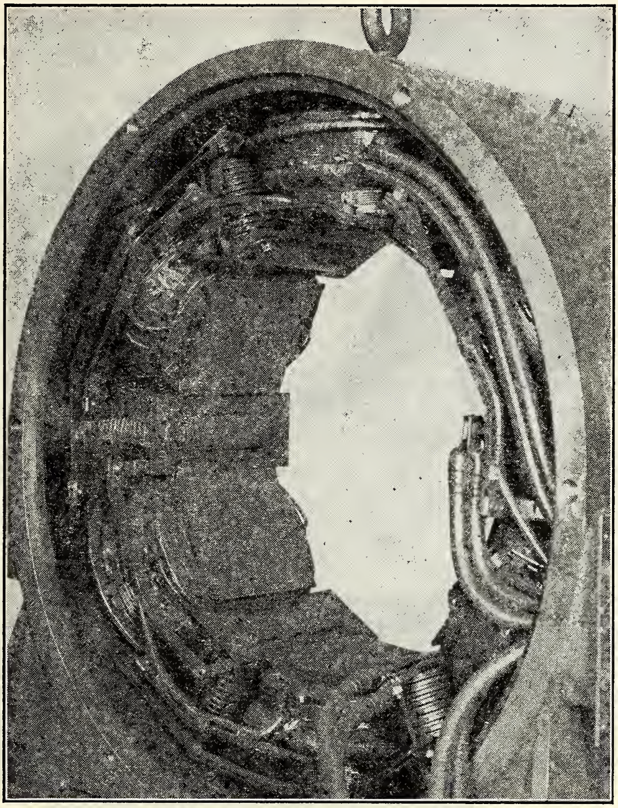


FIG. 5-3. The field structure of a modern d-c multipolar generator. *Courtesy of the Reliance Electric and Engineering Co.*

5. *Brushes and brush rigging*; the stationary portion for making a rubbing contact with the moving surface of the commutator.

6. *End frames*; members attached to the frame which serve to support the bearings; are used in all but very large machines.

7. *Bearings*; support the armature in proper alignment with respect to the field poles.

8. *Field windings*; supported by the field poles and set up the flux in which the armature conductors move.

9. *Armature winding*; seat of the generated voltage of a generator or torque in a motor.

3. Field Frame. Figure 5–1 represents the magnetic circuit of a simple *bipolar* machine, and Fig. 5–2 shows the magnetic circuit of a *multipolar* machine. In d-c machines, that part of the magnetic circuit constituting the frame and poles is stationary and is seen to be the part that carries

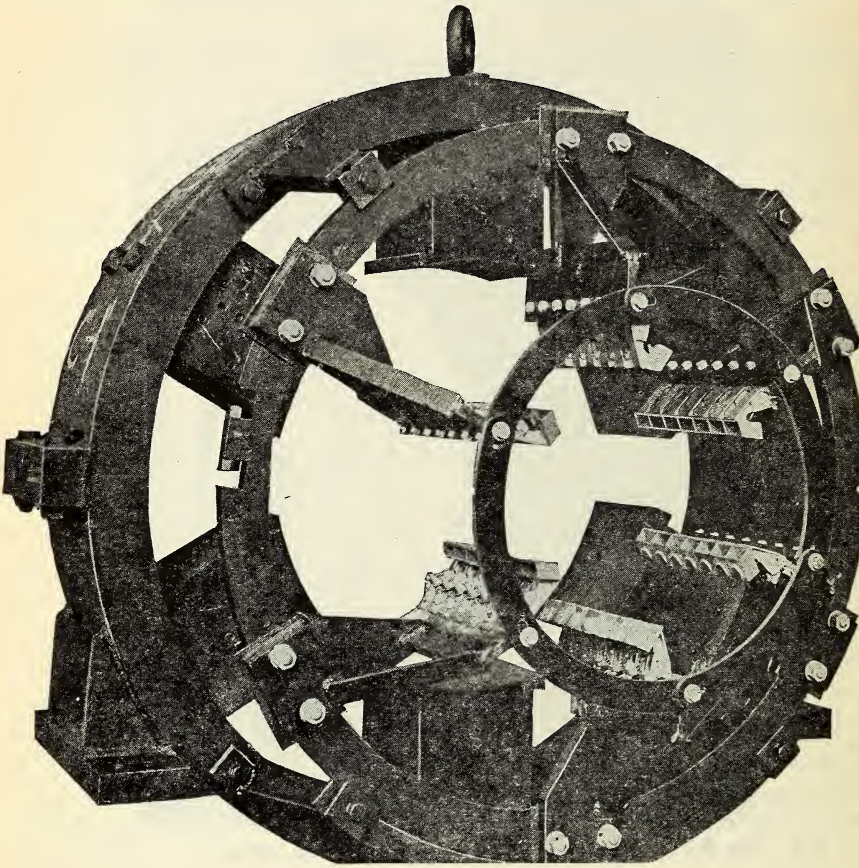


FIG. 5–4. A typical casting-less d-c stator. The entire frame and brush rigging are made from rolled steel shapes, cut by gas flame and assembled by arc welding.

the field coils, through which current is passed, to produce the magnetic flux. Figure 5–3 shows a large multipolar machine; the field frame is often split horizontally and the upper part fitted with an eye-bolt for lifting, to facilitate removal of the armature. The frame may also carry suitable footings by which the machine is firmly secured to its foundation.

According to present practice, the yoke is either a casting or is fabricated from open-hearth plates. In most machines a steel plate is bent

into the form of a cylinder, with the butting edges welded together. If a larger yoke is desired than can be made of a single plate, a number of plates are used and welded together. A completely fabricated frame, together with the necessary brush rigging, is shown in Fig. 5–4. Cast steel is preferred to cast iron because of its higher permeability, which allows the use of higher flux densities than are practicable with cast iron; because of this a field frame of steel, to carry a certain magnetic flux, is much lighter than one of iron. It is obvious that a yoke must contain enough metal to be rigid and self-supporting.

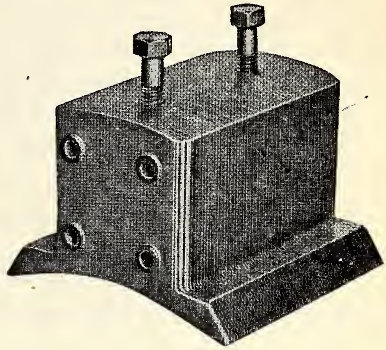


Fig. 5–5. The field poles of motors and generators are built up of laminations, perhaps $\frac{1}{16}$ inch thick, held together by several rivets which go through the laminations and are upset in the thicker end plates. *Courtesy of the Electrodynamic Works, Electric Boat Co.*

4. Material for Poles. The poles in modern machines are built up of sheet-steel laminations, punched out to proper form and held together tightly by bolts or long rivets, as shown in Fig. 5–5. The poles are attached to the inner side of the frame by means of bolts. The reasons for using laminated rather than solid poles will appear later. The poles are generally wider at the face next to the air gap than in the main body in order to spread the flux over a greater area where it crosses the air gap to the armature.

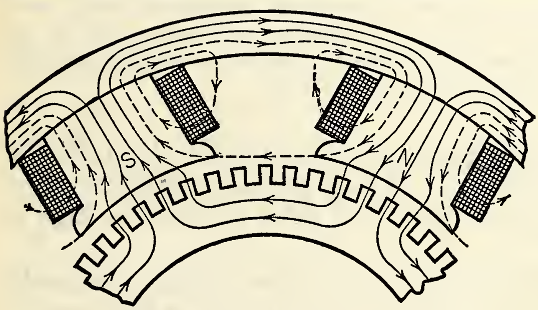


Fig. 5–6. Not all the flux set up by the field coils goes through the armature, where it is supposed to go. Flux which is distributed as shown by the dotted lines is useless; it is called leakage flux.

5. Leakage of Magnetic Flux. The function of the field coils and field frame is to produce a strong magnetic field in which the armature conductors may turn. Not all the magnetic lines generated by the field coils will pass through the armature core however; part of them will go by various so-called *leakage paths*, and this part is called the *leakage flux*. In Fig. 5–6 is shown the possible distribution of the leakage flux between two poles of a multipolar generator. The normal path of the flux is shown by the full lines, and the dotted lines show some of the leakage paths. In

a well-designed machine, the proportion of leakage flux to total flux is kept low; in modern machines it will vary from about 40 per cent in a small machine to perhaps 10 per cent in a large one.

6. Number of Poles. The number of poles of a machine depends in general upon its size, the speed at which it operates, and its voltage. Generator speeds are largely determined by the type of prime mover to be used; in the case of motors, the speed is determined primarily by the speed requirements of the load which the motor is to serve.

Whereas two poles are used for small machines, as the size increases the use of a multipolar frame results in shorter magnetic paths and therefore less magnetic leakage. It also results in a saving of material and in smaller field coils.

Machines of 5-hp capacity and less generally have a bipolar frame, i.e., have only two poles (there must always be at least two); for generators

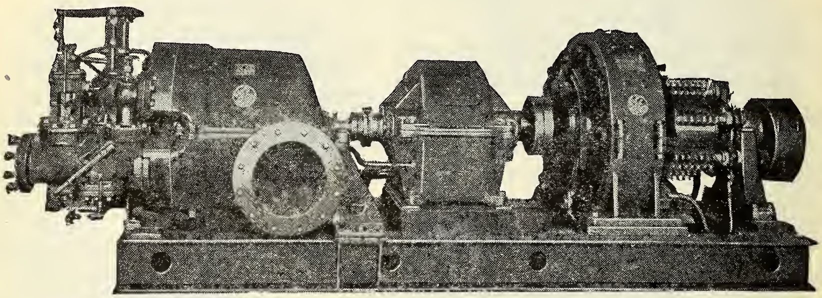


Fig. 5–7. The modern tendency is to drive all generators by turbine; by the use of reduction gears best speeds are obtained for both generator and turbine. In this cut the turbine is at the left, the gear case is in the center, and the generator at the right. *Courtesy of the General Electric Co.*

and motors of ordinary speeds, above 10 hp in capacity, the multipolar form of field frame is generally used. In large generators, driven by slow-speed reciprocating engines, the number of poles has been as high as twenty-four.

The multipolar frame, having more parts, is more expensive; generally low-speed or low-voltage machines will have more poles than high-speed or high-voltage machines of the same capacity. The reasons will appear later.

Small turbine-driven d-c generators up to 50 kw have been direct-connected and operate at speeds as high as 5000 rpm, but larger generators are connected to the turbine through a reducing gear, permitting both members to operate at their more economical speeds. Figure 5–7 shows a turbine-driven d-c generator in which a reducing gear is used.

7. Pole Tips. In earlier machines in which solid poles, cast integral with the frame, were used, *pole shoes* were attached to the inner faces of

the poles. The function of the pole shoes was to hold the field coils in place and also to spread the flux over a greater area where it left the pole and entered the armature, so that a greater number of armature conductors would lie in the flux at a given moment. The pole shoes therefore extended beyond the width of the pole, the extensions being known as the *pole tips.* The pole shoes were usually built up of laminations and were bolted to the inner faces of the solid poles after the field coils had been slipped over the poles. In modern machines the entire pole, including the pole tips, is built up of punched laminations, so that the pole shoe, as such, has disappeared, although the shape is much the same as it was and the pole tips are still relied upon to hold the field coils in place.

The pole tip under which a point on the armature first comes, as it rotates, is called the *leading pole tip*; the other is called the *trailing pole tip.* The term pole tip is used even when the pole has no projections, the edges of the pole itself constituting the pole tips in this case.

8. Air Gap. The minimum length of the air gap of a machine is fixed by consideration of mechanical clearance. The shorter the air gap, the lower will be the reluctance of the magnetic circuit, particularly since, air having unit permeability, the gap will constitute a large part of the total reluctance.

In the d-c machine the ampere-turns of the armature, when it carries load, exert a reaction on the main magnetic field, tending to oppose the field ampere-turns and causing twisting of the field, the effect being in proportion to the ratio of armature to field ampere-turns. It will appear later that, in a good machine, the field ampere-turns must always be greater than the armature ampere-turns. The number of field ampere-turns necessary to set up a required flux with an air gap of length just sufficient for mechanical clearance is considerably less than the armature ampere-turns. Therefore it is necessary to increase the field ampere-turns and, in order that the desired flux shall remain the same, the reluctance of the magnetic circuit must be increased. This is done by increasing the length of the air gap. The modern d-c machine has therefore an air gap longer than necessary for clearance; in large machines air gaps of a quarter inch or more may be found, even though they could be built with clearances of only a very small fraction of an inch.

9. Commutating Poles. Besides the main poles, most modern d-c machines have another set of poles which are very narrow in comparison with the main poles and are placed midway between them; these small poles are to help commutation and are called *commutating poles* (or sometimes *interpoles*). In small and medium-sized machines, commutating poles are usually solid; in larger machines they may be built up of laminations. A picture of a commutating pole for a small machine is given in Fig. 5–8. A frame equipped with commutating poles is shown in Fig. 5–9.

10. Armature Core. As previously stated, the magnetic circuit of any dynamo-electric machine may be considered as made up of two parts, one of which moves with respect to the other. The field frame carries the field windings; the other part of the magnetic circuit is called the *armature core.* On the armature core are placed the conductors which serve to generate the emf (if the machine is a generator) or to carry the current by which torque is developed (if the machine is a motor).

The armature core is always made of laminated iron, being built up by placing on the shaft a number of thin disks of iron and clamping them tightly together. The reason for making an armature core in this way, evidently much more difficult and expensive than if a solid piece of cast steel were used, will now be considered.

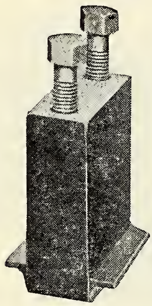


FIG. 5–8.

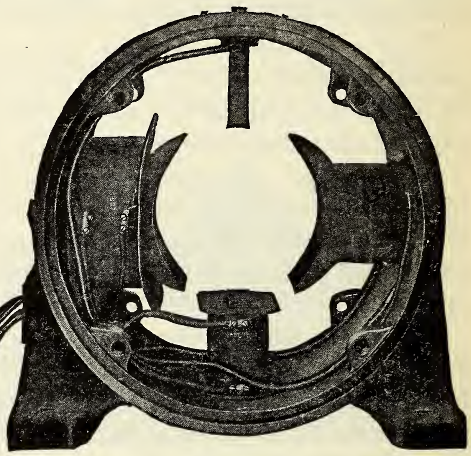


FIG. 5–9.

FIG. 5–8. Showing a commutating pole for a small machine. The small lugs, welded on, are used for supporting the coil.

FIG. 5–9. Nearly all modern machines have in addition to the main poles a set of narrow poles situated midway between the main poles. They are to facilitate commutation and are called commutating poles, or interpoles. *Courtesy of the General Electric Co.*

11. Experiment to Show the Advantage of Laminating the Core. Two armature cores, of exactly the same dimensions, were made for an experimental generator. One of the cores was of solid iron and the other was of thin iron plates clamped together; thin paper was put between every two plates. Then each armature core was in turn mounted in the field of the experimental generator, and the power required to rotate these bare armature cores at 1200 rpm, with a certain strength of magnetic field, was measured. When the laminated core was tested it was found that the required power (excluding friction) was 9 watts; the core was left running for some time, but it remained cool. The solid core was then substituted for the laminated one and the power consumption (again excluding friction) was 350 watts; after running for a few minutes it became so hot that

it had to be stopped, for fear of damaging the bearings. From this simple test it is evident that a solid armature core cannot be used in dynamo-electric machinery because, first, it requires so much power merely to rotate the armature core in the magnetic field and, second, the heat generated in the rotating core is so great that any windings on the armature core would be burned.

12. Losses in Armature Core. Hysteresis. The heat generated in any piece of iron rotating in a stationary magnetic field is due to two effects, namely, *hysteresis* and *eddy currents.* It is evident that as the armature core rotates it is magnetized first in one direction and then in the opposite direction, as a point on the core moves from under a north pole to a south pole. In Chapter II the question of hysteresis loss in a piece of iron going through magnetic reversals was explained. There is, then, this hysteresis loss in an armature core, and this loss may be kept small by using, for the core, iron that has a very narrow hysteresis loop, such as a specially treated alloy steel. After the sheet steel has been punched out in proper shapes for the core construction, these laminations are heated to a dull-red heat and allowed to cool slowly. This process, called *annealing,* results in a steel having a very narrow hysteresis loop.

13. Loss Due to Eddy Currents. Eddy currents (local currents in the material of the core itself) are produced in the rotating armature core because, as it moves through the magnetic field, voltage is generated in the iron. The direction of this voltage is parallel to the shaft, i.e., lengthwise of the armature core. Under a north pole it is in one direction and under a south pole in the opposite direction. As the solid iron core may be considered as one large conductor it is evident that these voltages will cause currents to circulate in the core. The direction of the voltages is shown by the solid arrows in Fig. 5–10, which represents a section taken parallel to the shaft; the path of the eddy currents is shown by the arrows and the dotted lines.

14. Reduction of Eddy-current Loss by Laminating the Core. Now, if the core is made of two parts, insulated from one another as in Fig. 5–11, the eddy currents will have to flow in two separate paths, as shown. The resistance of each of these paths is about twice as great as the resistance of the one path in the solid core, because, although the length of the eddy current path in Fig. 5–11 is about the same as that of Fig. 5–10 (more nearly true as thinner laminations are considered), the cross-sectional area of the path is about one-half as great as that of the solid core.

The voltage of each of the paths in Fig. 5–11 is one-half as great as that for the path of Fig. 5–10; hence the eddy currents in each lamination in Fig. 5–11 will be one-quarter as large as those of Fig. 5–10 (the voltage being halved and the resistance doubled). The I^2R loss of *each* path of Fig. 5–11 will therefore be one-eighth as great as that for Fig. 5–10, but

as there are two laminations to consider in Fig. 5–11 it is evident that the eddy-current loss of Fig. 5–11 is one-quarter as much as for Fig. 5–10. We thus arrive at the conclusion that *the eddy-current loss in an armature core varies as the square of the thickness of the lamination.*

In commercial machines the laminations are generally 0.014 inch thick. The laminations may be insulated from one another on large machines by a coat of insulating paint on each lamination; on small machines the oxide coating, formed on the iron sheet while it is being annealed, is relied upon for insulation. Owing to the oxide coating on the laminations, insulating paint, etc., only about 90 per cent (called the *stacking factor*) of the total

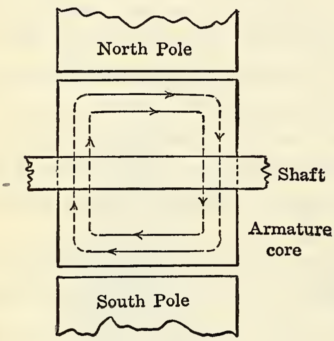


FIG. 5–10.

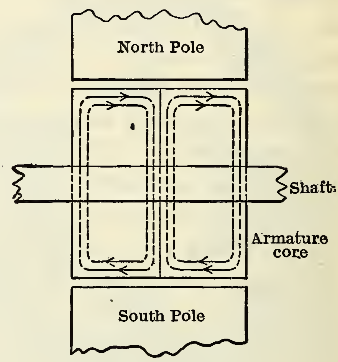


FIG. 5–11.

FIG. 5–10. If the armature core was a solid cylindrical piece of iron, eddy currents would flow in the revolving core as indicated here.

FIG. 5–11. By building the core of a number of disks the heating produced by the eddy currents is much reduced; the loss due to eddy currents varies as the square of the thickness of the laminations.

length of iron of an armature core, parallel to the shaft, is composed of iron.

15. Form of Laminations. In small armatures the laminations are solid disks, merely having a hole in the center for the shaft. In multipolar machines it is unnecessary to use solid disks, because in a large core made up of solid laminations there would be more iron than is necessary for the magnetic flux. The paths of the flux in a multipolar machine are shown in Fig. 5–12. It is seen that any iron inside the dotted circle is not utilized for the magnetic circuit.

The disks for large machines, therefore, are generally made ring-shaped; the iron which is punched from the inside of the disk may be used for the armature of a smaller machine. If the armature is very large (say more than 2 feet in diameter) the rings will not be in one piece; each lamination will be made up of several pieces. In building up an armature core of

such laminations, they are so assembled that the joints in one lamination do not come opposite those in the adjacent one.

16. Armature Spider. The ring-shaped armature core must be fastened to the shaft in some way, and for this purpose an armature spider is used.

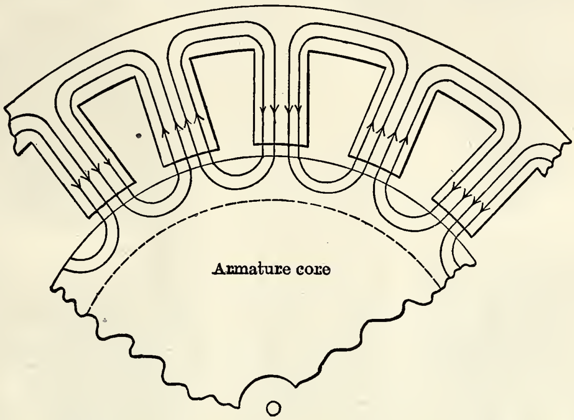


Fig. 5–12. For large machines it would be wasteful to use solid disks for building up the armature core; the interior of the core would carry practically no flux. For this reason the disks used are ring shaped, thus making the core a hollow cylinder.

The spider may consist of a cast-iron hub, through which the shaft is fitted, and from which radiates a set of spokes; on the ends of the spokes are lugs which are fastened by dovetailing keys to the inside of the armature core. The hub and spider may also be built up from plates and standard structural shapes, by welding. The armature punchings generally have dovetailed slots punched on their inside edge, in such a fashion that when the core is assembled they line up with one another and form a set of dove-tailed slots in the inside surface of the armature core, into which the keys in the spider fit.

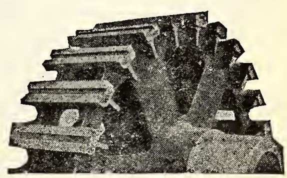


Fig. 5–13. Portion of a cast armature spider. The dovetailed lugs support the laminations and the extension of the hub supports the commutator.

In assembling such an armature, the spider, fitted with keys on the lugs, is laid horizontally on the shop floor (see Fig. 5–22). Then the laminations, one at a time, are slipped over these keys all the way around; when a complete ring of laminations has been placed, the next layer of laminations is put on in a similar manner, but so placed that the two rings "break joints", i.e., the joints between plates in the adjacent layers do not come opposite one another.

A cast spider is shown in Fig. 5–13; the extension of the hub furnishes a support for the commutator. A fabricated spider, supporting the arma-

ture of a large machine, is shown in Fig. 5–14. Another type of spider is shown in Fig. 5–41.

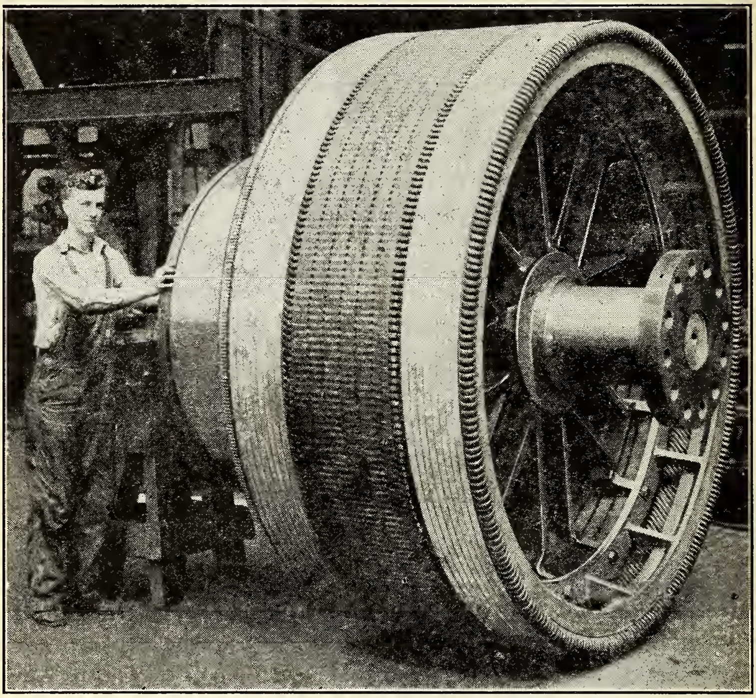


Fig. 5–14. Armature of a large generator showing the armature spider. *Courtesy of the Allis-Chalmers Manufacturing Co.*

17. Smooth and Slotted Cores. In the early types of dynamo-electric machines, the periphery (outside surface) of the armature core was *smooth*,

Fig. 5–15. The early machines had smooth cores, the winding being placed on the periphery and held in place by binding wires. Modern machines always use slotted cores, the windings being imbedded in the slots.

but in all modern machines the armature core is *toothed* or *slotted* (Fig. 5–15). In such a core there is a series of slots in the outer surface, running

parallel or with a slight skew to the direction of the shaft. This skew is introduced to make the machine more quiet in its operation. Slots parallel to the shaft often cause a whistle as they pass the poles and this is reduced by skewing the slots. These slots serve to hold the armature winding safely in place and protect it when the armature is out of the frame. In smooth-core machines the winding was held on the core by band wires laid

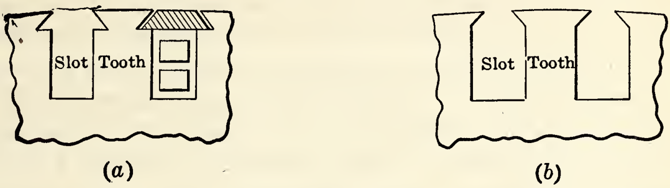


FIG. 5–16. Forms of teeth and slots; that shown in (a) is most generally used.

over the conductors. Frequently when such a machine was suddenly overloaded the winding would slip on the core.

18. Shape of Slots. The shape of the slot is generally determined by various factors in the design, and also, to some extent, by the kind of winding with which the armature is to be fitted. In Fig. 5–16 are shown representative forms of slots, that in (a) being known as an *open slot* with parallel walls, and that in (b) as a *semi-closed slot.* Practically all but very

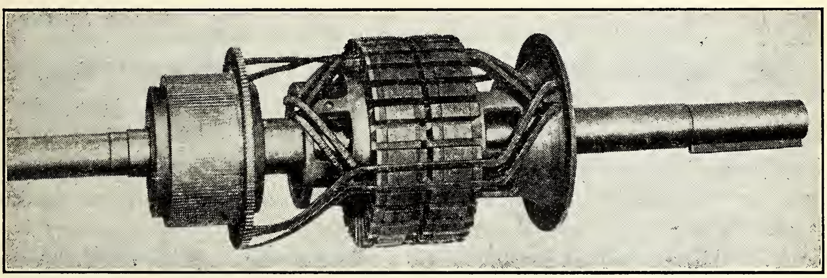


FIG. 5–17. Partially wound armature with open slots and showing two coils in place. *Courtesy of the Allis-Chalmers Manufacturing Co.*

small machines have open slots, the particular advantage of this type being that formed coils (previously bent up and insulated) can easily be inserted; they are held securely in the slot by a wooden or fiber wedge driven into place, as shown in the second slot of (a). (See also Fig. 5–65.) With open slots there are usually two coil-sides per slot in the winding, the coil-sides being placed in the slot one above the other, as indicated in (a). A partially wound armature with open slots is shown in Fig. 5–17; the method of placing the coils is apparent.

The semi-closed slot has the advantage of affording better protection

and support for the coils than does the open slot; also, as may be seen by reference to Fig. 5–18, the effective area of the air gap is much larger when a semi-closed slot is used, owing to the fact that the flux can fringe out from the edges of the teeth and thus use a greater percentage of the area under the pole face for carrying flux.

The fringing effect spoken of above also occurs on the edges of the ventilating ducts, the spaces left between laminations for the passage of air. Figure 5–19 shows a section of an armature having three ducts, and it is indicated how the flux, by fringing, actually uses that part of the air gap opposite these ducts. Ventilating ducts are discussed later on.

The coil may be fitted into the slot of Fig. 5–16*b* by having it only partially formed; that part of the coil which is to be inserted into the slot is left untaped, so that the wires forming the coil can be separated and

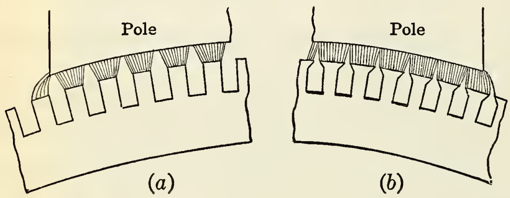


Fig. 5–18.

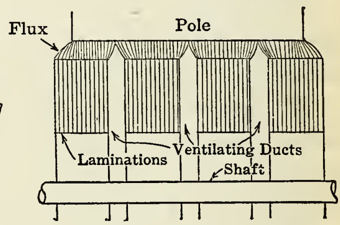


Fig. 5–19.

Fig. 5–18. A core with partly closed slots gives a larger useful area to the air gap, as shown by the flux distribution in these two diagrams.

Fig. 5–19. Section of a core showing the ventilating ducts and how the flux fringes across them in the air gap.

pushed into the slot one by one. This is generally done in the case of small, low-voltage machines, where the individual conductors of the coil are wire of small size and therefore flexible.

19. Pole-face Losses Produced by Slotted Cores. With the tufting of the flux opposite the teeth, as shown in Fig. 5–18*a*, any element in the pole face is subjected to considerable variation in its flux density, as the tufts of flux move across the pole face. Accordingly, voltages will be induced in the pole faces which in turn set up eddy currents, as represented in Fig. 5–20*a* by the dotted lines. On considering such an element of a supposedly solid pole face it is evident that the amount of flux through this area varies periodically as the teeth and slots alternately go by. This increase and decrease in flux will produce alternating currents, as indicated by the dotted lines, having a frequency equal to the number of teeth passing the pole face per second. By the use of laminated pole faces, these eddy currents are confined to the narrow paths available in each lamination, as suggested by Fig. 5–20*b*.

In addition to the heat generated by the eddy currents in the pole faces, a small amount is produced also by hysteresis, this being caused by the periodically changing flux density.

20. Ventilation. The question of ventilation of electric machinery is closely associated with the allowable operating temperature and the machine rating (see Chapter VIII). There are, however, certain features of construction of the armature core which are incorporated in the design to help keep the armature well ventilated and cool.

It is well known that any body will give off heat if it is hotter than its surroundings, and that the rate at which heat is given off increases as the temperature of the body above its surroundings increases. All the losses of an electric machine produce heat, and the machine must dissipate this

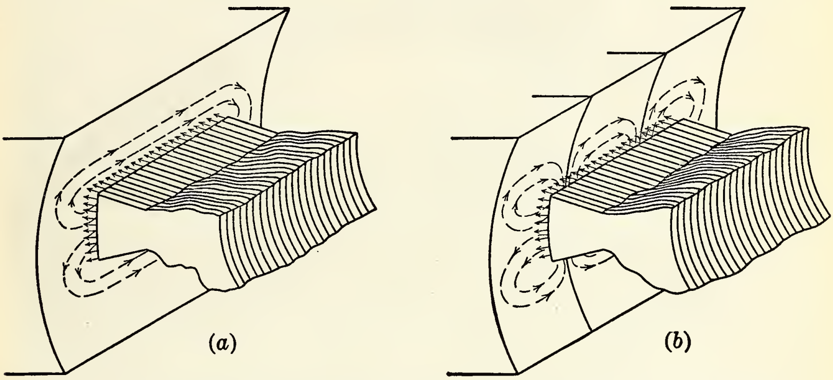


Fig. 5–20. If the pole face were solid it would have large eddy currents flowing in it, in the paths shown by the dashed lines in (*a*); by laminating the pole face the eddy-current paths are broken up and the heating the currents produce is much diminished, just as it is in the armature core. Actually the poles are of thin laminations instead of the few thick ones shown (for the sake of simplicity) in (*b*).

heat, in the equilibrium condition, at exactly the same rate as it is produced. This calls in turn for the machine to be at a certain equilibrium temperature; if less heat is dissipated than is produced, the temperature will rise, thus increasing the rate of heat dissipation, until the equilibrium temperature is reached. At this equilibrium temperature the rate of heat dissipation is just equal to the rate of production.

It is evident, since the machine insulation may be made of organic materials, that this equilibrium temperature must not be high enough to destroy the insulating material. Certain maximum allowable temperatures have been fixed for the commonly used materials (see Chapter VIII) and the machine must be rated so that the heat produced by its losses, when it is carrying its rated load, does not cause a higher equilibrium temperature to be reached.

One way of keeping down the temperature of a machine is continually to force cool air through it; the temperature of a machine may thus be kept down by properly ventilating it. Heat is generated within the armature core by the hysteresis and eddy currents, and it is necessary to carry off this heat to keep down the core temperature. For this purpose, ventilating ducts are built into the armature core. As the laminations are being assembled, spacers are introduced about every three inches of core. These spacers are placed radially, so that air passages from three-eighths to one-half inch wide are formed, through which air can circulate from the inside periphery of the core to the outside. As the armature revolves, cool air is drawn in through the ends of the core by the spider and is thrown

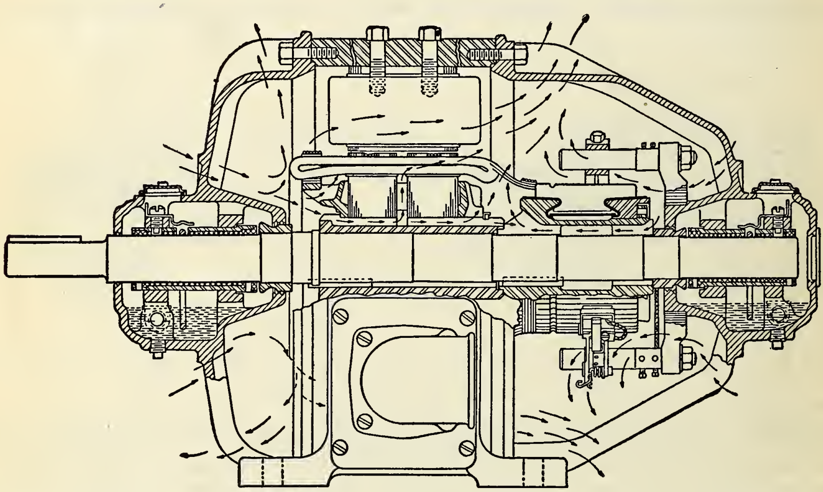


Fig. 5–21. Except in the smallest sizes, armature cores are assembled so that air ducts are formed. Air circulates through them as shown here, keeping the core much cooler than it would otherwise be.

by centrifugal force through these ventilating ducts out against the pole faces, etc. Figure 5–21 shows how these ducts are placed in an armature core and the paths taken by the air as it passes through them; Fig. 5–22 shows a partially assembled core, in which the air ducts may be seen, and on the top of the core may be seen one of the spacing laminations which has been specially formed with radial ridges in it. These ridges serve to space the laminations properly and so to form the air ducts as the core is built up. The effective length of the armature, so far as carrying flux is concerned, is of course less by the total width of the ducts present. Ventilating ducts may also be seen in the armatures of Figs. 5–14 and 5–17.

21. Commutator—Function. The emf generated by a coil of wire as it rotates in a magnetic field is an alternating one, i.e., the voltage is first

in one direction and then in the opposite direction, a complete reversal taking place as a coil-side moves by one pair of poles. Now, even if an armature winding is made up of a great many coils, connected together in any fashion whatever, the emf generated in this winding as it rotates in the magnetic field is an alternating one, and if the ends of the winding are connected to insulated rings on the shaft, with brushes bearing on the rings and connected to the external circuit, the voltage which such an

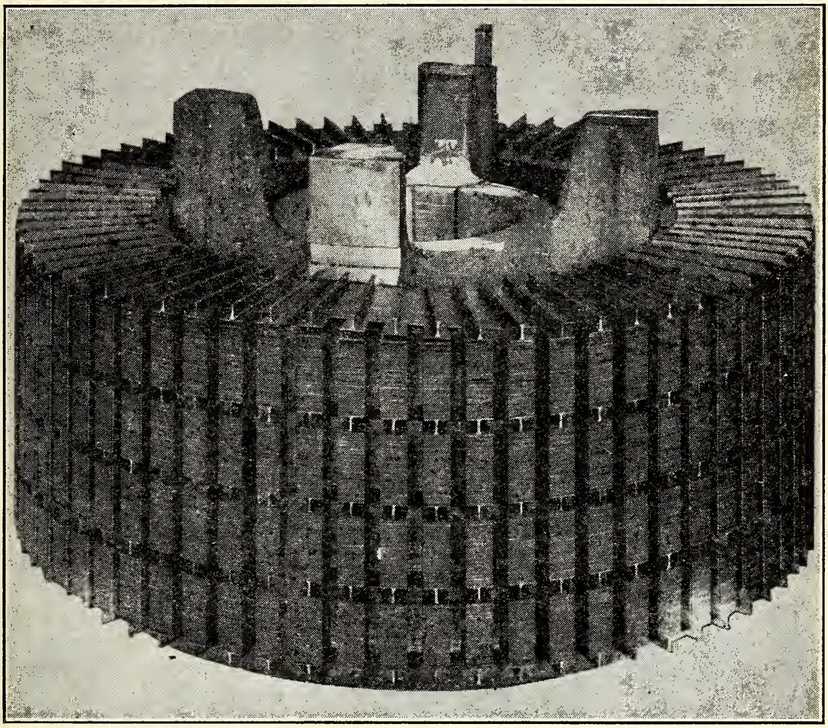


Fig. 5–22. This picture shows the stacking of the laminations upon the armature spider. The air ducts are formed by special laminations, one of which is shown on the top of the stack. *Courtesy of the General Electric Co.*

armature winding impresses on the external circuit is an alternating one. The current which would flow in such an external circuit would be an *alternating current*; the current would flow through the circuit in one direction for a short time, would stop flowing altogether, and would then reverse and flow in the opposite direction, etc. Such an alternating current is useful in many classes of service; but for many other classes of service, such as electroplating, electric railways, etc., a current which runs continuously in the same direction through the circuit is required.

The current through a circuit will not be continuously in one direction unless the machine which is forcing current to flow through the circuit impresses on the circuit a voltage which continuously acts in one direction.

The elementary generator, having slip rings and brushes connecting its windings to the external circuit, could not do this. The *commutator* is a device which serves to reverse continually the connection of the armature coils to the external circuit, so that, at the time the emf in the coils reverses, the connection of the coils to this external circuit reverses also, thus maintaining on the external line a uni-directional voltage.

22. Construction of the Commutator. The function and action of the commutator will be taken up in detail in Chapter VI; here we shall con-

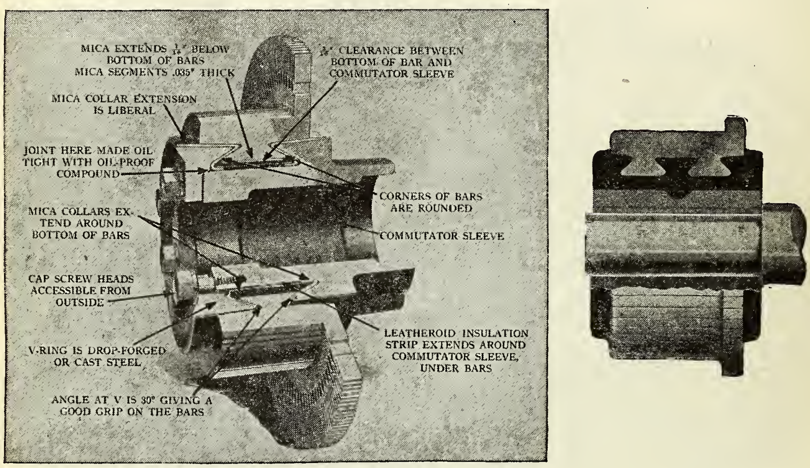


Fig. 5–23. Cross-section through two types of small commutators, showing on the left a built-up construction and on the right a form using molded insulation. The latter form is used only on very small machines. *Courtesy of the Reliance Electric and Engineering Co. and the Electrodynamic Works, Electric Boat Co.*

sider only the mechanical features of the commutator. It consists essentially of a set of copper bars of such a shape that, when assembled parallel to each other, they form a hollow cylinder. As they are assembled, insulation is put between adjacent bars so that every bar is well insulated from every other. Two forms of commutators for small machines are shown in Fig. 5–23. The commutator of a large armature may be seen at the left in Fig. 5–14.

In taking up the construction of the commutator we shall consider the *bars* themselves, the *insulation* used in the commutator, and the *commutator spider*, which serves to clamp the bars together and hold them on the armature shaft.

23. Form of Bars. The bars themselves are always made of copper, because it is a good conductor, is easy to shape properly, and wears well.

Generally, these bars are stamped or drop-forged from copper rods, the cross-section of the finished bar being trapezoidal, so that, as the bars are assembled side by side, they form a cylindrical ring. The angle between the two sides of a bar depends upon how many bars are to be used in the commutator.

The two forms of bars in Fig. 5–24 show different methods for connecting the armature coils to the bars. In Fig. 5–24*a* the coil ends are

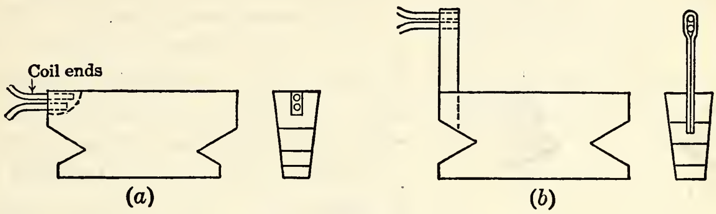


FIG. 5–24. Commutator bars showing coil connections.

slipped into grooves and soldered, a method suitable for small machines. As machine sizes increase, the diameter of the armature becomes greater than that of the commutator so that *risers*, or strips of copper attached to the bars, extend radially to the coil ends (Fig. 5-24*b*). The method of attaching the coil ends to the risers varies; often, in machines of medium capacity, the riser may be a part of the commutator bar.

24. Insulation. As the bars are assembled, sheets of insulation must be placed between every bar and its neighbor; insulation must also be used on the ends and the bottom of the bar, to keep it from contact with the device which clamps the bars together and supports the commutator upon the shaft.

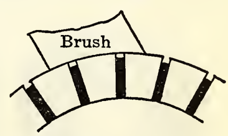


FIG. 5–25. In many modern machines the mica is undercut below the surface of the commutator.

Formerly a special grade of mica was used exclusively between the bars, a grade which had about the same wearing qualities as the copper bar itself. If the mica used was too tough and wore away more slowly than the copper, *high mica* would result, i.e., the mica insulation would soon project above the copper bars. High mica, presenting a rough surface to the brushes, resulted in excessive brush wear, and by holding the brush off the copper broke the circuit and caused sparking. In more recent practice the mica is cut away about one-sixteenth of an inch below the surface of the commutator, obviating the necessity of using mica of definite wearing qualities. Such an *under-cut* commutator is represented in Fig. 5–25.

The insulation between the clamping device and the bars is built up in the form of a ring-shaped piece or collar, with V-shaped cross-section. As

pure mica sheets are very brittle and are difficult to obtain in larger sizes, "*built-up mica*" or *micanite* is generally used. This is made by cementing very thin, small sheets of mica together by some such flexible binder as the natural resins (shellac, copal gum), synthetic resin (Glyptal*), varnishes, etc. By a proper drying and compressing process a flexible sheet of micanite is obtained, which, when warmed, may be formed into any desired shape.

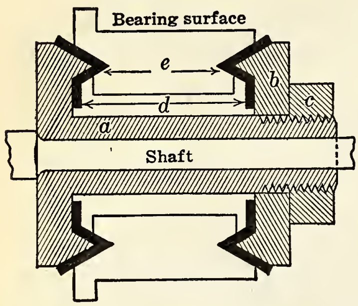


Fig. 5–26. Cross-section of a small commutator. As the nut *c* is tightened, all the pressure occurs on the surfaces *e*, so that the bars are forced into a compact cylinder.

25. Commutator Assembly. The assembly of a small commutator is shown in Fig. 5–26. It consists of a cast-iron sleeve, *a*, which is attached to the shaft after the commutator has been placed in its proper position, a loose end ring, *b*, and a lock nut, *c*. The micanite rings are shown at *d*. When the commutator has been properly assembled, the two surfaces marked *e* receive all the pressure as the lock nut, *c*, is tightened; in this way the lock nut serves not only to tighten the commutator bars endwise, but also to squeeze them tightly together by forcing them into a cylinder of smaller radius. If the distance between the sleeve, *a*, and the bottoms of the bars is small, additional insulation may be placed between them; an air space is shown in this region in **Fig. 5–26.** (See also Fig. 5–23.)

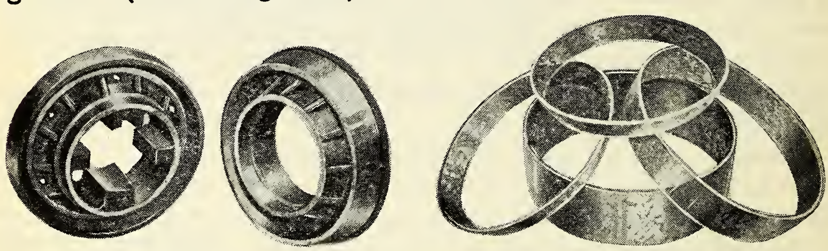


Fig. 5–27. Another type of small commutator. At the left are shown the rear and front commutator heads which support the commutator bars. On the right are shown the micanite rings which form the necessary insulation. *Courtesy of the Electrodynamic Works, Electric Boat Co.*

Another form of commutator construction is suggested in Fig. 5–27. Two heads with the bars between are drawn tightly together, the rear head being supported on the shaft. The necessary micanite insulating rings are shown on the right.

* General Electric Company.

As the commutator increases in diameter the sleeve also becomes larger in diameter than the shaft and the sleeve will be supported by some form of spider, of cast iron or of fabricated material.

26. Number of Commutator Bars. The number of bars to be used in a commutator depends principally upon the voltage for which a machine is designed. The proper number of bars for a given voltage is the result of a compromise between operating characteristics and the cost of building the commutator. The fewer the number of bars the cheaper will be the cost, but the operating characteristics of the machine will not be as good as if it had a great many bars.

In general it has been found that, *if the average voltage between commutator bars is less than* 15 *volts,* the machine will be satisfactory. As the full voltage of the machine exists between adjacent sets of brushes, this means that the

$$\text{Minimum number of bars} = \frac{\text{voltage of machine}}{15} \times \text{number of sets of brushes.}$$

For many reasons this rule is only approximate. The voltage between adjacent brush sets is not distributed uniformly among the commutator bars between them. Between a pair of adjacent commutator bars on one side of a brush the voltage may be 25 volts, and at the same time two bars on the other side of the brush may have a potential difference of only one or two volts. The formula considers only the *average voltage* between bars and so gives the *minimum* number of bars that should be used.

27. Effect of Using Too Few Bars. The mica insulation between adjacent bars is generally about 0.03 inch thick; such a thickness of mica will withstand a voltage of perhaps 10,000 volts, so that the limit of 15 volts per bar is evidently not fixed by the dielectric strength of the mica. The difficulty which arises if the voltage per bar gets too high is caused by *current leaking over the surface of the mica insulation.* The surface of the mica will have more or less dust rubbed into it and may become soaked with oil. If this oil becomes carbonized the layer of dust and oil between bars becomes a fair conductor. If the voltage per bar is too great under such conditions, current leaks over the surface of the commutator from one brush to the next and a "flash-over" may occur; i.e., a short circuit may occur between a pair of brushes, the short-circuit current following the surface of the revolving commutator, burning brushes, studs, etc.

Another effect of too few commutator bars is to increase the amplitude of the pulsations in the brush voltage of the machine. The function of the commutator is to change the alternating voltage of the armature coils to a uni-directional voltage at the brushes, and to achieve this the ends of the armature coils are connected to adjacent bars. To generate a given voltage will require a number of conductors upon the armature, these being

joined in series to form coils. The fewer the number of commutator bars, the more turns there must be per coil. As will be seen when commutation is studied, the number of coils and bars between brushes varies from instant to instant, so that the voltage of the machine will pulsate. These pulsations are more or less objectional and their amplitude increases as the number of commutator bars decreases. Furthermore, with a small number of coils, and therefore of bars, the number of turns per coil becomes large. This causes the self-inductance of the coils to reach values which lead to further commutation difficulties, as will be seen later.

28. Brushes. As the commutator, to which the armature coils are attached, revolves with the armature, it is necessary to have some stationary conductors for making a rubbing contact with the moving surface of the commutator, the external circuit being connected to these stationary conductors. Such conductors are called *brushes;* they are almost always made of *carbon blocks*, but may sometimes be made of *copper leaves*, *copper gauze*, etc. The choice between copper and carbon depends entirely upon the voltage for which the machine is designed. A low-voltage machine, such as is used in electroplating, must be equipped with copper brushes; carbon brushes would not serve at all. For machines of voltages of 100 and more, carbon brushes must always be used. The reason for this will appear later.

29. Contact Area and Safe Current-density of Brushes. The brushes and commutator form a moving contact surface, across which all the current which flows from the armature to the external circuit must pass, and it is necessary that this surface be kept clean.

The area of the brush where it comes in contact with the commutator surface is called the *contact area* of the brush. The current which can be carried safely by a square inch of contact area depends upon the two materials forming the contacting surfaces. If a copper brush is bearing on the commutator surface and the brush has 1 square inch of contact area, 150 to 200 amperes may safely be carried from the commutator by the brush, if the brush is made of carbon and has 1 square inch of contact area, not more than 40 to 60 amperes may be safely carried. If these values are exceeded, the brush and commutator will get too hot, owing to the resistance of the brush contact, as will be discussed later.

If either the commutator or brush surface becomes rough, the contact area is very much diminished. If a piece of dirt works into the contact surface of a brush and so makes a projection on this surface, it is evident that the brush will touch the commutator only at this projecting place, unless the brush is very *flexible.*

30. Flexibility of Brushes. A brush made of copper leaves or wires is very flexible, whereas one made of a carbon block is not flexible at all. Therefore, if a slight projection on the commutator lifts off one part of a copper-brush contact surface from the commutator, the rest of the brush

may stay in contact with the commutator owing to its flexibility. But if one corner of a carbon brush is lifted from the commutator, the whole brush leaves the commutator surface and the circuit is practically opened.

If the machine is delivering current to the external circuit when this happens, sparking will occur at the commutator surface, and if this continues for any length of time the surface of the commutator becomes so roughened as to be unserviceable. So far as flexibility is concerned, therefore, copper is preferable to carbon. The non-flexibility of the carbon brush is partly overcome by using, instead of one big brush, several small

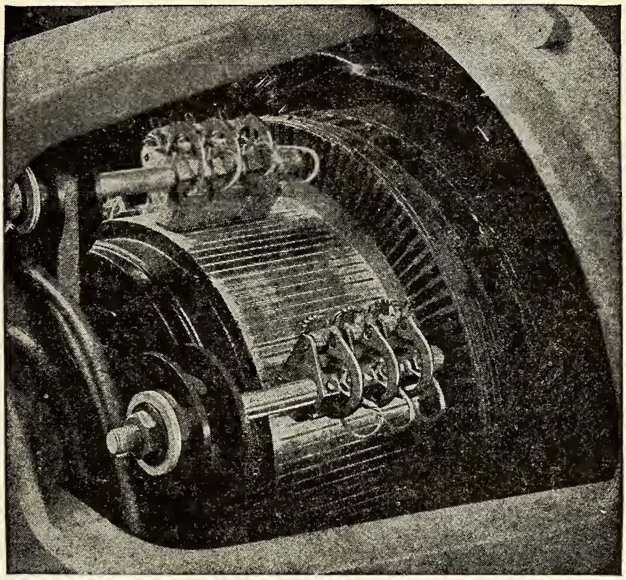


Fig. 5–28. To obtain flexibility, a carbon brush is generally made up of several small brushes, independent of each other. Each has its own spring to keep it in contact with the commutator. Two sets of brushes are shown above, with three brushes per set. *Courtesy of the Reliance Electric and Engineering Co.*

brushes, each capable of movement separately. Then, if one of these small brushes is lifted from the commutator accidentally, no open circuit is produced because the rest of the small brushes are still making contact. Figure 5–28 illustrates this construction; on each of the two brush-holder studs shown there are mounted three separate brushes, each of which is free to move by itself.

31. Brush Holders. The individual brushes are held in brush holders which in turn are attached to the brush-holder studs. The brush holder permits the brush to slide freely and follow any irregularities in the commutator surface, the brush being held against the commutator surface by a spring.

In order that there may be good electrical contact between the brush and the brush holder, and also to prevent the carrying of current by the spring, a copper pig-tail is used. If the steel spring carries too much current it will become hot and lose its temper. The pig-tail consists of many strands of fine wire braided into a single conductor, which is firmly molded into the brush at one end or clamped thereon, and screwed to the brush holder at the other, as in Fig. 5–29.

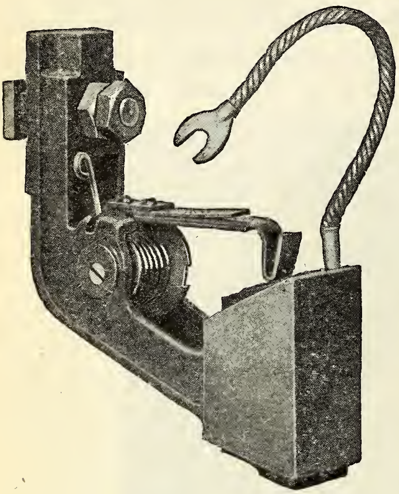


Fig. 5–29. To lead the current from the brush to the brush holder a flexible copper cable (called a pig-tail) is generally used. This is firmly imbedded in the carbon brush on one end and clamped securely to the brush holder on the other. *Courtesy of the General Electric Co.*

32. Number of Sets of Brushes. There must be at least two brush-holder studs, one at which the current leaves the machine and another at which it enters. With the type of armature winding commonly employed, as many sets of brushes (i.e., groups of brushes on the same stud and at the same point on the commutator) are required as there are field poles. A 12-pole generator would have 12 brush-holder studs, mounted rigidly on a *brush-holder yoke*, such as the one shown in Fig. 5–30 (also shown on the extreme left in Fig. 5–28 and on the right in Fig. 5–21). The brush-holder studs are equally spaced on the yoke, but insulated from it, and every other stud is electrically joined together by wires (Fig. 5–30), or, in large machines, by rectangular copper straps, called the *bus rings*. Wires attached to the bus rings form the two terminals of the generator.

Figure 5–29 shows a typical brush holder; Fig. 5–30 shows the brush rigging for a 4-pole d-c generator. Figure 5–4 shows the brush rigging used with a much larger machine.

33. Superiority of Carbon for Brushes. As previously mentioned, carbon brushes are used on all machines except those designed for electroplating, and this in spite of the fact that copper is a better conductor than carbon and requires only about one-quarter as much contact surface for a given current. There are two important reasons why carbon is preferred to copper for brushes: first, the mechanical wear on the commutator is much less with carbon than with copper; second, it is practically impossible to obtain sparkless commutation when copper brushes are used, on any but low-voltage machines.

Carbon brushes are made in varying degrees of hardness to suit the requirements of commutation. Graphite in a brush serves as a commutator lubricant. Low-voltage machines, such as the lighting generators and starting motors of automobiles, are generally equipped with brushes made of a mixture of carbon and metallic copper.

34. Sparkless Commutation. The commutation is said to be sparkless, or "black," when no sparking takes place at the contact surface between the brush and commutator. It is very important that sparkless commutation be obtained, because, under the action of sparking at the brush contact, the commutator very quickly roughens, which makes the sparking worse, and so the machine is soon rendered unfit for service. The explanation of this effect (i.e., sparking produced by copper brushes) will be taken up in a later chapter. It must be borne in mind here, however, that the sparking with copper brushes is not due to such causes as rough commutator, etc.; no matter how smooth the commutator may be, or how well fitted the brushes may be, this sparking cannot be eliminated.

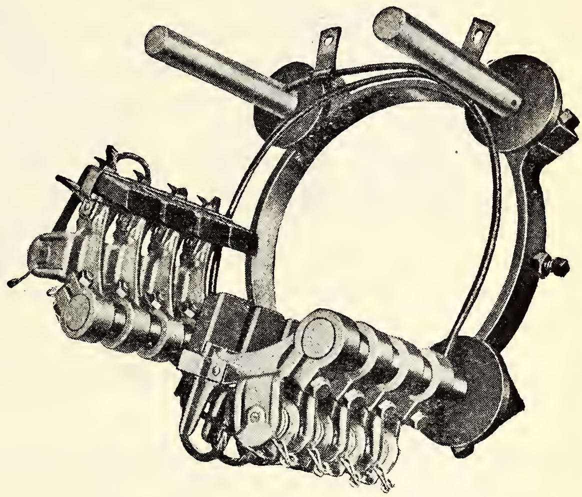


Fig. 5–30. Complete brush rigging for a four-pole machine. *Courtesy of the Electrodynamic Works, Electric Boat Co.*

35. Pressure of Brushes. The springs on a brush holder are adjustable, so that the pressure exerted by a brush on the commutator may be varied as desired. If too little pressure is used, the contact is not good, and the electrical resistance of the contact becomes high; hence the I^2R loss at this place becomes too great and the brush will overheat. If too much pressure is exerted by the brush on the commutator, the power used up by mechanical friction of the brushes becomes too great, and the brushes and commutator will get hot from this cause. It has been found that with carbon brushes the best results are obtained when the springs are adjusted to give a brush pressure of about 1.4 pounds per square inch of contact surface; this value, however, may be anywhere between 1 pound and 5 pounds, depending upon the quality of the brush, peripheral speed of the commutator, etc.

36. Resistance of Brush Contact. The resistance of the contact surface of a carbon brush and commutator is a variable depending upon the current density at the contact surface. As the current density increases, the resistance decreases; the variation of the resistance with the current takes place in such a manner that the *IR drop at the contact surface is nearly constant, and not dependent upon the current.* Although this *IR* drop is slightly different with different types of brushes and with the different grades of carbon employed, it is safe to assume that, on the average d-c machine, with the commutator in good condition, the drop is about *one volt per brush contact.* As there are always two brush contacts in series, the *total brush contact resistance drop in any d-c machine is about two volts.*

37. Brushes on Low-voltage Machines. It is because of this contact resistance drop that carbon brushes are never used on machines of very low voltage.

If, for example, one considers a machine delivering 100 amperes at 6 volts the reason will be obvious. The useful output of such a machine would be 600 watts. The power lost at the brush contact, if carbon brushes were used, would be the two-volt drop multiplied by the current of 100 amperes, or 200 watts. If there were no other losses the efficiency would be

$$\text{Per cent efficiency} = \frac{\text{power output}}{\text{power input}} \times 100 = \frac{600}{600 + 200} \times 100 = 75 \text{ per cent}$$

Actually other losses in the machine would reduce this still further and a very inefficient machine would result. As can be seen, this drop of 2 volts at the brush contacts is not such an important factor in determining the efficiency if the terminal voltage is high.

38. Bearings. The bearings of most generators and motors are of the self-lubricating sleeve type. As may be seen in Fig. 5–31 this type of bearing consists of a brass or cast-iron sleeve lined with babbitt metal as the wearing surface. Oil rings, suspended from the shaft, dip into an oil reservoir below the shaft, and drag oil with them, as they rotate, to the top of the shaft from where it runs down into the bearing.

Ball and roller bearings are also widely used for motors. These bearings are packed with grease and have the advantage that they can be made dustproof more easily and require less attention.

39. Field Windings. The field windings of a dynamo-electric machine serve to force the flux through the whole of the magnetic circuit. The field coils are always placed on the poles; the mmf produced by these field coils must be sufficient to force the magnetic flux through the poles, yoke, air gaps, and armature core. If the length, area, and *B-H* curve of each portion of the magnetic circuit are known, the number of turns required in one field coil and the current necessary can easily be calculated.

40. Magnetic-field Calculation. The method used for calculating the ampere-turns required to set up a given total flux in a magnetic circuit has already been given in section 38, page 54. It was there pointed out that the practical method is to use the *B-H* curve instead of the method involved in Eq. (37), page 52.

It is the practice of American designers to work with inches and to

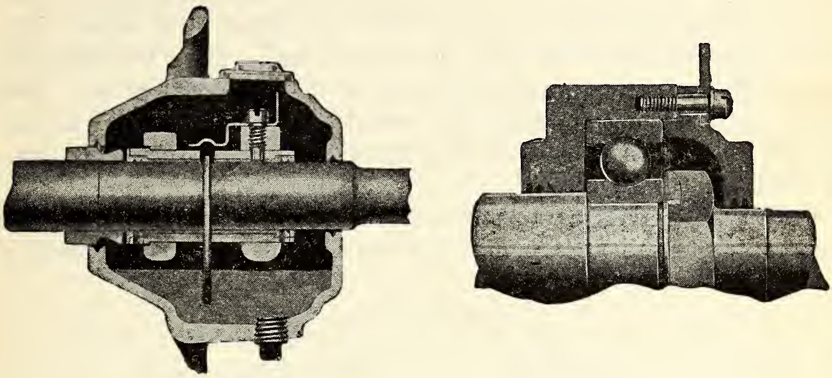


FIG. 5–31. On the left is shown a cross-section of a sleeve bearing with the oil ring hanging in the lubricating oil. As the shaft turns, oil is carried up onto the shaft by the ring and flows into the bearing. On the right is shown a cross-section of a ball bearing.

express densities in lines per square inch. A set of magnetization curves plotted between flux density in lines per square inch and ampere-turns per inch is given in Table X of the Appendix.

The first step in designing the magnetic circuit is to choose suitable flux densities for the various parts. The following table gives the limits for the values generally used in American practice:

TABLE A

Part	Material	Kilolines per Square Inch
Field yoke	Cast steel	70–100
Field yoke	Cast iron	35– 70
Field yoke	Open-hearth plates	70–110
Pole core	Steel, or laminations	70–100
Air gap	Air	40– 70
Armature teeth	Steel laminations	80–125
Armature core	Steel laminations	60– 90

It will be realized that the design of electrical machinery becomes a compromise between many factors. The designer, guided by the cost and performance of machines previously built, tentatively decides upon the constants and dimensions of a new machine and then, after several refinements, gradually reaches values which best meet the required conditions.

Example 1. Consider the bipolar field frame for a modern small machine, as shown in Fig. 5–32. The magnetic circuit consists of a double yoke, two poles, two air gaps, and the slotted armature, all in series. The field is to be excited by two coils, one on each pole. These two coils give mmf's which act in the same direction in the magnetic circuit. The ampere-turns required for the whole circuit will be calculated as though only one coil were to be used; one-half this number will be the proper number of ampere-turns per coil.

In the yoke of this circuit the flux from the pole divides, half going one way and half the other. The flux in going through the armature teeth is

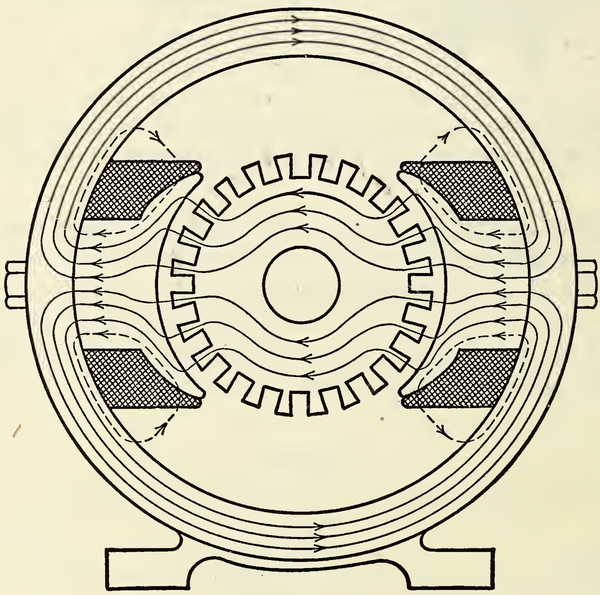


Fig. 5–32. Cross-section of the essential parts of a bipolar machine. Only a very few flux lines are shown, the leakage lines being drawn in dotted lines.

very dense, and although the length of path is small it is better to figure this as a separate part of the magnetic circuit, instead of treating the teeth as part of the armature core.

The diagram of Fig. 5–32 shows the location of the field coils, leakage lines, etc. The poles are made of laminated steel and are bolted to the

cast-steel yoke. The area of the teeth is taken as being approximately equal to the cross-sectional area of as many teeth as lie under the pole face. Practically all the flux leaving the pole face crowds into the teeth, very little of it going down to the armature core by the slots. Although it is incorrect, as an approximation we shall consider the area of the air gap as equal to that of the pole face; the useful area is actually somewhat less, owing to the effect of the slots.

Suppose that the necessary flux in the air gap entering the armature core is 1,200,000 lines. There must be more flux than this through the poles and yoke, because of the leakage lines. If the leakage factor is taken as 1.25, the flux through the yoke and poles is $1{,}200{,}000 \times 1.25 = 1{,}500{,}000$ lines.

In this example the cross-sectional areas of the different parts of the magnetic circuit are given in the following table as well as the lengths of the paths. From the flux and cross-section, the density in each part is calculated, and then from the magnetization curves the ampere-turns per inch are obtained; this quantity, multiplied by the length of path in that part of the circuit, gives the required ampere-turns for that part. The sum of the ampere-turns for each part gives the number required for the complete magnetic circuit, and one-half this number is to be supplied by each field coil.

The number of ampere-turns required for the air gaps is best calculated as was shown on page 56.

$$H = \frac{0.4\pi NI}{l}$$

In air, $B = H$, so that

$$B = \frac{0.4\pi NI}{l} \tag{1}$$

where B is the flux density in lines per square centimeter and l is the length in centimeters.

To determine the ampere-turns required per inch of air gap, Eq. (1) may be converted; since $B = B'/(2.54)^2$ and $l = l' \times 2.54$, where B' is in lines per square inch and l' is in inches, we have

$$\frac{B'}{(2.54)^2} = \frac{0.4\pi NI}{l' \times 2.54}$$

and

$$NI = \frac{B'l'}{0.4\pi \times 2.54} = 0.313B'l' \tag{2}$$

The number of ampere-turns required per inch of air gap is therefore equal to the flux density in lines per square inch multiplied by 0.313.

Tabulating the data, we have:

Part	Material	Average Length in Inches	Average Area in Square Inches	Flux	Flux Density	Ampere-turns per Inch	Ampere-turns
Armature core	Commercial sheet steel	6.0	19	1,200,000	63,160	4	24
Teeth	Commercial sheet steel	0.5 on each side	12	1,200,000	100,000	65	65
Air gap	Air	0.1 each	20	1,200,000	60,000	18,780	3756
Poles	Commercial sheet steel	4 each	15	1,500,000	100,000	65	520
Yokes	Cast steel	20	10 each	750,000 each	75,000	25	500

Total ampere-turns required.....................................4865

$$\text{Ampere-turns per pole} = \frac{4865}{2} = 2433$$

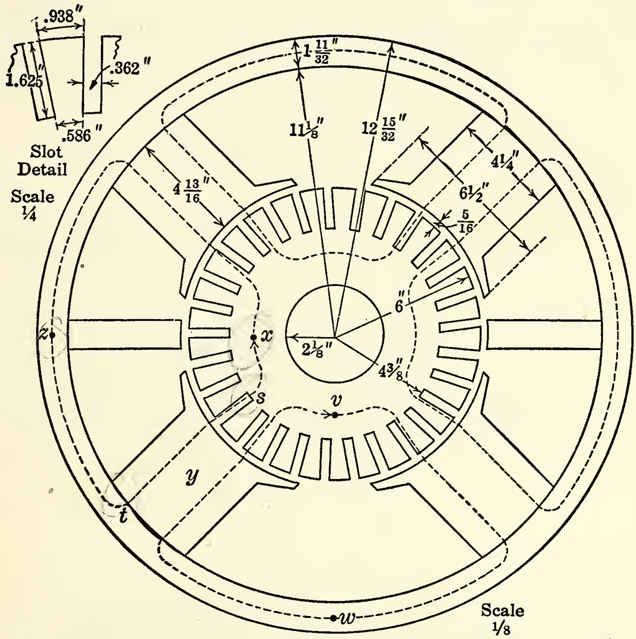
In the above example, the area of the flux path through the armature was taken as 19 sq in, this being the area of a vertical section, *on both sides* of the shaft. Now it might also have been considered that there were two parallel paths through the armature, one above and the other below the shaft. The area of each path, 9.5 sq in, would have been that between the shaft and the bottoms of the slots; and the flux per path 600,000. The same results for flux density and ampere-turns would be obtained as before; the reason for the second viewpoint will appear in the next example.

Example 2. The next problem will be to calculate the necessary ampere-turns for a 30-kw, 4-pole, 230-volt, 1800-rpm generator. A diagram of the magnetic circuit is given in Fig. 5–33, and it will be seen that the pole flux divides, one-half going each way in both the yoke and the armature core, in the same way as the flux divided in the problem of Fig. 2–46. This problem may be treated in the same way.

As some engineers design their machines by calculating the required ampere-turns for the field, per pole, we shall do it this way, determining the ampere-turns required per pole to set up the flux lines *zyx* and *wyv* from *z* to *x* and *w* to *v*, respectively. The required air-gap flux per pole is 1,300,000 lines, and, if the leakage coefficient is 1.22, the flux in the poles and yoke is 1,586,000 lines.

The lengths of the flux paths in the armature from *s* to *x* and in the yoke from *t* to *z* are taken from Fig. 5–33. The area of the armature core is the depth below the slots multiplied by the effective length of the arma-

ture (see below, Fig. 5–33), or ($4\frac{3}{8}$ inches $-$ $2\frac{1}{8}$ inches) $\times$ 4.05 inches = 9.1 square inches. The area of the teeth is equal to the average number of teeth under a pole multiplied by the mean area of a tooth. The pole arc is $6\frac{1}{2}$ inches, and this distance is usually increased by twice the length of the air gap to allow for fringing, or $6\frac{1}{2}$ inches + $2(\frac{5}{16}$ inches) = 7.125 inches. The width of a tooth plus a slot is 1.3 inches, so that the average number of teeth under a pole is 7.125/1.3 = 5.48. The mean area of a tooth is usually taken as that one-third of the distance from the narrowest end.



Length of Armature, overall = 4¾″ **Ventilating Ducts, 1 of ¼″**
" " " , effective = 0.9 (4.75″–0.25)=4.05″ Total Slots = 29
" " Pole, along Shaft = 4¼″ **Width of Yoke, = 8⅛″**
Inductors per Slot = 10 **Total Commutator Bars = 145.**

FIG. 5–33. Cross-section of the essential parts of a four-pole, 30-kw generator.

In this case, the mean width of a tooth is $\{0.938 \text{ inch} + 2(0.586 \text{ inch})\}/3 = 0.703$ inch, and this mean width multiplied by the effective length of the armature gives 0.703 inch $\times$ 4.05 inches = 2.85 square inches as the mean area per tooth. The mean area of 5.48 teeth is 5.48 $\times$ 2.85 = 15.6 square inches. Actually at some instants there are only 5 teeth under a pole face and then there may be 6; thus the reluctance of the air gap changes as the armature rotates. The average flux, however, is the same as though there were 5.48 teeth (that is, 5 teeth and part of another) under the pole face.

The area of the air gap is roughly taken as the area of the pole face. In an exact design this will be modified to consider flux fringing. If the

area of the air gap is taken as the area of the pole face, the gap area becomes 6.5 inches × 4.25 inches = 27.6 square inches. The area of the poles is 4.25 inches × 4.25 inches = 18 square inches, and of the yoke is $1\frac{11}{32}$ inches × $8\frac{1}{8}$ inches = 10.9 square inches.

Tabulating our data as before, and using the magnetization curves of Table X of the Appendix, we have:

Part	Material	Length in Inches	Area in Square Inches	Flux	Flux Density	Ampere-turns per Inch	Ampere-turns
Armature core	Commercial sheet steel	s to x 2.6	9.1	650,000	71,430	7	18
Teeth	Commercial sheet steel	1.625	15.6	1,300,000	83,330	12	20
Air gap	Air	0.312	27.6	1,300,000	47,100	14,742	4606
Poles	Commercial sheet steel	4.812	18.0	1,586,000	88,100	17	82
Yoke	Open-hearth plates	t to z 8.5	10.9	793,000	72,750	22	187

Total ampere-turns required per pole 4913

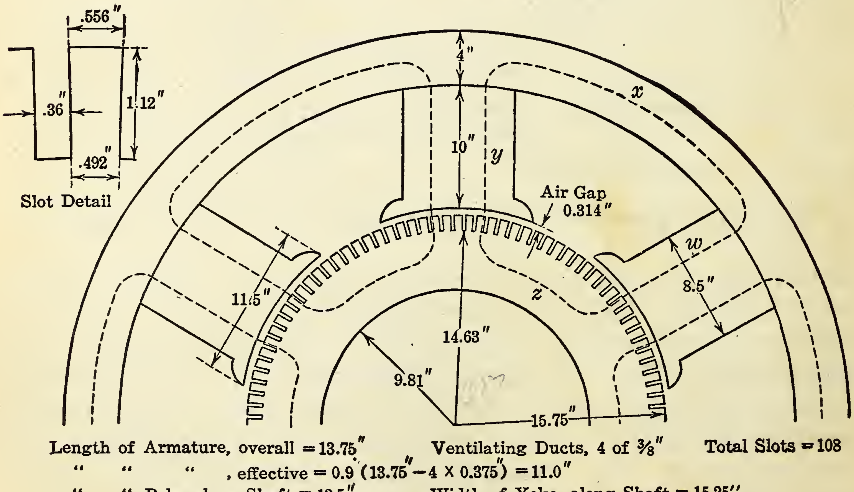


FIG. 5-34. Giving the essential dimensions of the magnetic circuits of a multi-polar machine; this is the kind of a sketch the designer works from.

Example 3. We shall next figure the necessary ampere-turns for a 300-kw, 6-pole generator, which is to furnish 275 volts when running at 900 rpm. A diagram of its magnetic circuit is given in Fig. 5–34, from which it will be seen that the pole flux divides, one-half going each way in both the yoke and the armature core. As any one complete magnetic circuit involves a pair of poles, we shall in this problem determine the ampere-turns required for a pair of poles. The leakage factor for the machine is taken as 1.18; if the required air-gap flux is 8,800,000 lines per pole, the flux through the poles and yoke is then 10,400,000 lines.

Tabulating our data as before, we have, considering a pair of poles:

Part	Material	Length in Inches	Area in Square Inches	Flux	Flux Density	Ampere-turns per Inch	Ampere-turns
Armature core	Commercial sheet steel	13.0	53	4,400,000	83,020	12	156
Teeth	Commercial sheet steel	1.12 each	75	8,800,000	117,330	315	706
Air gap	Air	0.314 each	155	8,800,000	56,770	17,769	11,159
Poles	Commercial sheet steel	10 each	115	10,400,000	90,430	21	420
Yoke	Cast steel	27	61	5,200,000	85,250	41	1,107

Total ampere-turns required, per pair of poles........................13548

$$\text{Ampere-turns per pole} = \frac{13548}{2} = 6774$$

These three examples will serve to show how the number of field ampere-turns is roughly determined. In an actual design other factors have to be taken into account; the effect of the armature mmf on the field strength has to be considered, and the problem is somewhat more complicated than we have indicated.

It is to be noticed that, when *the flux* of a machine is spoken of, the flux entering the armature from the air gap per pole is always intended.

41. Proper Size of Wire for a Shunt-wound Field. The size of wire to be used in winding the shunt-field coils is now to be determined. A *shunt* field is wound of relatively small-sized wire and is shunted across, or connected in parallel with, the armature. Generally, all the field coils of the machine are connected in series, and whatever voltage exists across the armature is the source from which the field current is to be taken.

The average length of one turn of the field coil is estimated from the known size of the field pole and the assumed depth of the coil. Suppose that the length of one turn close to the field pole was 10 inches, and that

we assumed a winding depth of 1 inch; the length of an outside turn would be approximately 18 inches (if the pole was of rectangular cross-section) so that the *average length* of a turn would be 14 inches.

Let E = the voltage of the field-current supply;
NI = the total ampere-turns required on the machine
= NI per pole $\times$ number of poles;
l = the average length of one turn, in feet;
r = the resistance per foot of the proper-sized wire; this is to be determined;
I = the current through the field circuit;
R = the resistance of the field circuit;
N = the total number of turns on all coils in series.

Then,

$$R = Nlr \tag{3}$$

and

$$I = \frac{E}{R} = \frac{E}{Nlr} \tag{4}$$

Multiplying both sides by N

$$NI = \frac{NE}{Nlr} = \frac{E}{lr}$$

or

$$r = \frac{E}{lNI} \tag{5}$$

Now NI has been calculated, E is known, and l has been approximately determined from the assumed size of the field coil. Therefore, r, the resistance, per foot, of the proper-sized wire to use, is determined. From the wire table, the diameter of the wire and its cross-sectional area in circular mils can be obtained.

The proper-sized wire and the number of required ampere-turns being known, the proper number of turns is now to be determined. Here we at once notice a peculiarity in the problem. *So far as the magnetizing effect of the coil is concerned, with a given supply voltage, it makes no difference how many turns are used for the field coil; no matter how many turns are used, the mmf of the coil will be the same.*

Suppose that we put on 1000 turns and that this number of turns has a resistance of 50 ohms. If the voltage of the field-current supply is 100 volts, the field current will be 2 amperes and the ampere-turns of the coil will be 2000. If, now, 2000 turns are used, the resistance of the coil increases to 100 ohms, the field current decreases to one ampere, and the ampere-turns in the coil will be 2000, as before.

42. Proper Number of Turns. There must be some method of determining the proper number of turns to use, *and this is fixed by the safe allowable temperature rise in the coil.* It is apparent that, if, for example, only one turn were used in the field coil, the proper number of ampere-turns would be obtained, but the current through the one turn would be so great that the wire would melt. As the number of turns is increased the required current decreases; when the number of turns has been increased to such an extent that the current which flows through the coil does not heat the wire more than 55 C above room temperature, and never to a temperature in excess of 95 C (the limit fixed by the AIEE), we may say that the field coil has been properly designed.

The process of predicting the temperature rise from the power used up as heat in the coil (I^2R loss) and the calculated radiating surface requires judgment and experience, because some radiating surfaces are more effective than others, etc.

It has been found, however, from the results of many tests, that a safe figure to use in fixing the number of turns is obtained by allowing about 1000 *amperes per square inch,* or about 1300 *cir mils per ampere* in the winding. As the proper-sized wire has already been found, the cross-section in circular mils is known. Suppose it was 2600 cir mils; this wire could safely carry 2 amperes, so that if 3000 ampere-turns were required per pole the proper number of turns = 3000/2 = 1500 per coil. The figure 1300 cir mils per ampere is good only for field coils where the wires are packed tightly together and the heat cannot easily escape. In armature windings, and for series-field coils, where the ventilation is better, it is customary to allow 1500 amperes per square inch in large machines and 3000 in small machines, or 850 and 425 (see Table IX of the Appendix) cir mils per ampere, respectively; if the wire were stretched in the open, probably 200 to 300 cir mils would be a sufficient allowance.

43. Series-field Winding. In practice, to improve the operating characteristics of electrical machines, an additional field winding is fitted to each pole, this extra winding, consisting of a few turns, being called the *series field.* It derives its name from the fact that it is connected in series with the armature instead of being shunted across it, as in the shunt-field winding.

44. Construction of Field Coils. Shunt-field coils are usually made of double cotton-covered (dcc) wire, or, particularly in the case of small machines, of enamel-covered wire. The wire is wound either on a frame arranged to slip over the pole core (Fig. 5–35), or else on a properly shaped wooden form from which the coil is taken off and taped, and then dried and impregnated with some insulating compound; when dry, the coil is ready for assembly on the machine (Fig. 5–36).

Series and commutating-field coils, since they carry larger currents, are

made either of large-sized insulated wire or of copper strap wound on the spool edgewise, the turns, in the latter case, being separated by spacers of insulating material (Fig. 5–37).

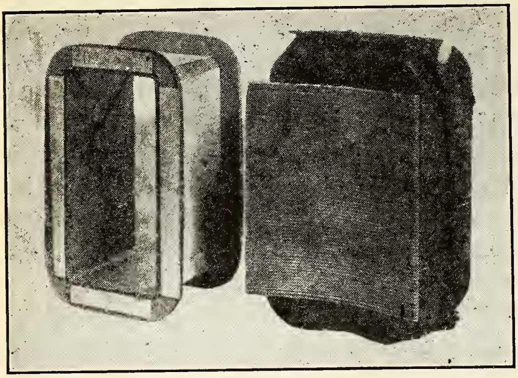


Fig. 5–35. Field coil wound upon a rigid insulating frame. It is then slipped over the pole as shown. *Courtesy of the Westinghouse Electric and Manufacturing Co.*

When a shunt and a series coil are used together on the same pole, the series coil may be generally wound over or under the shunt coil; such *compound* field coils are shown in Figs. 5–36 and 5–38.

In larger machines it becomes necessary to afford some ventilation for the field coils. When frames are used, they can be made with a double wall, permitting air to circulate between the coil and the pole core. In the case of compound

Fig. 5–36. Views of a compound field coil which was wound on a form. After removal from the form it was taped, to give it the requisite rigidity, mounted on a rigid insulating frame, and impregnated. *Courtesy of the Reliance Electric and Engineering Co.*

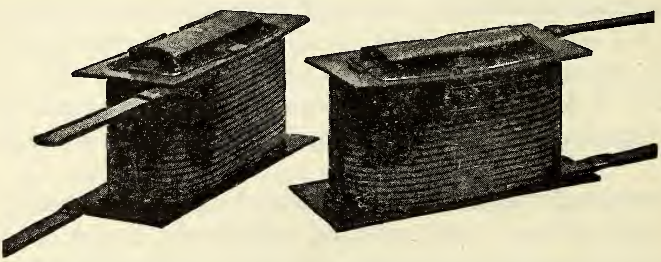


Fig. 5–37. Commutating poles wound with heavy copper strap, the different turns being separated by spacers of insulating material. *Courtesy of the Reliance Electric and Engineering Co.*

field coils, a space may be left between the shunt and series coils; the use of copper strap, wound edgewise, with spaces, serves the same purpose.

The terminals of the field coils are usually brought out on opposite sides to facilitate connections between coils on adjoining poles.

45. Armature Windings. The question of armature windings is a very broad one, and only an elementary outline and analysis of some of the various forms employed in different d-c generators can be given here. By the term "armature winding" is meant the group of conductors placed on the armature core, which revolve with it and cut the magnetic field. The different ways in which these conductors may be placed on the armature core, and interconnected, lead to innumerable winding schemes. The armatures of motors and generators are wound in exactly the same way; the same factors have to be kept in mind, no matter whether the armature is to be used for one purpose or the other. We shall speak of the different windings as generator windings, but they serve just as well for a motor armature.

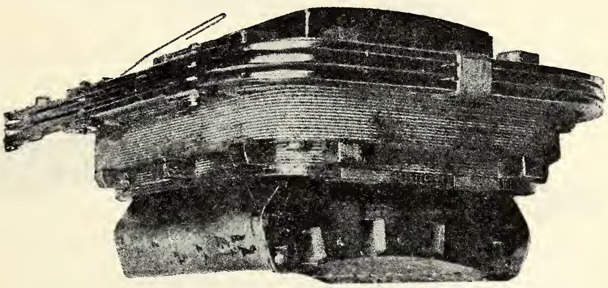


Fig. 5–38. A compound field winding. The series winding is frequently made with turns sufficiently large that they may be fitted right over the shunt coil. The slots in the pole face are for a pole-face winding.

The armature windings for a-c generators are quite different from those intended for a d-c generator; we shall consider here only the d-c windings, the first two general divisions of which are the *ring winding* and the *drum winding.*

46. Ring Winding. In the ring winding, the coils are wound around the armature in such a way that half the coil is on the *inside of the armature core.* This may be seen by reference to Fig. 5–39, which shows the armature of an early type of generator. The winding of these armatures is not so easy as the winding of a drum armature, nor can a ring winding be insulated as well; but a more important objection is the amount of "dead wire" on the armature. The conductors on the inside of the core do not cut any flux as the armature revolves, and hence they can generate no emf. This extra wire is detrimental to the machine, not only because of the extra cost for wire but also because the *resistance of the armature winding is much larger* than it should be; this high resistance means high I^2R and a high heat loss in the generator. As a result, the efficiency is lowered.

For the reasons given above, the ring winding is used very little at present. It would require no further discussion were it not for the fact that the drum winding is not easy to show by a diagram. In the analysis of the action of a drum winding, therefore, the drum winding will often be pictured as a ring winding. The ring winding is easier to represent, and the deductions made regarding its action apply equally well to the drum winding.

47. Drum Winding. The drum winding is distinguished by the fact that all the armature conductors *are on the outside surface of the armature core;* the coils do not thread through the inside of the armature as they do in the ring winding. The drum winding *does not contain so much inactive wire,* therefore, as the ring winding; also, the operation of winding the

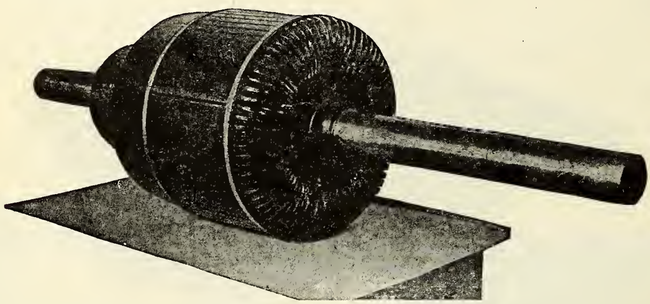


Fig. 5–39. A ring-wound armature, practically never used in modern machines; the coils wind right through the center of the armature core.

armature is much more easily accomplished with the drum than with the ring type and results in a better-insulated armature. These are two of the advantages which make the drum winding so universally used. A typical medium-sized drum-wound armature is shown in Fig. 5–40; it may be seen that all the winding is placed on the periphery of the armature. The shaft for the armature of Fig. 5–40 is shown in Fig. 5–41. The supports to hold the armature laminations and the commutator are built up of standard forms, properly bent and then welded together.

48. Series and Multiple-circuit Windings. The next classification is made according to the number of paths by which current can flow through the armature winding to the external circuit. In one type of winding, called the *series, two-circuit,* or *wave winding,* there are two paths through the armature, no matter how many poles the generator may have. This type of winding is used principally for motors; it is used for generators only in small sizes, say up to 50 kw.

In another type of winding, there are as many paths through the armature winding as there are poles on the generator; this is called the *multiple-*

circuit, or *lap winding*. By far the greater percentage of generators are equipped with this type of winding.

When a machine is bipolar, the series winding and the multiple-circuit winding have the same number of paths, namely, two. Two is the smallest number of paths which can be obtained with a closed-circuit winding.

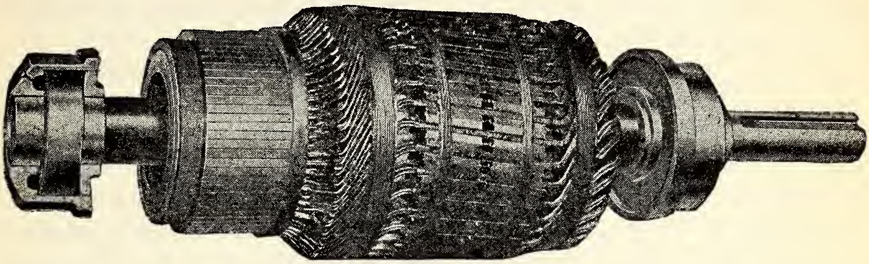


Fig. 5–40. A drum-wound armature; both sides of the coils are fitted on the outer periphery of the core. This armature is to rotate in ball bearings. *Courtesy of the Electrodynamic Works, Electric Boat Co.*

In discussing armature windings, we shall call any conductor in which voltage is induced an *inductor*; the rest of the conductors making up the winding we shall call simply conductors. The wires on the outer side of the coil of a ring winding are inductors, because when the armature rotates, these wires move through the magnetic field, and voltage is induced in them. Those wires which are on the inside of the armature core and on the ends are merely conductors.

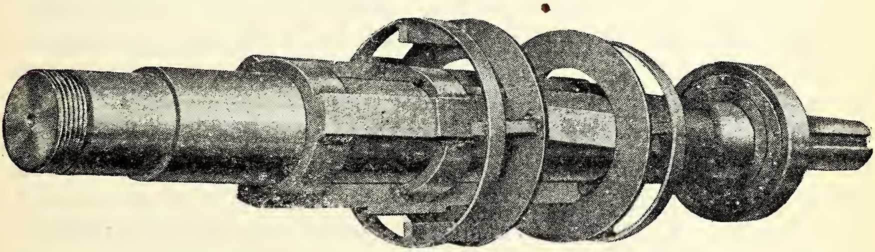


Fig. 5–41. The shaft of the armature shown in Fig. 5–40. The supports for the laminations and the commutator are of structural pieces welded together. *Courtesy of the Electrodynamic Works, Electric Boat Co.*

49. Superiority of Multiple-circuit Winding for Generators. One reason why the multiple-circuit winding is used so much more for large machines than the series, or two-circuit, winding will be apparent when we consider the voltage generated by a given number of armature inductors, first when connected so as to give a two-path winding and then when connected to give a multiple-path winding.

Suppose we have a 12-pole generator having 1500 active inductors on the armature altogether. The average voltage generated per inductor might be 2 volts. If these inductors were arranged in a series winding, all of them would have to be used in only two paths, so that there would be 750 inductors in series in each path. The voltage generated per path would be 1500, and this would be the voltage of the machine. While this machine would be suitable for a 1500-volt railway system, its voltage would be too high for lighting purposes.

Suppose now that the inductors were arranged in a multiple-circuit winding; as the machine has 12 poles there would be 12 paths in the winding, and therefore the number of inductors in series per path would be 125. The voltage per path, which is the same as the voltage of the generator, would be 250, which would be right for a three-wire lighting generator.

A machine of the size we have in mind could not use a smaller number of inductors than 1500, because in that case the inductors would have to be of such large cross-section that they could not be bent readily, and therefore the operation of winding the armature would be too difficult. Of course, the two windings would have the same capacity in kilowatts; the multiple-path winding which generates one-sixth as much voltage as the wave winding could deliver six times as much current (the same current per path), making the output of the two windings the same.

The series winding has certain advantages, however, and most motors are wave wound. For one thing, a series winding requires only two sets of brushes, no matter how many poles the machine has. In certain cases, notably railway motors, the inspection and adjustment of brushes are thereby made much easier than they would be with a multiple-circuit winding with many sets of brushes. Also, if the bearings of a machine become worn, letting the armature down so that it is eccentric with respect to the pole pieces, the flux from the lower poles into the armature becomes greater than that from the upper poles. If the winding of such an armature is multiple-circuit, such an inequality in flux produces overheating and sparking at the commutator; if the armature is series wound no such effect exists. As motors are small compared to generators, the idea brought out above, about the voltage of a winding and the number of inductors, is not important when the machine in question is a motor; moreover, motors are generally installed in some corner and are rarely inspected for bearing wear, etc. Hence the preponderance of series windings for motors.

The six-path ring winding shown in Fig. 5–42 shows how the six paths are formed in a multiple-path winding, and also how the voltage of the machine is the same as the voltage generated in one path.

50. Examples of Simple Lap or Multiple-circuit Armature Windings. In Fig. 5–43 is shown a lap winding in very elementary form. Fourteen

inductors are to be arranged in a closed-circuit winding and *must be connected together so that all conductors in series in a path give voltage in the same direction in that path;* we might state it differently by saying that in any path there must be *no opposing* emf's.

In order to do this, a certain order of "picking up" the inductors must be followed. If two inductors under the same pole are joined at the same end of the armature, the voltage of the inductors will be in opposition in the circuit and will neutralize each other; but if two inductors under opposite poles are connected at the same end of the armature, their voltages

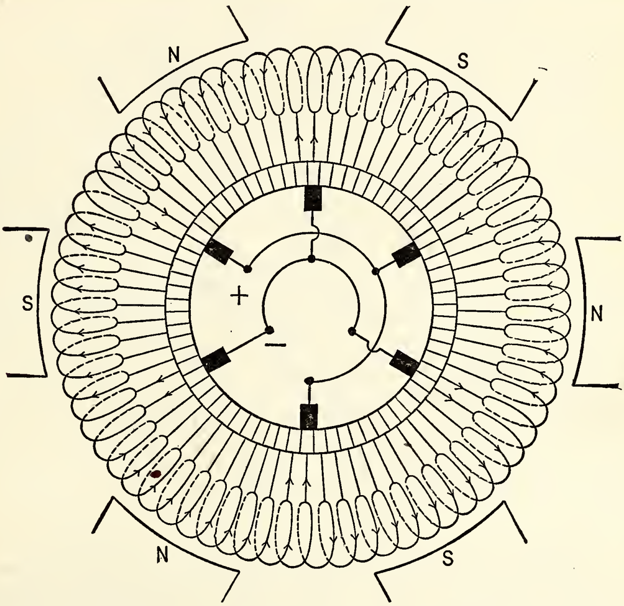


FIG. 5–42. A ring winding. When placed in a six-pole frame with six brushes on the commutator, there are six current paths through the armature.

will be added together. In the example shown, starting at bar *b*, we pick up, in order, inductor 3 under the south pole and inductor 10 under the north pole, returning to bar *a*. From bar *a*, we go back to pick up inductor 1 under the south pole and then inductor 8, and so on. Following this order the winding finally closes upon itself, as may be seen from the winding table of Fig. 5–43.

The two paths between the brushes, starting from the negative brush, are

$$e\text{–}9\text{–}2\text{–}d\text{–}7\text{–}14\text{–}c\text{–}5\text{–}12\text{–}b$$

and

$$f\text{–}6\text{–}13\text{–}g\text{–}8\text{–}1\text{–}a\text{–}10\text{–}3\text{–}b$$

It will be seen that, in going from one brush to the other through either path of the winding, all inductors give voltage in the same direction. The coil 4-11 appears in neither path; it is momentarily short-circuited by the negative brush.

The winding just considered is made up of 7 coils of one turn each, and it will be noticed that the commutator has as many segments as there are coils on the armature. This will generally be the case; if a machine

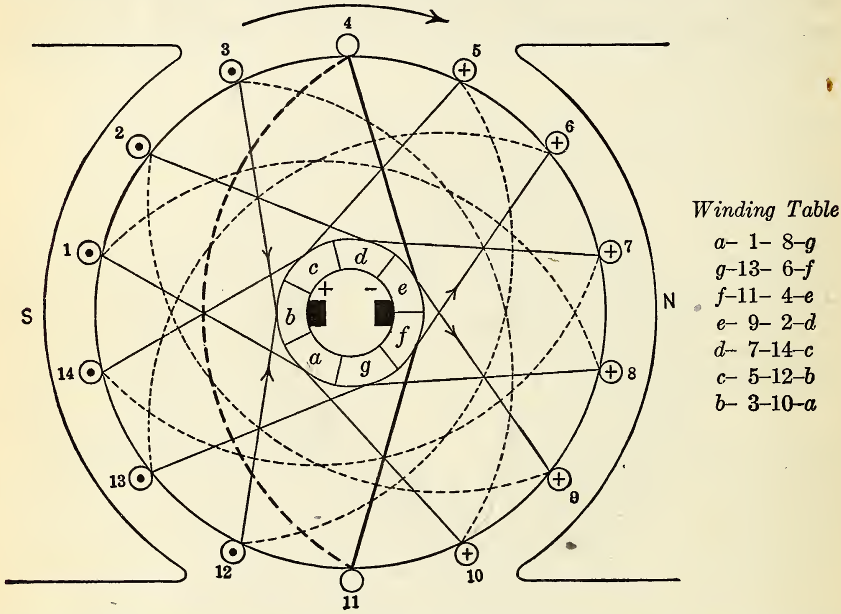


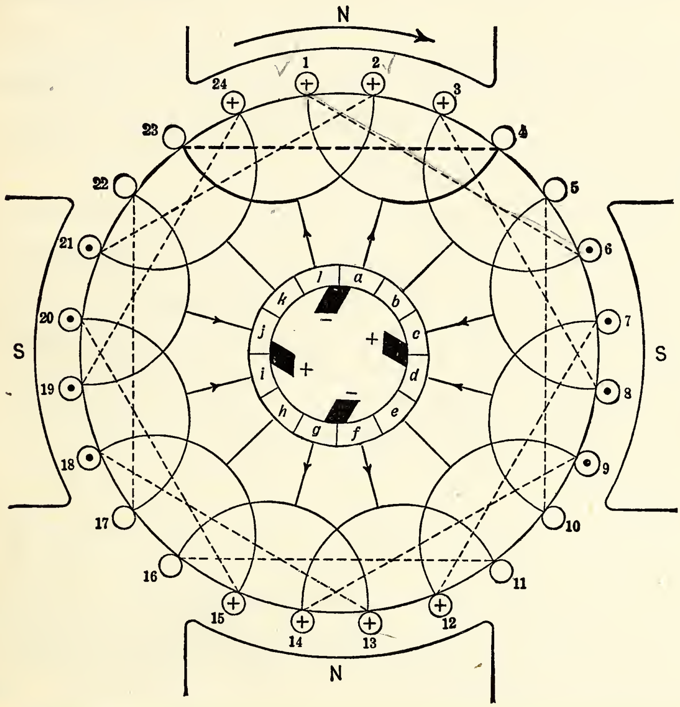
FIG. 5-43. Elementary bipolar winding to show how the inductors are connected together and to the commutator bars. On the actual machine the brushes rest on the outer surface of the commutator, but they are here shown on the inside for clearness in the diagram. The dots and crosses in the inductors signify voltages, not currents. Coil 4–11, shown in heavy lines, is short-circuited by the negative brush.

has 64 coils there will be 64 commutator segments, no matter how many turns there are per coil or how the coils are connected.

In the multiple-circuit or lap winding, the two ends of any one coil are connected to adjacent commutator bars. In Fig. 5-43, for example, the coil made up of inductors 4 and 11 has its terminals connected to bars *e* and *f*, which are adjacent. At the moment shown, coil 4–11 is short-circuited by the negative brush; one-fourteenth of a revolution later, in the direction indicated by the arrow, coil 3–10 will be short-circuited by the positive brush; and so on.

In Fig. 5-44 is shown a 4-pole machine having 24 inductors on a lap-wound armature, which more clearly indicates the mode of connecting the

inductors in a multiple-circuit winding. Starting at bar *a* and picking up inductor 1 under the upper north pole and then inductor 6 under the right-hand south pole, it is now necessary to pick up the next inductor under a north pole. This next inductor could be picked up by going back to the upper north pole after leaving inductor 6 or proceeding to the next north pole. This choice decides the type of winding. In Fig. 5–44 the winding proceeds from inductor 6 to bar *b* and then back to inductor 3 under the preceding north pole, and a lap or multiple-circuit winding re-



Winding Table

a– 1– 6–*b*
b– 3– 8–*c*
c– 5–10–*d*
d– 7–12–*e*
e– 9–14–*f*
f–11–16–*g*
g–13–18–*h*
h–15–20–*i*
i–17–22–*j*
j–19–24–*k*
k–21– 2–*l*
l–23– 4–*a*

FIG. 5–44. A four-pole multiple-circuit winding. An actual winding is never as simple as this one.

sults. But if after leaving inductor 6 the next inductor had been picked up under the lower north pole, a wave or series winding would have been started.

In the winding of Fig. 5–44 there are four parallel paths and 4 brushes are required. There is one turn per coil, and so 12 coils; the commutator has 12 segments.

51. Brush Position and Voltage in Short-circuited Coils. If at the time a coil is short-circuited by a brush, an appreciable voltage is being generated in the coil, a large current will circulate through the path made

up of the coil, the two brush contacts, and part of the brush itself. This *short-circuited current,* as it is called, will produce disastrous sparking at the brush contact if it is large; hence *the brushes must be placed on the commutator so that the coil which they short-circuit occupies such a position in the magnetic field that no voltage is induced in it while it is short-circuited.* (This statement will be qualified later under the discussion of commutation.) Evidently the position of the brushes shown in Figs. 5–43 and 5–44 satisfies this requirement, because at the time a coil is short-circuited the inductors are moving in the interpolar space, where the flux is practically zero.

After a lap or multiple-circuit winding is laid out, the position of the brushes is determined from the coils which do not generate any voltage. Thus in Fig. 5–44 coil 23–4 lies in the interpolar spaces and is symmetrically placed with respect to the upper poles. The upper negative brush is then located so as to span bars *l* and *a* exactly.

52. Pitch. The term pitch is frequently used in connection with armature windings. The pitch of a coil is measured by the number of inductors passed over between coil-sides; the pitch so determined is called the *back pitch* (referring to the back of the armature) and determines the spread of the coil (see Figs. 5–46 and 5–47). Thus, in Fig. 5–44 inductors 1 and 6 form a coil, and the back pitch is 5, that number of inductors having been counted over in connecting inductors 1 and 6. The *front pitch* (or pitch at the commutator end) is formed by the number of inductors counted over between the two connected to the same commutator bar. In Fig. 5–44 the front pitch is 3.

It will be noted that in the examples of windings so far considered the back and front pitches are both odd. This must be so, in order that odd- and even-numbered conductors may be picked up alternately. If both pitches were even, only half the conductors on the armature would be picked up; if one pitch were even and the other odd, the winding would not progress, as may easily be verified.

The distance between pole centers expressed in slots is often called the pole pitch; and, when the back pitch is so chosen that one coil-side is under the middle of one pole while the other coil-side is under the middle of an adjacent pole and thus equal to the pole pitch, the winding is said to be *full pitch.* But if the back pitch is less than the number necessary to give a full-pitch winding, then a *short-pitch* or *fractional-pitch* winding results. Full-pitch and short-pitch windings are compared further in section 55.

53. Developed Windings. Instead of representing an armature winding as we have done in Fig. 5–44, a so-called *developed winding* may be given. In such a diagram, the winding of the armature is represented in a plane surface, just as though the winding had been peeled off from the

armature and laid on a flat surface; the poles are represented in their proper positions, and the commutator is shown in developed form also. The winding of Fig. 5–44 is shown in developed form in Fig. 5–45.

54. Commercial Windings. None of the windings given so far represent actual practice. In a slotted armature there are always at least two inductors per slot, and these inductors are not parts of the same coil; they form sides of two different coils. A winding with two coil-sides per slot is usually called a *two-layer* winding, and one with only one coil-side per slot is called a *single-layer* winding. The former is the usual type, using as many coils as there are slots.

In order to build up the voltage of a machine, it often is necessary to use more than one turn per coil (i.e., more than two inductors per coil).

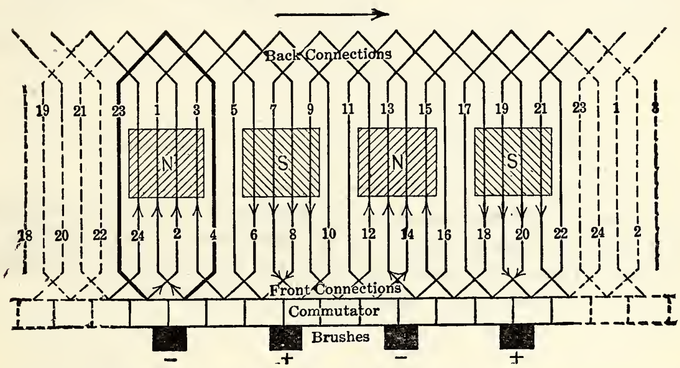


FIG. 5–45. A so-called developed winding; this shows about the appearance the winding of Fig. 5–44 would have if it were stripped from the armature and laid out flat.

All the turns are usually taped together to form a single coil so that there will still be two coil-sides per slot in a two-layer winding.

Figure 5–46 represents a two-layer winding of more than one turn per coil. It will be seen that the left-hand coil-sides (odd-numbered) form the upper layer, and the right-hand coil-sides (even-numbered) the lower layer. The kinks in the coils are for the purpose of allowing the end connections to pass each other neatly and compactly.

In Fig. 5–47, coils of several turns per coil are represented, that marked (*a*) showing a coil of 2 turns for a lap winding, and that marked (*b*) a coil of 3 turns for a series or wave winding.

If there are as many coils as there are slots, there must be the same number of commutator bars. Frequently, however, two or more single coils may be taped together to form a *double coil* or *polycoil,* as shown in Fig. 5–48, and inserted into the slots as before. Such an arrangement requires as many commutator bars as there are single coils, or twice the number of slots.

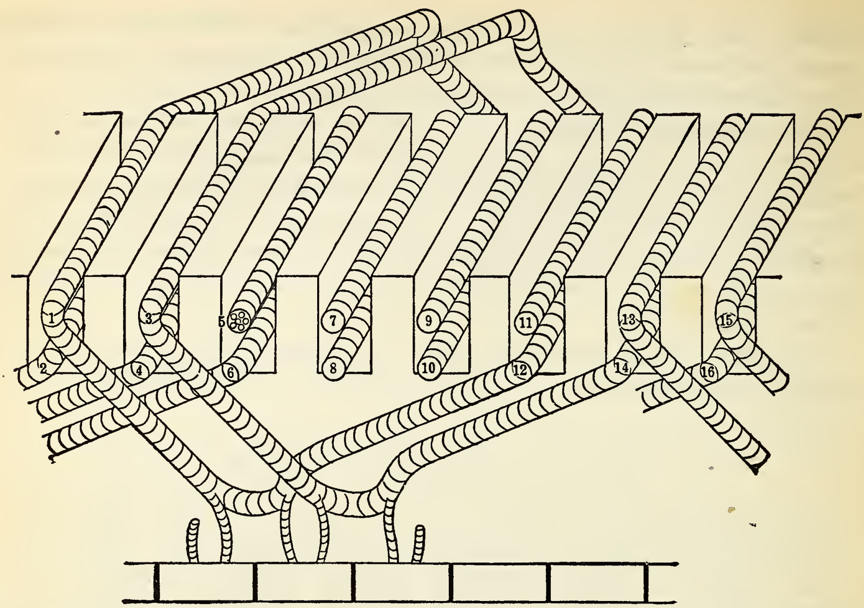


FIG. 5-46. Formed coils are shaped so that one side of a coil fits in the bottom of one slot and the other coil side fits in the top of a slot distant from the first by nearly the distance between poles. The ends of the coils are formed so that they fit neatly one into the other. Back pitch is 11; front pitch is 9.

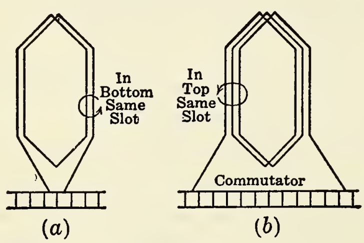


FIG. 5-47.

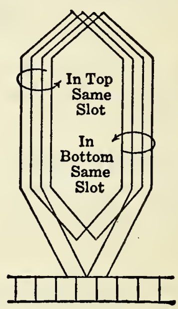


FIG. 5-48.

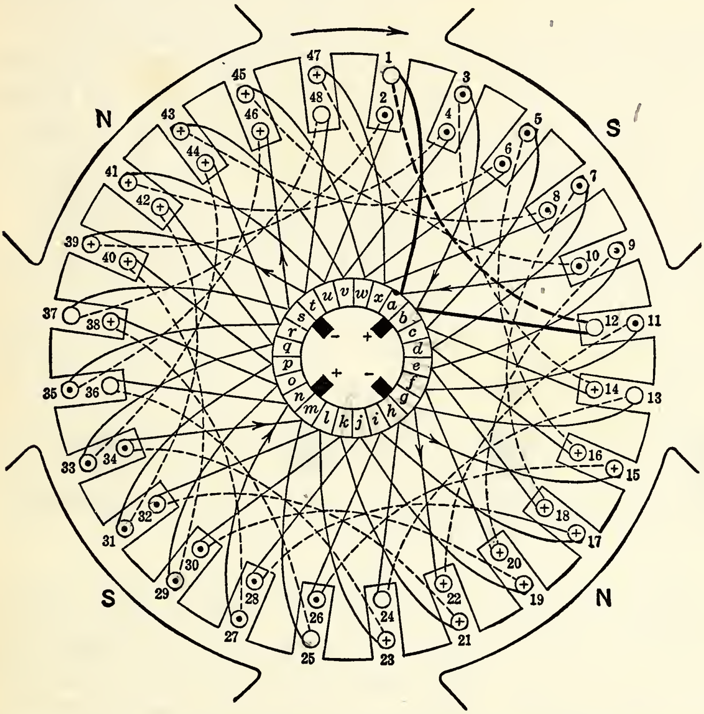
FIG. 5-47. Coils generally consist of more than one turn; (a) shows a two-turn coil for a lap winding and (b) shows a three-turn coil for a wave winding.

FIG. 5-48. Showing how a double coil or polycoil is connected to the commutator bars.

55. Multiple-circuit Windings. In Fig. 5-49 is shown a two-layer multiple-circuit, or lap, winding, for a 4-pole machine having 24 slots on its armature. As is customary in winding diagrams, only one turn per coil is shown even though there are more, as we are interested only in the end connections of the coils. It is evidently a lap winding, as one coil "laps over" its neighbor. The proper position for the brushes is fixed by

locating a coil in which no voltage is generated and then placing one brush on the two adjacent segments to which the coil ends are attached. The other three brushes are located symmetrically with respect to this one.

In multiple-circuit windings, the *average pitch* (z) is generally considered as equal to the average of the front and back pitches. Then, as the back and front pitches must be different and both be odd, it follows that the



Winding Table

a– 1–12–*b*
b– 3–14–*c*
c– 5–16–*d*
d– 7–18–*e*
. . . .
. . . .
s–37–48–*t*
t–39– 2–*u*
u–41– 4–*v*
v–43– 6–*w*
w–45– 8–*x*
x–47–10–*a*

1–12

Fig. 5–49. A two-layer multiple-circuit winding for a four-pole machine. One short-circuited turn is shown in heavy lines. Back pitch is 11; front pitch is 9.

average pitch must be even. In a simple winding the back pitch will be $(z + 1)$ and the front pitch $(z - 1)$.

For a full-pitch winding the conditions to be satisfied are expressed by the formula

$$Z = pz = sb \tag{6}$$

where Z = total number of coil-sides or bars, which is twice as great as the number of coils;
p = number of poles;
z = average pitch;
s = total slots on armature;
b = number of coil-sides per slot.

In the winding of Fig. 5–49 the back pitch is 11 and the front pitch is 9; the average pitch is therefore 10, and it is a short-pitch winding. To be a

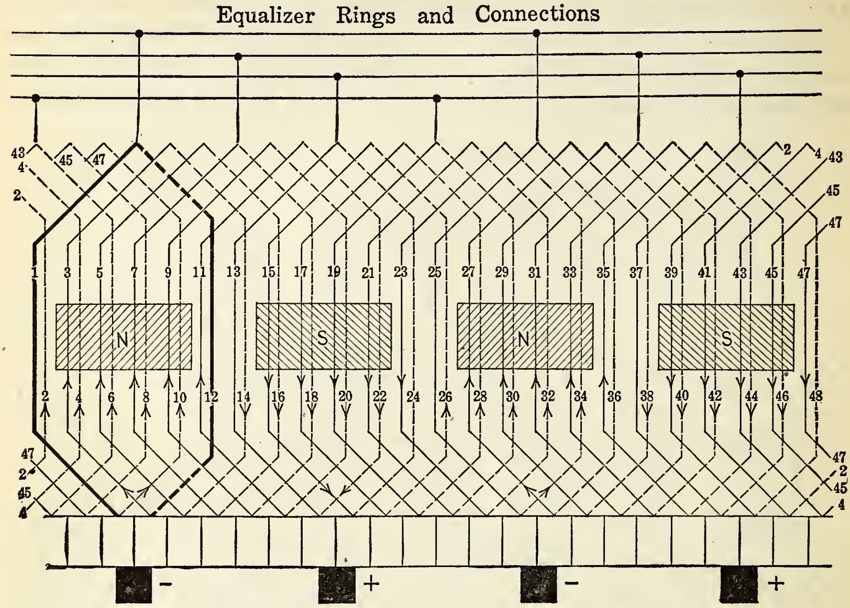


FIG. 5–50. The winding of Fig. 5–49 in developed form. Equalizer connections also are shown.

full-pitch winding, the average pitch would have to be 48/4 = 12, the back pitch therefore 13 and the front pitch 11. If the average pitch is shortened too much, in every path there will be some inductors which generate voltage opposite to that generated by the rest of the inductors. This is a disadvantage, but an advantage is gained by having shorter end connections when the average pitch is less than the number of inductors between pole centers. Shorter end connections mean less wire and less armature resistance, and so short- or fractional-pitch windings are much used. However, as will be evident when the subject of commutation is studied, a short-pitch winding requires as many commutating poles as there are main poles; in a full-pitch winding, half as many commutating poles as there are main poles is sufficient.

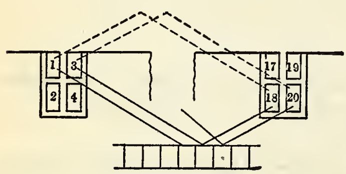


FIG. 5–51. Showing how inductors are numbered. The back and front pitches must be odd because they span from the top layer (odd-numbered inductors) to the bottom layer (even-numbered inductors), or vice versa. The figure shows a polycoil made up of two single coils.

In Fig. 5–50, the two-layer, multiple-circuit winding of Fig. 5–49 is

shown in developed form. In Fig. 5–51, a multiple-circuit winding with polycoils or four coil-sides per slot is suggested. The back and front pitches are 17 and 15, respectively.

56. Procedure in Laying Out a Multiple-circuit Winding. If it were required to lay out the 4-pole lap winding of Figs. 5–49 and 5–50, with 48 inductors in 24 slots, the easiest method is first to draw and number the inductors without any connections.

If the winding is to be full-pitch it is evident that, since there are 24 slots and 4 poles, the coils must span from one slot to another 6 slots farther on. This means that the odd-numbered inductor in one slot, say inductor 1, must be joined to the even-numbered inductor in the sixth slot farther on, i.e., inductor 14. The back pitch is therefore 13 and the front pitch 11.

Having the full pitches, a short-pitch winding results by merely reducing both pitches by 2, 4, 6, etc. In this example, pitches less than 11 and 9 would be too short, but with more inductors the pitches may be shortened by more than 2. Inspection of the winding diagram will indicate when the pitches are too short.

With the pitches decided upon, the diagram may be completed and the poles blocked in. The number of slots between pole centers is equal to total slots divided by the number of poles, i.e., the pole pitch, and each pole may be drawn in to cover about 70 per cent of the distance between pole centers. It is well to locate the first pole center in the middle of some one coil, for then the short-circuited turns will show up more clearly and facilitate the location of the brushes.

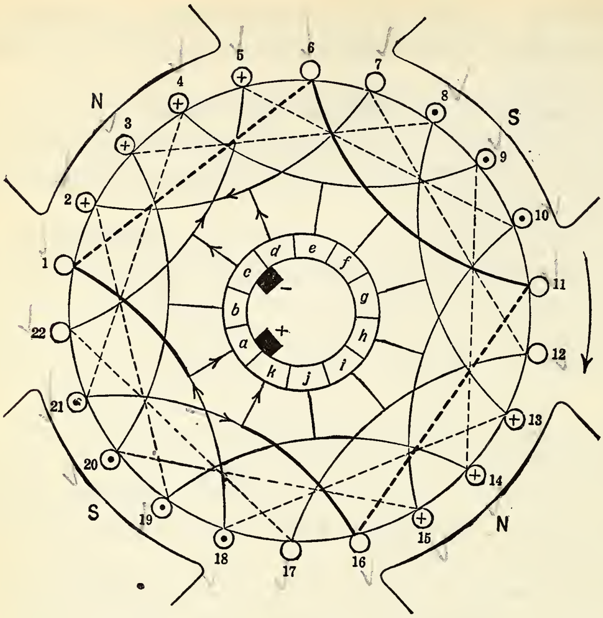
57. Simple Wave or Series Windings. A simple 4-pole wave winding with 22 inductors and 11 commutator bars is shown in Fig. 5–52. The method of connection may be seen by starting with bar *c* and then following through inductor 5, inductor 10, bar *h*, inductors 15 and 20, bar *b*, inductors 3 and 8, and so on. Each successive inductor picked up lies under the next pole ahead. The winding of Fig. 5–52 is shown in developed form in Fig. 5–53.

This winding requires but two brushes and will be found to have two parallel paths with the brushes placed 90° apart in a 4-pole winding. The paths may be seen by tracing through as follows:

$$\text{Negative brush}\left\{\begin{matrix} d\text{–}2\text{–}19\text{–}j\text{–}14\text{–}9\text{–}e\text{–}4\text{–}21\text{–}k \\ c\text{–}5\text{–}10\text{–}h\text{–}15\text{–}20\text{–}b\text{–}3\text{–}8\text{–}g\text{–}13\text{–}18\text{–}a \end{matrix}\right\}\text{positive brush}$$

Coils 1–6 and 11–16 in series are short-circuited by the positive brush across bars *a* and *k*; coils 7–12 and 17–22 in series are short-circuited by the negative brush across bars *c* and *d*.

The front and back pitches of the winding of Fig. 5–52 are each 5; both pitches of a series winding must be odd but need not be the same.



Winding Table

a– 1– 6–*f*
f–11–16–*k*
k–21– 4–*e*
e– 9–14–*j*
j–19– 2–*d*
d– 7–12–*i*
i–17–22–*c*
c– 5–10–*h*
h–15–20–*b*
b– 3– 8–*g*
g–13–18–*a*

FIG. 5–52. A wave, or two-circuit, winding for a four-pole machine; in the heavy lines are shown the two coils, connected in series, which are short-circuited by the positive brush. Coils 7–12 and 17–22 in series are short-circuited by the negative brush.

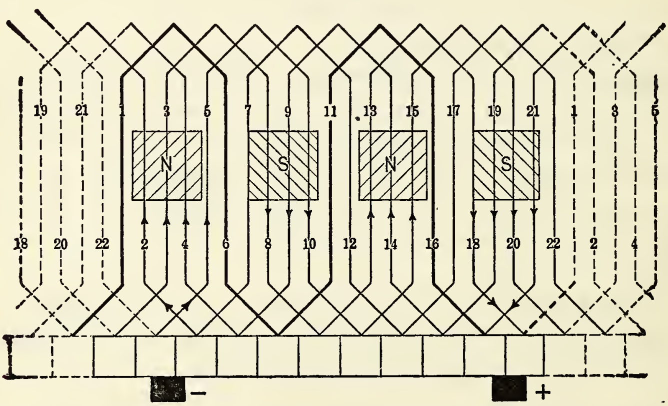


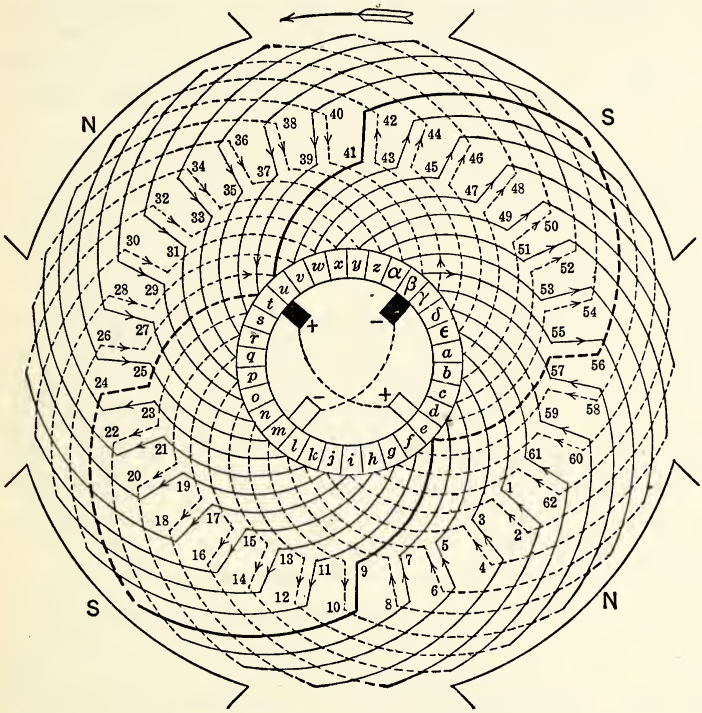
FIG. 5–53. The wave winding of Fig. 5–52 shown in developed form.

58. Commercial Wave or Series Windings. In Fig. 5–54 is shown a two-layer series, or wave, winding for a 4-pole armature having 31 slots. In this diagram the armature coil-sides are shown radially, the two coil-sides in the same slot being represented close together and the lower coil-sides in broken lines. In Fig. 5–55 the same winding is shown in developed

form. This winding requires only two brushes, 90° apart (shown black), as may be seen by tracing it through, as follows:

Negative brush
- 42–27–12–59–44–29–14–61–46–31–16–1–48–33–18–3–50–35–20–5–52–37–22–7–54–39
- 55–8–23–38–53–6–21–36–51–4–19–34–49–2–17–32–47–62–15–30–45–60–13–28–43–58–11–26

positive brush



Winding Table

a– 1–16–p
p–31–46–ε
ε–61–14–o
o–29–44–δ
. . . .
l–23–38–α
α–53– 6–k
. . . .
i–17–32–x
x–47–62–h
h–15–30–w
. . . .
r–35–50–b
b– 3–18–q
q–33–48–a

FIG. 5–54. A two-layer wave winding for a four-pole machine, in a commonly used method of representation. The back and front pitches are both 15.

It will be seen further that the positive brush short-circuits two coils in series, namely, 41–56 and 9–24 (shown heavy), and the negative brush short-circuits coils 57–10 and 25–40.

If additional brushes are put on the machine, they can be placed 90° from the others, as indicated in outline. The two positive brushes are separated electrically only by the short-circuited coils 41–56 and 9–24, which, lying in the interpolar space, are generating no voltage. Similarly, the two negative brushes are joined by turn 25–40, the additional negative brush now also short-circuiting turn 55–8. The two positive brushes and the two negative brushes are thus practically in parallel, respectively.

The advantage in adding the extra brushes is that a shorter commutator may be used. If with two sets of brushes, four brushes per set were necessary to carry the current, the commutator would have to be long enough to accommodate the four brushes. However, if four sets of brushes are used, but two brushes per set would be needed so that the length of the commutator is reduced nearly 50 per cent.

Study of Figs. 5–54 and 5–55 will disclose that, in tracing the inductors, each time we come around the armature we arrive at a slot next to the one we started from. Thus, starting with inductor 1, we pick up in succession 16, 31, 46, 61. (See winding table.) Now inductor 61, in slot 31, lies in the next slot from inductor 1; so, in going from 1 to 61, we have just not

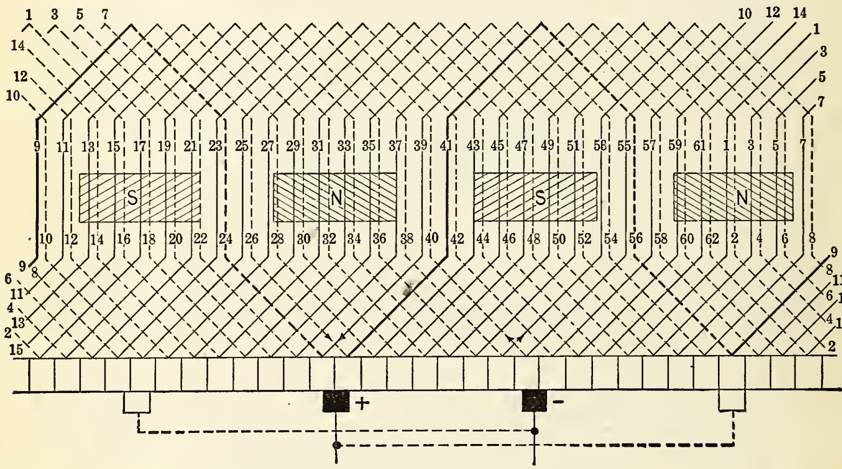


FIG. 5–55. The wave winding of Fig. 5–54 shown in developed form.

gone fully around. From 61 we proceed through inductors 14, 29, 44, to 59, etc. Evidently such a winding will build up in a backward or *retrogressive* fashion.

Had the winding been arranged for 58 inductors or 29 slots, using the same pitches, we would, starting with inductor 1, have picked up 16, 31, 46, and then 3, thus crossing over inductor 1. Such a winding would build up in a forward or *progressive* fashion, and, though perfectly feasible, would have relatively longer end connections.

In general, the conditions to be fulfilled in laying out a series winding are rather complicated, and any complete consideration is beyond the scope of this book. A few general statements which apply to simple windings are in order, however.

In series windings, both back and front pitches, must be odd and they may be the same. The number of slots, however, must be carefully selected.

With 4-pole machines, if only two inductors per slot are used, a winding may be arranged with any odd number of slots. It is not possible to arrange a series winding with 4 or 8 inductors per slot and use all inductors; the latter arrangement would have an even number of coils. If 6 inductors per slot are to be used, a winding is possible using 29, 33, 37, 41, etc., slots.

With 6-pole machines, any number of inductors per slot may be used, provided that the total number of slots is not divisible by 3.

59. Procedure in Laying Out a Simple Four-pole Wave Winding. If it is required to lay out the 4-pole winding of Fig. 5–55 with 31 slots and 62 coil-sides, an average pitch is first obtained by dividing the number *of coil-sides, less two*, by the number of poles, giving, in this case, 15. It has been shown, in a 4-pole wave winding, that after 4 inductors have been picked up the fifth inductor must lie in the slot just before that in which the first is placed. Thus the order of connecting inductors in this case with both back and front pitches of 15 is 1–16–31–46–61, which is satisfactory. Centers of poles must lie $7\frac{3}{4}$ slots apart and, although they may be arbitrarily placed in a diagram, it is preferable to locate the pole centers symmetrically with respect to the slots. The exact location of the brushes may be obtained by study of the coils which are located in the interpolar spaces.

If a 4-pole winding for 33 slots and 66 coil-sides were required, the average pitch would be $\frac{64}{4} = 16$. Since both pitches must be odd, a back pitch of 17 and a forward pitch of 15 may be used. The inductor order would then be 1–18–33–50–65.

For a 4-pole winding with 29 slots and 58 coil-sides, the average pitch would be $\frac{56}{4} = 14$, so that pitches of 15 and 13 would be satisfactory.

60. Duplex-lap or multiple-circuit Windings. In the lap or multiple-circuit windings described in sections 50 to 56, the front and back pitches differed by two, and there was one armature circuit between any one brush and the next adjacent one, or as many parallel circuits through the entire armature as there were poles. Such windings are designed as *simplex* windings.

In Fig. 5–56 there is shown a winding with 24 slots and coils, so that there are two coil-sides per slot. The back pitch is 13 and the front pitch is 9, i.e., the back and front pitches differ by four. If this winding is traced (in heavy lines) starting with inductor 1, it will be seen that only alternate slots are filled and only alternate commutator bars are used, and that it closes from inductor 10 to inductor 1. The table for this winding is

$$a\text{–}1\text{–}14\text{–}c\text{–}5\text{–}18\text{–}e\text{–}9\text{–}22\text{–}g\text{–}13\text{–}26\text{–}i\text{–}17\text{–}30\text{–}k\text{–}21\text{–}34\text{–}m\text{–}$$
$$\text{–}25\text{–}38\text{–}o\text{–}29\text{–}42\text{–}q\text{–}33\text{–}46\text{–}s\text{–}37\text{–}2\text{–}u\text{–}41\text{–}6\text{–}$$
$$\text{–}w\text{–}45\text{–}10\text{–}a$$

The winding thus closes upon itself or is *re-entrant* after going once

around the armature. It is complete, as if it were a simplex winding with 12 coils.

As the winding used only alternate slots and commutator bars, another winding, the duplicate of the first, may be placed in the vacant slots with the same pitches, and connected to the unused commutator bars. This second half of the entire winding (shown in light lines) is also a simplex winding by itself and has a separate closing from the first.

Although the two parts of the entire winding described are separate so far as the armature is concerned, if brushes, wide enough to span more than two commutator bars, are used, there will then be two parallel paths between any two adjacent brushes, or a total of *twice* as many paths through

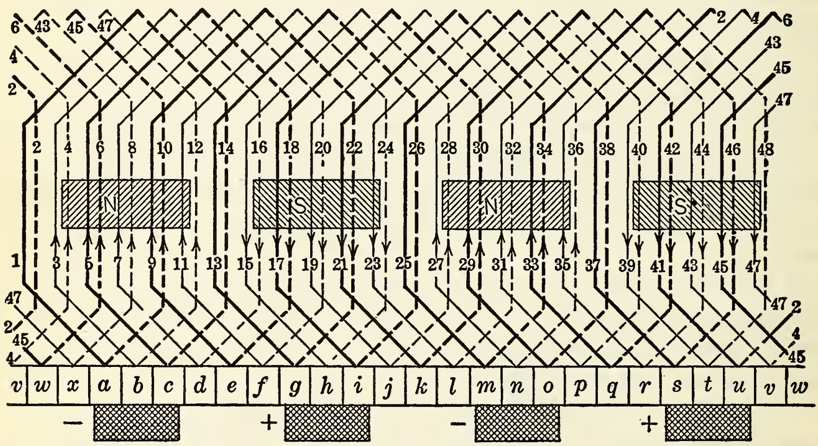


Fig. 5–56. A duplex, doubly re-entrant, lap or multiple-circuit winding. It is duplex because there are two circuits in parallel between adjacent brushes; it is doubly re-entrant because it closes upon itself in two places.

the winding as there are poles. For this reason the entire winding of Fig. 5–56 is designated as a *duplex winding.*

Further, since the entire winding of Fig. 5–56 has two separate closings, it is said to be *doubly re-entrant.* A simplex winding can have but one closing and is therefore singly re-entrant.

The 24-slot drum winding of Fig. 5–56 has been redrawn as a 2-pole ring winding in Fig. 5–57, the two parts being again shown, one in heavy lines and the other in light lines.

Suppose that the armature previously discussed had had 23 slots and 23 commutator bars instead of 24. If a winding with the same pitches were started it would proceed according to the table

$$a\text{–}1\text{–}14\text{–}c\text{–}5\text{–}18\text{–}e\text{–}9\text{–}22\text{–}g\text{–}13\text{–}26\text{–}i\text{–}17\text{–}30\text{–}k\text{–}21\text{–}34\text{–}m\text{–}$$
$$\text{–}25\text{–}38\text{–}o\text{–}29\text{–}42\text{–}q\text{–}33\text{–}46\text{–}s\text{–}37\text{–}4\text{–}u\text{–}41\text{–}8\text{–}w\text{–}45\text{–}12\text{–}b$$

The winding has gone around the armature once but has not closed upon itself as in the last case because of the removal of one slot (with the coil sides 47 and 48) and one commutator bar (bar x).

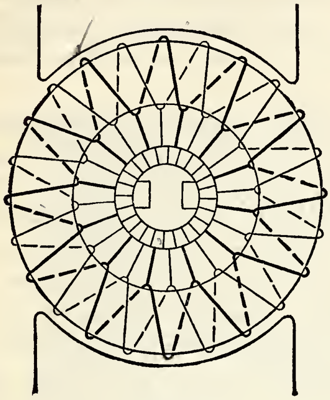


Fig. 5–57.

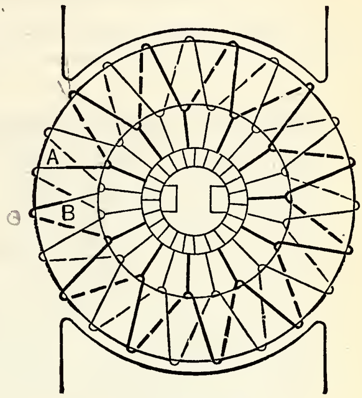


Fig. 5–58.

Fig. 5–57. A duplex, doubly re-entrant ring winding. There are four parallel paths through the armature between brushes.

Fig. 5–58. A duplex, singly re-entrant ring winding. It also has four parallel paths through the armature between brushes.

The winding will proceed, if we continue, according to the table

$$b\text{–}3\text{–}16\text{–}d\text{–}7\text{–}20\text{–}f\text{–}11\text{–}24\text{–}h\text{–}15\text{–}28\text{–}j\text{–}19\text{–}32\text{–}l\text{–}23\text{–}36\text{–}n\text{–}$$
$$\text{–}27\text{–}40\text{–}p\text{–}31\text{–}44\text{–}r\text{–}35\text{–}2\text{–}t\text{–}39\text{–}6\text{–}v\text{–}43\text{–}10\text{–}a$$

and after once more going around the armature closes with the first winding.

The 23-slot winding just described is shown as a bipolar ring winding, in Fig. 5–58. If the heavy-line part is considered as starting at A and building up clockwise, it will end at B, at which point it joins the light-line portion and finally closes at A. This winding, having but one closing, is singly re-entrant. If the brushes in Fig. 5–58 also span more than two commutator bars, there will be four parallel paths through the armature, or twice the number of poles. The winding of Fig. 5–58 is designated as duplex, singly re-entrant.

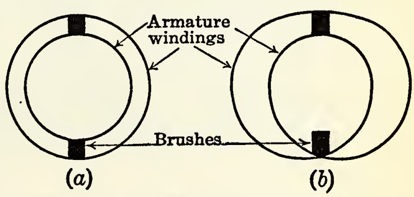


Fig. 5–59. Diagrammatic representation of (a) a duplex, doubly re-entrant winding, and (b) a duplex, singly re-entrant winding. The circles represent the armature windings.

The ideas as to duplex, doubly re-entrant, and duplex, singly re-entrant, windings are again presented in different form in Fig. 5–59. It is evident

that, by proper choice of pitches, numbers of slots, and still wider brushes, windings may be extended to become triplex, triply re-entrant, etc.

Since the use of multiplex, multiply re-entrant windings, with multiple-circuit windings, increases the current-carrying capacity of a winding with a given number and size of inductors, but also reduces the voltage, their use is restricted to heavy-current, low-voltage machines, such as are used in the electrochemical and electrometallurgical industries.

61. Multiplex Series or Wave Windings. Although but infrequently used, it is possible to arrange an armature winding as a duplex, doubly re-entrant, series winding, using two simplex windings in alternate slots. Or by the proper choice of slots and pitches, a duplex, singly re-entrant series winding is also possible. With either arrangement for a four-pole machine, if the brushes span more than two commutator bars, there will be four parallel paths through the armature. Such a winding for a four-pole machine would have no advantage over a multiple-circuit winding, except that the conductors in each parallel path would lie under all poles, so that the behavior of the machine would not be affected by worn bearings.

A multiplex wave winding might be employed where it is desired to provide an armature winding which has more than two paths in multiple, but not as many paths as the number of poles. Thus, if it were desired to build a six-pole machine with four multiple armature paths, a duplex winding, with a given number of inductors, would give a higher voltage than a multiple-circuit winding, but less than a simplex wave winding.

62. Equalizer Connections or Equipotential Connections. It has been shown that in multiple-circuit windings there are as many paths in parallel as there are brushes, and, further, that each circuit is always under the influence of the same pole. In accordance with Kirchhoff's laws, if each circuit is to carry its proportionate share of the total armature current, it must at all times generate the same voltage and have the same resistance as all the other paths. If, with no external load on the armature, one path were to generate more voltage than another, an equalizing current would flow through them over the connecting brushes, resulting in unnecessary heating of the winding and sparking at the brushes. With load on the armature, the external current will divide unequally between brushes of the same polarity and also between the parallel paths of the armature winding. Unequal distribution of current between brushes of the same polarity will overload some of them, thus limiting the possible output of the machine, and it may also result in sparking.

Unequal generated voltages in the parallel paths of the armature winding may be due to various causes. (1) Slight inequalities in the air gap are caused by irregularities in construction or by wear of the bearings; such irregularities in the air gap cause some of the poles to carry more flux than others. (2) If the various parallel magnetic paths of the machine

are not all alike in construction, even though the air gaps are identical, the flux per pole will be different; such inequalities may be caused by poor joints between the pole cores and the yoke or, in large machines, by poor joints between the upper and lower portions of the yoke.

By careful workmanship, the effects of constructional defects may be minimized; but, to overcome the effect of unequal current distribution

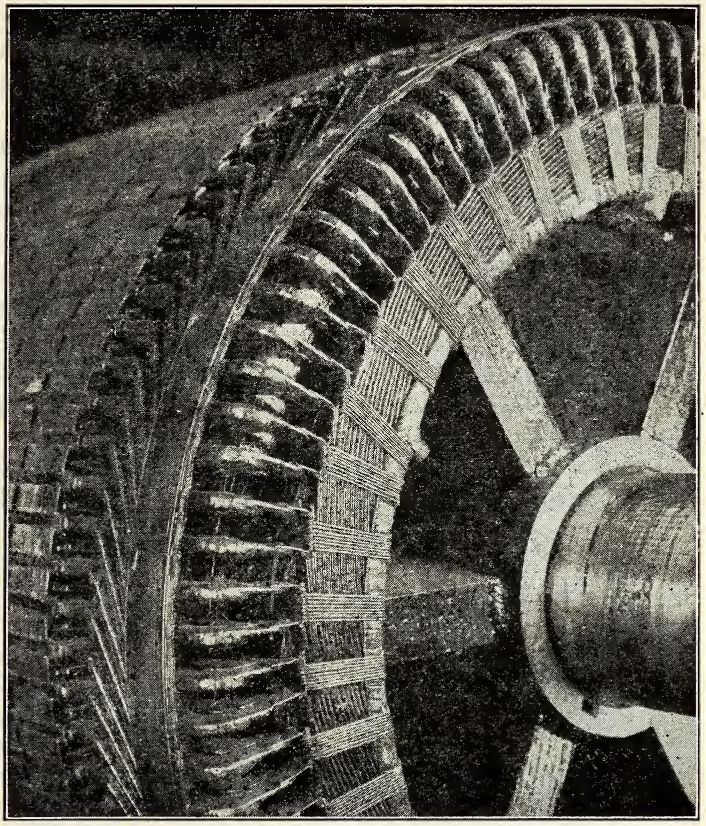


FIG. 5–60. Equalizer connections at the back of a large armature; connections are made to each coil in this case. *Courtesy of the Westinghouse Electric and Manufacturing Co.*

between brushes, *equalizer connections* or *equipotential connections* are used. These consist of low-resistance ring-shaped conductors, which are tapped into the winding at points which, assuming each pole exactly the same, are theoretically at the same potential. If the points are not actually at the same potential, equalizing currents will flow directly over the equalizer connections from one armature circuit to another and thus balance the currents carried by the brushes.

Equalizer connections may be placed either at the back of the armature or under the end connections of the winding between the core and the commutator. In large machines the equalizer connections are made up of large rings, generally at the back of the armature, which are tapped into the winding at appropriate places. These rings are referred to as *equalizer rings.*

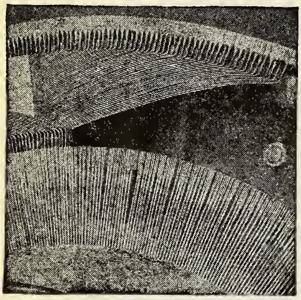


FIG. 5–61. Involute equalizing commutator risers. One layer of risers has been partly cut away. *Courtesy of the General Electric Co.*

In Fig. 5–50, four equalizer rings are shown, each ring being connected to armature coils of the same potential. In Fig. 5–60 is shown an armature with as many equalizer connections on the back end as there are coils.

In series windings, no equalizer connections are necessary, since each of the two parallel paths through the armature is acted upon by all the poles.

A method of providing equalizer connections at the front end of the armature is the use of *involute equalizing commutator leads,* or *involute risers,* as shown in Figs. 5–61 and 5–62. As shown in Fig. 5–62, the winding is displaced 180° from corresponding commutator bars, that is, bars 1 and 2, instead of being placed directly under coil ends *a* and *b*, are moved 180° away. Coil *cfd* is at exactly the same potential as coil *aeb*, and hence ends *c* and *a*, as well as *d* and *b*, may

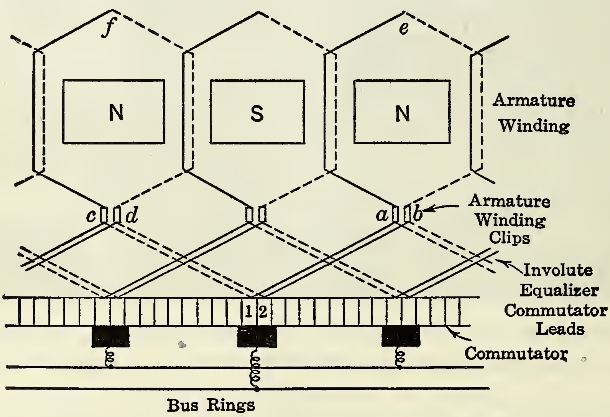


FIG. 5–62. Showing how involute risers are connected. Each commutator bar is joined to two adjacent coil ends of the same potential.

be joined. This is effected by joining ends *c* and *a* to bar 1 and ends *b* and *d* to bar 2. In other words, each commutator bar is joined to two coil ends of the same potential. There are thus two layers of commutator risers as shown in Fig. 5–61, where one layer has been partly cut away.

The advantages of this latter method include better current equalization with a minimum of soldered joints in the various paths, and because the connections are close to the commutator there is a minimum of resistance between brushes of the same polarity. Further, there is an equalizing connection to every armature coil. The disadvantages are that the winding must be full-pitch and that damaged risers are more difficult to repair.

63. Current Capacity of a Winding. The current capacity of an armature is fixed by the safe allowable rise in temperature, just as the safe current for a field coil is fixed. The armature is much better ventilated than the field coils, however, because the fan action of the armature comes into play as it revolves. The current density allowable in armature conductors depends upon ventilation. In Table IX of the Appendix mean values are given for machines of different sizes. If the armature of a 10-kw, 110-volt generator has four parallel paths, and the conductor comprising each path is No. 10 wire, the approximate safe current capacity per path is found by allowing 425 cir mils per ampere. This gives about 23.5 amperes per path. As there are four paths in parallel on the armature, and as each path can safely carry 23.5 amperes, the armature may safely furnish 94 amperes to the external circuit. If the same-sized wire was used for a wave winding, in which there would be only two paths, the safe capacity of the armature would be 47 amperes, but with the same number of inductors the voltage of the generator would be 220.

64. Formed Coils. So far we have said nothing about the actual process of winding the armature. The older machines were all *hand-wound,* but practically all modern machines are wound with *formed coils.* Suppose that an armature is to be wound with a coil pitch (i.e., back pitch) of 11 slots, and that there are to be six turns per coil. Instead of six turns of suitably insulated wire actually being wound in the proper slots (hand winding), the wire may first be wound upon a wooden or metal form, shaped in such a fashion that when the formed coil is taken off this form it is of the right shape and size to be slipped, complete, into the proper slots.

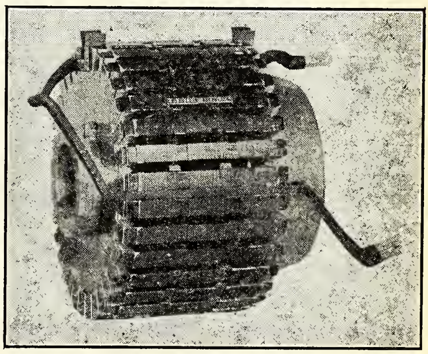


FIG. 5–63. A formed armature coil of one turn placed in the proper slots. *Courtesy of the General Electric Co.*

A formed coil placed in the armature slots is shown in Fig. 5–63. It will be seen that the ends of the coil are so bent that those of neighboring coils fit snugly against each other when assembled on the armature core. This is also shown in Fig. 5–17.

65. Shape of Armature Conductors. In small machines, the armature conductor is generally a round wire, which can easily be bent and shaped in sizes up to about No. 7; if it is necessary to use conductor of larger cross-section, several smaller wires in parallel or a rectangular conductor of the proper size may be used. Round wires, unless quite small, do not utilize the space in a slot very economically, so that rectangular conductor is generally used on large machines. Flexibility may be gained in using several rectangular conductors in parallel to form an inductor.

66. Insulating Materials Used in Machinery.* The principal materials used in electrical machinery consist of cotton, wood fiber, mica, glass, etc., in various reconstructed and treated forms, combined and impregnated with varnishes, waxes, oils, and various other synthetic products. The subject is complicated because of the great number of materials and is further confused by trade names. Only the most important materials will be presented.

Cotton is one of the most important of the insulating materials, being used in natural or impregnated form and also as the base in various reconstruction and chemical processes. In natural form, cotton appears as yarn, tape, or cloth. As yarn it is a much-used covering for smaller sizes of wire as used on armatures, one or two coverings being common. Cotton tape is used as a covering for armature coils, being wound over the coil with lapping joints. When used in natural form cotton serves as a support or separator of conductors; its insulating value is considered equivalent only to the same thickness of air. After impregnation with insulating liquids or compounds, the insulation of cotton is improved.

Rag-base papers, known by various trade names, are tough insulating papers made from cotton rags. They are used where both insulation and resistance to mechanical damage is likely, as in the slots of armatures. A special type of this form of paper is known as *fish paper,* being made by a special process from old cotton rags. It is very hard and tough in both directions, and so may be used in thin pieces. It is much used for slot insulation, with and without paraffin impregnation or varnish treatment.

Fullerboard, or *pressboard,* is made from 75 per cent rag stock and 25 per cent wood pulp, in large sheets up to about 0.5 inch thickness. It is particularly useful under or in contact with oil, may be formed into channels and angles, and is much used for barriers, washers, etc., in machinery. *Kraft fullerboard,* made entirely from wood pulp, is similar to rag fullerboard and is cheaper; it has, however, greater shrinking and warping tendencies and is not so readily formed.

Kraft paper, brown in color, is made from wood pulp, in thickness 0.0005 to 0.5 inch. As it may be made extremely pure and uniform and is

* Condensed from Pender and Del Mar, *Electrical Engineers' Handbook,* John Wiley & Sons, 1936.

readily impregnated, it is one of the most useful of electrical papers. Its chief use is in capacitors, but it is also useful as slot insulation and as backing for mica tape.

Varnish-treated cloth, known also as *varnished cambric* or *empire cloth,* is a muslin or unbleached cotton cloth which has been given several impregnations or treatments with tan or black varnish. After each treatment, the cloth is dried and heated. Its use for cables has already been mentioned, and it is also used in machinery as a coil wrapping and for slot insulation.

Fiber, which appears under various grades and trade names, is made by various chemical treatments from pure cotton cellulose. It is a hard, tough, bone-like material that may easily be formed and machined, and which has high insulating value when dry. It may be made into large sheets of almost any desired thickness.

Mica, besides its application in commutators, is reconstructed into plates, tubes, etc., which may be molded into appropriate shapes. It is also built up from thin flakes, with thin paper or cloth as a backing, into tape. This tape is sometimes used as coil wrapping in d-c mill-type motors which are built to withstand temperatures as high as 125 C. Mica tape is much used in high-voltage a-c machines.

Asbestos is sometimes used in felted form, as a tape or as a braid for coil wrapping in d-c mill-type motors.

Glass in the form of long thin fibers may be woven into tape and used for coil wrappings. It may also be spun or braided upon wires in one or more layers. So used, glass is always impregnated with varnish. Glass is extensively used in mill-type motors which may operate at temperatures of 125 C, as a substitute for mica or asbestos.

Cellophane is a product made from wood pulp by chemical treatment, and is obtainable in the form of sheets, tubes, tape, etc. It gives promise of possible use as a wire insulation and coil wrapping.

Cellulose acetate is a product made by chemical process from cotton fiber or wood pulp. It comes in forms similar to Cellophane and appears to have similar applications.

Plastics, which have made such great advances in recent years, are much combined with cloth, paper, and other fibers to form many useful varieties of insulating materials. Some of these find application in machinery.

67. Armature Insulation. As previously mentioned, it is common practice to form and insulate the armature coils before they are inserted into the armature slots. The conductor is wound upon a proper form which gives the coil the proper spread and the necessary end twists. If the conductor is of small cross-section it will carry a single or a double wrap of cotton before it is applied to the form. Large rectangular conductors will usually be bare when formed into coils and then a strip of

paper, pressboard, fish paper, etc., will be laid between the turns to insulate one conductor from the other. After the coil is removed from the form it will be wrapped with varnished cambric or plain cotton tape or even both. The amount of insulation so far placed upon the coil depends upon the voltage for which the winding is designed; the higher the voltage the greater the amount of insulation.

The armature coils are now impregnated, the process being either a simple dipping in insulating compounds and air drying, or a heating and

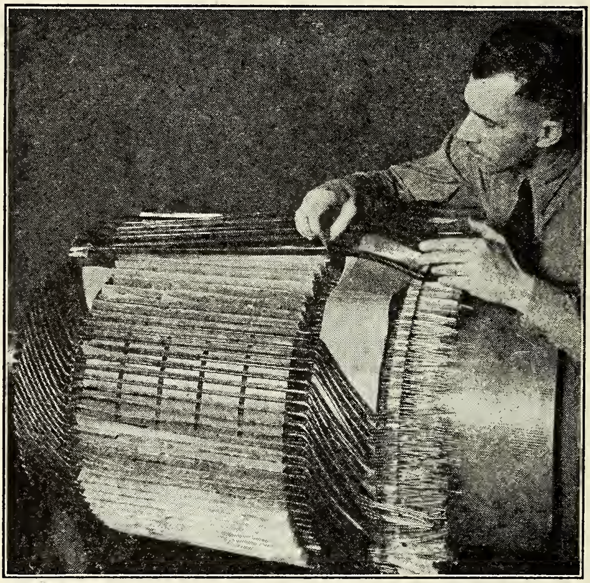


Fig. 5–64. Winding the armature of a 250-hp, 230-volt, 400- to 1200-rpm motor. The winding is of the wave or series type and the slot insulation may be seen. The ends of the coils will be soldered to the commutator risers and the excess cut off. *Courtesy of the Allis-Chalmers Manufacturing Co.*

vacuum process. The amount of treatment the coils receive will depend upon the voltage and the amount of insulation desired. In the vacuum process the coils are placed in a tank and heated in a vacuum to draw off all moisture. The tank is then flooded with an insulating liquid and put under pressure for a time. After the insulating liquid is drained from the tank the coils are heated and dried. These processes may be repeated several times.

68. Slot Insulation. The process of winding the armature consists in first inserting the slot armor. This consists of a channel or trough of fiber, pressboard, or fish paper, as may be seen in Fig. 5–64. These troughs when first inserted extend beyond the ends of the armature core, so that,

when the formed coils are hammered into place, there may be no risk of sharp edges on the iron core cutting through the insulation of the coil, and so *grounding* the winding to the armature core.

In Fig. 5–65 are shown cross-sections of typical slots in which the conductor and coil wrappings, etc., are shown. These diagrams are not drawn to proper scale, the thickness of insulation having been exaggerated in the interest of clearness.

Between the upper and lower coil-sides a strip of insulation is inserted because there will be considerable difference of potential between them. This insulation may be of paper, fiber, pressboard, fish paper, etc., depending upon the voltage between coil-sides. After the upper coil-sides are

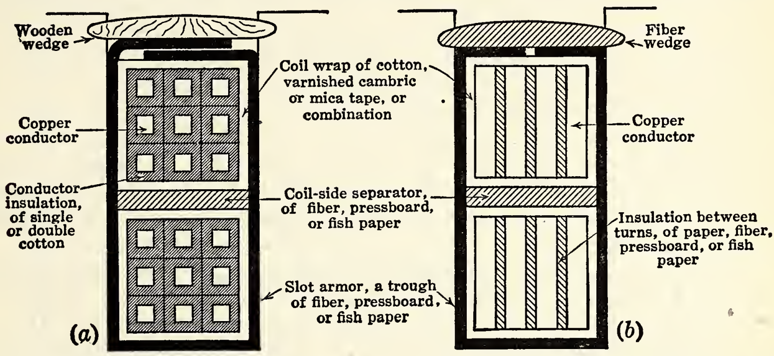


Fig. 5–65. Slot cross-sections showing how coils fit in place, how they are insulated and how they are kept in place by wedges. The dimensions of the insulation are exaggerated.

in place, the ends of the slot armor, which up to now have extended beyond the armature periphery, are trimmed, if necessary, and bent over. A wooden or fiber wedge is then inserted into notches in the teeth to hold the winding in place. Finally the armature is banded by winding phosphor-bronze binding wire in bands around the armature periphery. When the wires are in place, solder is run between them, joining them into a band. Several bands of binding wire may be used, as may be seen in Fig. 5–40.

On small machines, using straight-sided slots, wedges often are not used. After the coil-sides are snugly in place, a strip of stiff, insulating material is placed into the top of the slot as the armature is banded; the bands then hold the winding in place.

PROBLEMS

5–1. An armature core is in the form of a hollow cylinder of 10 inches inside diameter and 18 inches outside. Its axial length is 20 inches. If used in a 4-pole field at 900 rpm, the flux density in the air gap being 10,000 gausses, what is the probable hysteresis loss in ergs per second, and in watts? See section 47, page 76, for data on hysteresis loss. The poles are 9 inches wide.

5–2. If the speed of the machine of problem 5–1 is increased 100 per cent and the flux density is cut down 50 per cent, the voltage generated by the armature will be the same. By what percentage is the hysteresis loss changed? Increased or decreased?

5–3. The loss in the armature of a certain turbo-generator is 7500 watts. If the temperature of the ventilating air at input is 20 C and upon exhaust is 55 C, how many cubic feet of air per minute must be supplied to the core?

5–4. In a certain armature core the hysteresis loss is 550 watts and the eddy-current loss is 180 watts; the core, because of these losses, rises 35 C. If the flux is held constant and speed is increased 25 per cent, what will be the temperature rise, assuming radiation, etc., per degree rise, to be the same at the higher speed as it was originally? What will it be if the radiation per degree rise varies directly with the speed?

5–5. In problem 5–4 what will be the effect of the increased speed upon the full-load armature I^2R loss, full-load current being considered the same for both speeds?

5–6. In problem 5–4 the flux density at the higher speed is to be decreased sufficiently to maintain the core loss at the value it had at the lower speed. How much must it be decreased?

5–7. A 110-volt, 500-kw generator has six sets of brushes. What must be the contact area of one set if a current density of 40 amperes per square inch of contact area is allowed?

5–8. What should be the contact area of one set of brushes in a 250-volt, 50-hp, 4-pole motor, the efficiency of which is 85 per cent? Allow 50 amperes per square inch of contact area.

5–9. In a magnetic circuit similar to that given in Fig. 5–32, the leakage factor is 1.26, and the flux required through the core is 1.2×10^6 lines. Armature core and poles are sheet steel; yoke is cast steel. Core is 8 inches long and 22 square inches in cross-section; teeth are 0.6 inch long, and useful cross-section is 12 square inches. Air gap is 0.18 inch long and 18 square inches in cross-section; poles are 5 inches long and 14 square inches in cross-section; yoke is 24 inches long and 12 square inches in actual cross-section. How many ampere-turns are required per pole?

5–10. If the required air-gap flux per pole of the machine in Fig. 5–33 is to be 1.5×10^6 lines with a leakage factor of 1.25, how many ampere-turns per pole would be required?

5–11. The air-gap flux in a bipolar motor is to be 1.3×10^6 lines with a leakage factor of 1.28. The yoke is a plate and the poles are made of sheet steel. With the following dimensions in inches, how many ampere-turns are required per pole?

Part	Core	Teeth	Gap	Poles	Yoke
Length	8.4	0.65 each	0.2 each	5.4 each	27
Area	24	13	20	18	13 each

5–12. If all the dimensions of the machine in Fig. 5–33 were reduced 10 per cent, how many ampere-turns per pole would be required to furnish 1.3×10^6 lines, with a leakage factor of 1.22?

5–13. The required air-gap flux of a 4-pole machine similar to Fig. 5–33 is to be 1.4×10^6 lines with a leakage factor of 1.23. How many ampere-turns per pole are needed if the poles are of sheets and the yoke of plates, with the following dimensions in inches and square inches? Lengths of core and yoke are from pole to pole.

Part	Core	Teeth	Gap	Poles	Yoke
Length	5.5	1.48 each	0.3 each	5.0 each	17
Area	8.5	12.5	21.5	19.5	10 each

5–14. The air-gap flux in a 4-pole generator is to be 1.5×10^6 lines, the leakage factor 1.25. How many ampere-turns are required if the poles are of sheet steel and the yoke of plates, with the following dimensions in inches? Lengths of core and yoke are from pole to pole.

Part	Core	Teeth	Gap	Poles	Yoke
Length	6	1.5 each	0.3 each	5 each	18
Area	10	13.5	25	20	11 each

5–15. The required air-gap flux for a 4-pole motor similar to Fig. 5–33 is to be 2.90×10^6 lines with a leakage factor of 1.20. If the poles are of sheets and the yoke is of cast steel, how many ampere-turns per pole are needed with the following dimensions in inches? Core and yoke lengths are from pole to pole.

Part	Core	Teeth	Gap	Poles	Yoke
Length	11	1.67 each	0.156 each	6 each	22
Area	16.7	26.3	66	40.5	25 each

5–16. How many ampere-turns per pole would be required in the machine of Fig. 5–34 if the required air-gap flux per pole were 9.6×10^6 lines?

5–17. A 10-pole generator requires an air-gap flux of 10^7 lines per pole. Leakage factor is 1.18. How many ampere-turns are required if dimensions are as follows in inches? Core and yoke lengths are from pole to pole. Yoke is of cast steel and poles are laminated.

Part	Core	Teeth	Gap	Poles	Yoke
Length	14	1.8 each	0.35 each	12 each	32
Area	60	91	150	131	72 each

5–18. The required air-gap flux per pole of a 12-pole generator is to be 11×10^6 lines with a leakage factor of 1.16. If the poles are made of sheets and the yoke is of cast steel, how many ampere-turns per pole are needed with the following dimensions? Core and yoke lengths are from pole to pole.

Part	Core	Teeth	Gap	Poles	Yoke
Length	18	1.3 each	0.40 each	12.5 each	35
Area	70	100	165	160	75 each

5–19. To set up the required flux in a 2-pole, 110-volt motor, 4730 ampere-turns per pole are needed. If the average length of turn is 22 inches and 95 volts are available for field excitation, what size wire should be used and how many turns of it are required per pole?

5–20. In a small bipolar, 250-volt generator the required number of ampere-turns per pole is 5000. The mean length per turn of the field winding is 12 inches. The field rheostat drop will be 20 per cent of the available voltage. Determine proper size of wire and the necessary turns per pole. The field rheostat is in series with the field.

5–21. In a 4-pole, 30-kw, 125-volt generator the required ampere-turns per pole are 4800. If the mean length of turn of the field winding is 25 inches and 15 per cent of the voltage is to be used up in the rheostat in series with the field winding, what size wire should be used and how many turns per pole will be proper?

5–22. If the field ampere-turns required in a 4-pole, 230-volt, 30-kw generator are 6200, what size wire should be used in the field and how many turns of it are required? The average length of turn is 25 inches and 215 volts are available for the field.

5–23. In a 4-pole, 50-kw. 250-volt generator the required ampere-turns per pole are 8100. The mean length per turn of the shunt-field winding is 30 inches. If 15 per cent of the voltage is to be used up in a rheostat in series with the shunt-field winding, what size wire should be used and how many turns of it must be used per pole?

5–24. In a 6-pole, 275-volt generator, 8420 ampere-turns per pole are required to set up the necessary flux. If the field rheostat, in series with the field, is allowed not more than 15 per cent of the armature voltage, and the average length per turn of the field winding is 58 inches, what size wire should be used, and how many turns of it must be used per pole? Calculate the power lost in the field alone, using the wire selected.

5–25. A 300-kw, 10-pole, 275-volt generator requires 8000 ampere-turns per pole. The mean length per turn is 53 inches. The field rheostat drop is to be 15 per cent of the voltage. Determine proper size of wire and the necessary number of turns. The field rheostat is in series with the field.

5–26. The series field in a 6-pole, 300-kw, 275-volt generator is to supply 1636 ampere-turns per pole. How many series turns are required per pole? What cross-section should the series turns have, in circular mils, if 850 cir mils per ampere are allowed? If the series winding is made of strap $\frac{1}{4}$ inch thick, how wide should it be? If the average length of turn is 58 inches what will be the full load I^2R loss in the series field?

5–27. The series field of a 4-pole, 230-volt, 30-kw generator is to supply 1300 ampere-turns per pole. If 1500 cir mils per ampere are allowed in the series field, how many series turns per pole are required? What area in square inches should the series turns have? If the average length of turn of the series field is 25 inches, what will be the full-load I^2R loss in the series field?

5–28. The field circuit of a 110-volt, shunt-wound motor uses a current of 2.6 amperes. This value of current flows when the motor is just started, the resistance in the field rheostat in series with the field being 9.6 ohms. For how large a temperature rise of the field windings can the field current be maintained constant by cutting out the rheostat? Assume a starting temperature of 20C.

5–29. Give the winding diagram of a multiple-circuit winding for the armature of a 4-pole machine, with 36 inductors and 18 commutator bars. (The number of bars and inductors here is evidently too small for a commercial winding.)

5–30. Lay out the diagram of a full-pitch, multiple-circuit winding for a 4-pole generator with 48 inductors and 24 commutator bars. What are the front and back pitches? What fractional pitches seem feasible?

5–31. Lay out completely the diagram for the multiple-circuit armature winding of a 4-pole machine with 56 inductors and 28 commutator bars. What are the front and back pitches? Is the pitch you selected full or fractional?

5–32. Give the winding table and the developed winding for a 4-pole, multiple-circuit armature with 48 commutator bars and 48 slots.

5–33. Give the winding table and a portion of the developed winding for a 4-pole, multiple-circuit armature with 99 coils and 33 slots.

5–34. Give the winding table and a developed diagram of the winding covering 2 poles for a 6-pole, multiple-circuit winding with 120 inductors and 60 commutator bars.

5–35. Lay out a multiple-circuit winding for a 4-pole machine with 84 slots and 84

commutator bars. Show the complete winding for a little more than one pole. What are the pitches and are they full or fractional?

5–36. Lay out the complete winding for a 4-pole wave winding with 38 inductors and 19 commutator bars.

5–37. Lay out a two-circuit or wave winding for a 4-pole motor with 46 inductors and 23 commutator bars. What are the pitches?

5–38. Give the winding diagram of a series winding for the armature of a 4-pole machine, with 54 inductors and 27 commutator bars.

5–39. Give the winding table and developed winding for a 4-pole, wave winding with 45 slots and coils.

5–40. Lay out a two-circuit winding for a 4-pole motor with 51 slots and coils. What are the pitches?

5–41. Give the winding table and the developed winding for a 4-pole, wave-wound armature with 53 slots and two inductors per slot.

5–42. Give the winding table for a 4-pole wave winding with 55 slots and coils. What are the pitches?

5–43. Give the winding table for a 4-pole, two-circuit winding with 41 slots, 123 coils, and 123 bars.

5–44. Lay out a duplex, doubly re-entrant winding for a 4-pole generator with 48 slots and 48 commutator bars.

5–45. Lay out a duplex winding for a 4-pole generator with 47 slots and 47 commutator bars.

5–46. In winding the armature of a 6-pole machine, 1250 feet of No. 12 copper wire are used. What is the armature resistance at 70 C if it is (*a*) a wave winding and (*b*) a multiple-circuit winding?

CHAPTER VI

THE DIRECT-CURRENT GENERATOR

1. Voltage Form for an Elementary Generator. Consider a ring-wound armature with only one coil, the two ends of the coil being connected to two slip rings as in Fig. 6–1. Although represented as one inside the other, these rings would be of the same size and would be placed side by side on the armature shaft.

As is the case with all d-c machines, the air gap is assumed as of constant radial length, so that the flux under the pole face is then of uniform

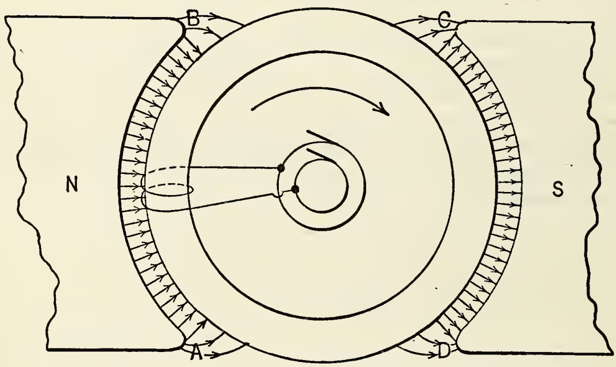


Fig. 6–1. Conventional representation of a single coil on a ring armature, revolving in a bipolar field of practically uniform density.

density. At the pole tips, the density changes from that under the poles to zero, this region being known as the *pole fringe.*

As the coil moves from A to B, its voltage will rise rapidly from zero to a value which will remain constant while the coil is under the pole face and moving through uniform flux, and then again decrease rapidly to zero in the pole fringe at B. In moving from B to C, no flux is cut; hence no voltage is generated. From C to D the coil is again cutting the flux, but now the voltage will be *in the opposite direction* to that generated when the coil was moving from A to B, the relative direction of motion and flux being opposite to that when moving from A to B. Then, as the coil moves from D to A, again no voltage is generated.

If the voltage wave is represented by using the angular position of the coil for abscissae and the magnitude of the generated voltage for ordinate,

a curve will be obtained, such as is shown in Fig. 6–2. Calling the position zero when the coil is at A, it is evident that when the coil has revolved 180° the voltage will be in the opposite direction and will be represented

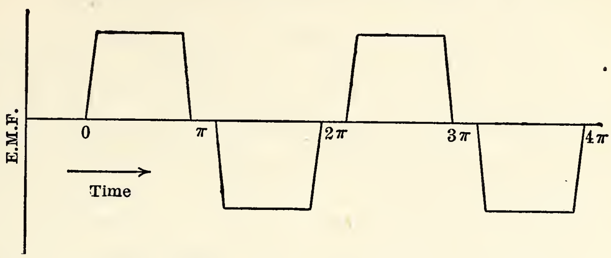


Fig. 6–2. The coil shown in Fig. 6–1 would generate a voltage wave of about the form shown here. Actually, however, the corners of the wave would not be so sharp, as it is impossible to make the flux in the air gap change from a high value to zero value so abruptly.

by negative values on the curve; when the coil has rotated 360° (i.e., back to A) the cycle of values begins over again. Since the flux in Fig. 6–1 is uniform under the pole faces, the voltage wave is flat-topped. An oscillo-

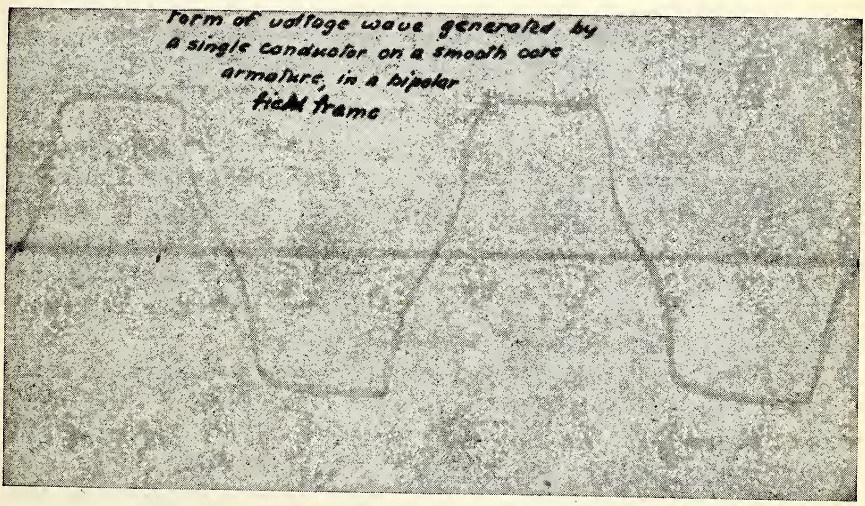


Fig. 6–3. An oscillogram of the voltage generated by a single conductor on a smooth-core armature in a bipolar field. If the armature had been slotted there would have been ripples on the wave as may be seen from Fig. 6–41, which gives a wave similar to that above, using however a slotted-core armature.

gram of the voltage of such a one-coil winding on an actual machine is given in Fig. 6–3; it is seen to be of approximately the same shape as that given in Fig. 6–2.

Such a machine, then, gives a voltage which alternates in direction once every time the coil moves past a pole. If an external circuit, such as a load of lamps, is connected to the brushes, an alternating current of the same shape as the voltage wave of Fig. 6–2 would flow.

2. Action of Commutator. To make the simple machine of Fig. 6–1, give a voltage which, *at the brushes, is uni-directional,* it is necessary to connect the ends of the coil to the two segments of a two-part commutator, and so place the brushes on the commutator (diametrically opposite to each other) that, *at the same time that the voltage of the coil reverses, the connection of the coil to the external circuit reverses.* When this is done, the voltage at the brushes will be uni-directional, as shown in Fig. 6–4, and will give uni-directional current in a circuit connected to the brushes.

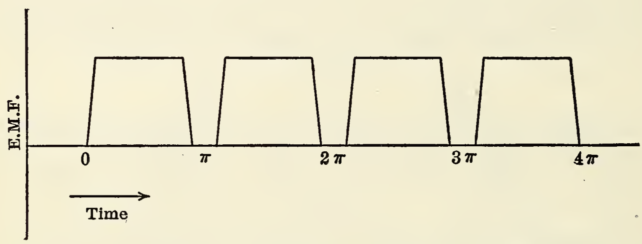


FIG. 6–4. If the voltage wave of Fig. 6–2 were reversed in polarity every other alternation, the voltage would be uni-directional, but not uniform; the coil of Fig. 6–1, if connected to a two-part commutator, would give, at the brushes, a voltage of about the form shown here.

The commutator and proper position of the brushes to give uni-directional voltage at the brushes are shown in Fig. 6–5, where the brushes are shown inside the commutator for clearness of representation; they really bear on the outside surface of the commutator. It will be seen that, as the coil passes through the interpolar space when its voltage is zero, its connections with respect to the brushes are reversed.

If another coil is wound on the armature in the same direction as the first, and if the coils are connected as in Fig. 6–6, a two-coil, *closed-circuit* winding results. The direction of the induced voltages is marked on both coils by arrows, and it is seen that the voltages of both coils act in the same direction in the *external circuit,* but that the two voltages oppose each other in the local circuit made up of the two coils only. In this two-circuit winding, *the voltage of the generator is evidently the same as the voltage per path, but the current capacity of the machine is equal to twice the current capacity per path.* The voltage wave form of the armature shown in Fig. 6–6 would be exactly the same as that shown in Fig. 6–4, which was for a single-coil armature.

The voltage wave of Fig. 6–4 is not suited for ordinary purposes of

lighting, running motors, etc. A uniform, non-pulsating voltage of higher value is desired and requires an armature with many coils and many commutator bars. Consider a four-coil, closed-circuit armature with a four-

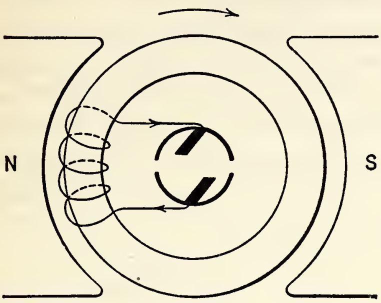


FIG. 6–5.

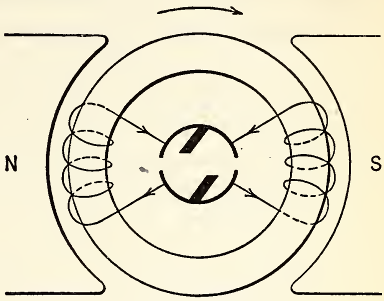


FIG. 6–6.

FIG. 6–5. The connection of the coil of Fig. 6–1 to a two-part commutator is shown here; brushes, diametrically opposite, rest on the commutator and serve to connect the rotating coil to the outside circuit.

FIG. 6–6. If two coils were used with a two-part commutator, they would be connected as shown here, being in parallel for delivering current to the brushes.

part commutator, connected as in Fig. 6–7. As the armature revolves, every coil will generate a wave of voltage of the same shape and magnitude as that of any other coil, but these waves will be behind one another by a

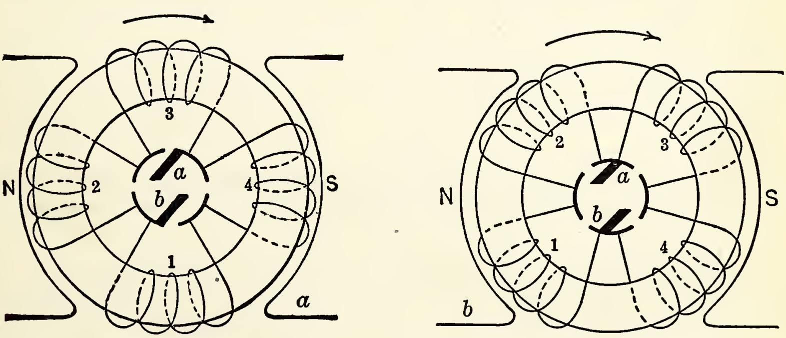


FIG. 6–7. A four-coil winding would require a four-part commutator, and there would be two coils in series in each part of the armature winding. At the time shown in (*a*), only one coil is generating voltage in either path of the winding, the other coil in each path being in the interpolar space and so generating no voltage. After the armature has rotated a little, as shown in (*b*), both coils in either path are active in generating voltage.

time interval equal to that required for one-quarter of a revolution of the armature. The voltage of the machine is obtained at any time by adding together the voltages of the two coils that are in series with each other in

the path considered. In Fig. 6–7*a*, for the instant shown, coils 1 and 4 are in series to form one path through the winding, and coils 2 and 3 are in series to form a second path. The two paths are obviously in parallel. In Fig. 6–7*b* coils 1 and 2 constitute one path and coils 3 and 4 the second.

3. Voltage Form of a Multicoil Armature. The voltages of the different coils of the armature of Fig. 6–7 are shown in Fig. 6–8*a*, the curves being numbered to correspond with the coils. Thus at time = 0 (Fig. 6–7*a*) coils 1 and 3 are generating zero voltage, while coil 2, moving under the north pole, and coil 4, moving under the south pole, generate voltages which are in the same direction with respect to the external circuit. An eighth

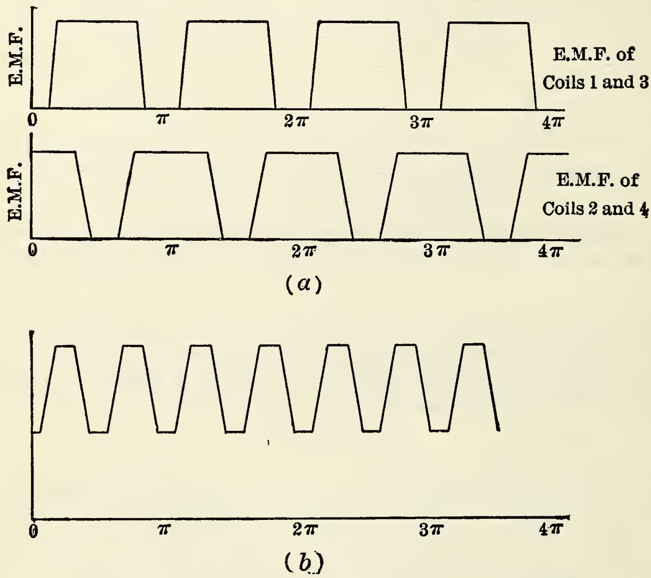


FIG. 6–8. A four-coil armature, with a four-part commutator, gives at the brushes a voltage more nearly uniform than that delivered by the two-coil armature.

of a revolution later, as is indicated in Fig. 6–7*b*, coils 1 and 2 are both generating voltage in the same direction and are in series to form one path between brushes, the line voltage being the sum of the two coil voltages. Similarly coils 3 and 4 together generate an equal voltage in the *same direction with respect to the external circuit* and together form the second path through the armature.

The line or brush voltage, obtained by adding the coil voltages in an armature path, is shown in Fig. 6–8*b*. Although the brush voltage is not yet regular, it never goes to zero value as with the two-coil winding (Fig. 6–4), and the fluctuation is but 50 per cent of the maximum value. By considering in the same way an eight-coil armature with an eight-part commutator, it is found that, as the number of coils is increased, the pulsa-

tion in the brush voltage continually decreases. On an ordinary commercial machine having perhaps 20 coils between brushes, the variation can be detected only by some sensitive instrument like a telephone receiver. Besides decreasing in magnitude as the number of coils is increased, the fluctuations of the voltage also increase in rapidity, until on commercial generators they are in the neighborhood of 1000 or more per second.

Thus a multiple-coil armature, equipped with a multiple-part commutator, will produce a uni-directional brush voltage of practically constant magnitude. The machine is therefore called a continuous-current or direct-current generator. The voltage in the individual coils is alternating, but, with the commutator changing the connection of the coil to the external circuit, the alternating coil voltage becomes uni-directional at the brushes.

4. First Method for Calculating the Voltage of a Generator. In determining the voltage of a generator, it is necessary only to calculate the rate at which flux is being cut by the conductors on the armature and to know the method of interconnection of these conductors.

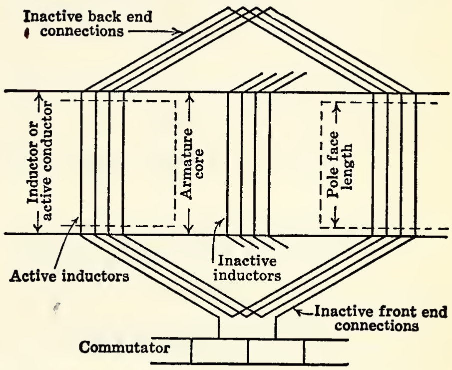


Fig. 6–9. An armature coil showing the portions which are active or inactive in the generation of voltage. Only the inductors which lie under the pole faces will generate voltage.

An armature coil may consist of a number of turns in series. The term *inductor* often is used to denote that part of a turn which is capable of cutting flux; it is generally the portion of the turn which lies within the slots of the core, as distinguished from the front and back end connections, which are inactive so far as voltage generation is concerned. The inductor portion of a turn being practically the turn-side, the term inductor is often used synonymously. Since there are two sides to a turn, it is often said that there are two inductors per turn; a coil of four turns would have eight inductors, etc. (Fig 6–9).

The term *active inductors* is often used to indicate those inductors which are actually cutting flux at any time. Evidently not all the inductors on an armature are active at a given moment, for, while some of them lie under the pole face and generate voltage, others must be situated in the interpolar space, where there is no flux to be cut, and, hence, where they can generate no voltage. Generally, 60 to 70 per cent of the inductors of a machine are active at any one instant.

It must also be considered that some of the active inductors lie in a

weaker field than others. The field may be considered in two parts: that part directly under the pole face where the field has normal density; and the pole fringe, the part of the field near the edge of the pole or pole shoe where the flux falls to zero and the density of the field is less than normal. If the voltage of the machine is obtained by calculating the voltage generated by all the inductors in a path, it is necessary to find the voltages generated (*a*) by inductors under the pole and (*b*) by inductors in the pole fringe, and then to add the two voltages so obtained.

5. Example of Voltage Calculation for a Bipolar Generator. Consider a bipolar armature wound with 44 coils of 8 turns each of No. 12 wire. The total length per turn is 2 feet, of which 1 foot is active, each inductor being 6 inches long. The peripheral speed of the armature and therefore the velocity of the inductors is 3000 feet per minute, 60 per cent of the inductors are active (i.e., lie under the pole face at the same time), and the flux density in the air gap is 10,000 lines per sq cm, the pole fringe not considered. It is desired to find: (1) the generated voltage; (2) the safe current capacity; (3) the armature resistance; (4) the full-load terminal voltage.

As the machine will have a two-path winding, there will be 22 coils per path or $22 \times 8 = 176$ turns per path. (It is evident that there are 352 total inductors, each 6 inches long.)

$$\text{Total length of inductor per path} = 176 \times 12 = 2112 \text{ inches}$$

$$\text{Active length of inductor per path} = 2112 \times 60 \text{ per cent} = 1267.2 \text{ inches}$$

The velocity of the inductors is 3000 feet per minute

$$= 3000 \times 12/60 = 600 \text{ inches per second}$$

The flux density being given in lines per square centimeter must be expressed in lines per square inch, or the above quantities must be changed to centimeters. Changing the flux density to lines per square inch:

$$\text{Flux density} = 10{,}000 \times 6.452 = 64{,}520 \text{ lines per square inch}$$

Flux cut per second in one path

$$= 64{,}520 \times 1267.2 \times 600 = 491 \times 10^8 \text{ lines}$$

Hence the generated voltage per path is 491 volts, and, since the two paths are in parallel, this is the generated voltage of the machine.

6. Allowable Armature Current. For this small armature it is safe to allow one ampere per 400 cir mils cross-section of the armature conductor, according to Table IX of the Appendix. As a No. 12 wire has a cross-section of 6530 cir mils, the safe current to allow is 16.33 amperes per path. As there are two paths in parallel in this armature, and as each can safely carry 16.33 amperes, the armature can carry 32.65 amperes. The output

of the machine will then be a little less (owing to the IR drop in the armature, as explained later) than $32.65 \times 491/1000 = 16$ kw, so that the assumed value of 400 cir mils per ampere of armature conductor from the table is about correct. If we had supposed a more poorly ventilated armature than is considered in the table, and allowed 600 cir mils per ampere, the safe current would have been 21.76 amperes.

7. Calculation of Armature Resistance. The resistance of the armature is obtained by calculating the resistance per path and then dividing by the number of paths in parallel in the armature. In the machine above there are 22 coils of 8 turns each, and, since each turn is 2 feet long, the length of wire per path is therefore 352 feet. The resistance of 352 feet of No. 12 wire is 0.559 ohm at 20 C. As there are two paths in parallel, the armature resistance is 0.559/2 or 0.279 ohm. At 75 C the armature resistance is 0.339 ohm.

8. Calculation of Terminal Voltage. The full-load IR drop in the armature winding at 75 C is, therefore,

$$0.339 \times 32.65 = 11.07 \text{ volts}$$

The drop at the brush contacts, with carbon brushes assumed, is 2 volts. Therefore the total drop in the armature circuit with full-load current is $11 + 2 = 13$ volts.

The full-load terminal or brush voltage = 491 volts (generated) − 13 volts (IR drop) = 478 volts, and the actual output is $32.65 \times 478/1000 =$ 15.6 kw, instead of 16 as found above.

9. Calculations for a Multipolar Generator. Consider a 12-pole generator with a multiple-circuit armature having 240 coils on the armature, 4 turns per coil, each turn consisting of two No. 8 wires in parallel. The length per turn is 4 feet, and the length of inductor per turn is 20 inches. The armature is 4 feet in diameter and has a speed of 200 rpm. Fifty per cent of the inductors lie in a field of 58,000 lines per square inch and 20 per cent of them lie in the pole fringe where the average flux density is 32,000 lines per square inch. It is desired to find the same quantities as in the previous problem.

As the machine has a multiple-circuit winding:

Coils per path $= 240/12 = 20$

The length of inductor per path

$$= 20 \times 4 \times 20 = 1600 \text{ inches}$$

The length of inductor per path in the dense field

$$= 1600 \times 50 \text{ per cent} = 800 \text{ inches}$$

The length of inductor per path in the pole fringe

$$= 1600 \times 20 \text{ per cent} = 320 \text{ inches}$$

$$\text{The peripheral speed} \quad = \frac{4 \times 12 \times \pi \times 200}{60} = 502.7 \text{ inches per second}$$

Voltage generated in inductors under pole face

$$= 800 \times 58{,}000 \times 502.7/10^8 = 233.3 \text{ volts}$$

Voltage generated in pole fringe

$$= 320 \times 32{,}000 \times 502.7/10^8 = \underline{51.5 \text{ volts}}$$

$$\text{Total voltage per path} = 284.8 \text{ volts}$$

The conductor of which the winding is formed is a double No. 8, and so has a total cross-section of 33,020 cir mils. Allowing 700 cir mils per ampere (Table IX of the Appendix) gives a current capacity per path of 47.17 amperes. With 12 armature paths the whole armature has a capacity of $12 \times 47.17 = 566$ amperes. The approximate output of the machine is 161 kw, so that the assumption of 700 cir mils per ampere (Table IX of the Appendix) is about correct.

The length per turn of the winding is 4 feet and the length per coil is 16 feet. Since there are 20 coils in series per path, the length per path is $16 \times 20 = 320$ feet.

The resistance of 320 feet of No. 8 wire, at 20 C, $= 0.2010$ ohm

The resistance of 320 feet of double No. 8, therefore, $= 0.1005$ ohm

Hence the armature resistance, at 20 C, is $0.1005/12 = 0.00838$ ohm, which becomes 0.0102 at 75 C.

$$\text{Full load } IR \text{ drop} = 566 \times 0.0102 = 5.77 \text{ volts}$$

With 2 volts allowed for the drop at the brush contacts, the full-load terminal voltage is $284.8 - 5.8 - 2 \qquad = 277$ volts

It is to be noted that the data used in the last two sample problems were not taken from actual machines. In commercial usage certain voltages have been more or less standardized for certain classes of service. Voltages of 110–125 and 220–250 are ordinarily used for lighting generators, and voltages of 600–700 and 1200–1500 for railway generators. Generators intended for electrolytic purposes are built for voltages as low as 6 volts.

10. Rectangular Armature Conductors. If it had been stated that the armature conductor of the above machine was rectangular 0.08 by 0.30 inch, instead of a double No. 8, the determination of the armature resistance would best be made by the use of Eq. (20), page 94.

The cross-section of the conductor $= 80 \times 300 = 24{,}000$ square mils

$= 24{,}000 \times 1.273 = 30{,}550$ cir mils

The resistance, per path of 320 feet, at 0 C

$$R_p = \frac{9.56 \times 320}{30{,}550} = 0.1001 \text{ at } 0 \text{ C}.$$

The armature resistance is then $0.1001/12 = 0.00834$ ohm at 0 C or 0.0111 ohm at 75 C.

11. Second Method for Calculating the Voltage of a Generator. A formula, in the derivation of which the voltage is considered from the standpoint of the *average voltage* of an inductor, rather than its instantaneous value as in the first method, and in which the quantities involved are differently stated, can be obtained as follows:

Let Z = total number of *inductors* on the armature,
p = number of field poles,
m = number of parallel paths in the armature winding,
Φ = total useful flux per pole, entering armature,
n = revolutions per minute of armature.

Since there are Z/m inductors in series, per path, and as the total flux cut by one inductor in one revolution is $p\Phi$, the average flux cut per second by one inductor is $p\Phi n/60$.

$$\text{Average voltage generated by one inductor } \frac{p\Phi n}{60 \times 10^8} \tag{1}$$

The voltage generated per path, which is also the voltage of the machine, E_g, is then

$$E_g = \frac{p\Phi n}{60 \times 10^8} \times \frac{Z}{m} = \frac{p\Phi n Z}{m \times 60 \times 10^8} \ * \tag{2}$$

* The procedure in calculating the voltage of the generator in terms of the instantaneous voltage of an active inductor (section 5) was based upon the expression $BlV/10^8$ (Eq. (9), page 38), in which B is the flux density, through which a total length of inductor per path, l, was moving with velocity V.

If C = total number of coils upon the armature,
T = turns per coil,
L = inductor length per coil,
$L/2$ = armature length = pole-face length (parallel to shaft),
P = fraction of active inductors = ratio of armature periphery covered by poles, to the entire armature periphery,
D = armature diameter,
n = rpm,
m = number of parallel paths,
p = number of poles,

12. Commutation. The function and construction of the commutator have been taken up in previous paragraphs, and mention has been made of the fact that there may be sparking at the contact surface between the brush and commutator. The reasons for this sparking will now be apparent, the superiority of the carbon brush over the copper brush will be shown, and the purpose of the commutating poles will be discussed.

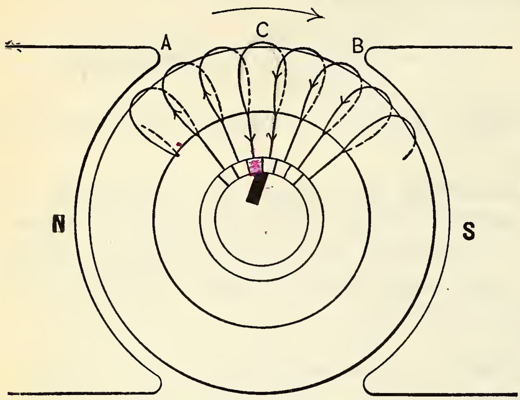


FIG. 6–10. By inspection of this diagram, giving the direction of current flow for coils on both sides of the brush, it is evident that the current in any one coil must reverse its direction as it passes the brush.

The direction of the current through any coil is reversed as the commutator bars, to which the coil ends are attached, move under the brush. This must be so, because, before the coil reaches the brush, as at A, Fig. 6–10, the current is in one direction, and when the coil is at B the current is in the opposite direction in the coil. This reversal of current takes place just as the coil is short-circuited by the brush, as in

---

Z = total armature conductors = $2CT$,
Φ = total flux entering the armature per pole.

Then the voltage of the generator is

$$E_g = B\left(\frac{CTLP}{m}\right)\left(\frac{\pi Dn}{60 \times 10^8}\right) \tag{3}$$

If this equation is regrouped, and numerator and denominator multiplied by 2, and placed equal to Eq. (2), it may be proved that corresponding terms in parentheses are equal.

$$E_g = \left(\frac{BP\pi DL}{2}\right)\left(\frac{n}{60 \times 10^8}\right)\left(\frac{2CT}{m}\right) = (p\Phi)\left(\frac{n}{60 \times 10^8}\right)\left(\frac{Z}{m}\right)$$

Since P, the fraction of active inductors, represents the ratio of armature inductors lying under a pole face to the total inductors, it will also represent the ratio of the armature periphery covered by the poles to the entire periphery. If the armature circumference is πD, the portion covered by poles is $P\pi D$. The area of the armature periphery covered by poles, or the combined area of all the pole faces, is $P\pi DL/2$. The total flux entering the armature from all poles will be the last area multiplied by the flux density B, i.e., $BP\pi DL/2$. If Φ is the flux per pole and p the number of poles, the total flux $BP\pi DL/2 = p\Phi$.

The total number of inductors upon the armature will be twice the number of coils times the turns per coil. ($Z = 2CT$.) With m parallel paths, the inductors per path will be $2CT/m = Z/m$.

position C, Fig. 6–10. The reversing of the direction of current in the coil as the coil moves under the brush is called commutation. If this reversal takes place with no visible sparking at the brush contact, the machine is said to have black, or sparkless, commutation.

13. Effect of Self-induction on Commutation. The reversal of the current in a short-circuited coil during commutation is effected by several complicated electrical and magnetic factors, so that a complete discussion of the subject of commutation is beyond the scope of this book. Since the important factor is the self-induction of the armature coils, and since all the other factors act largely in the same way, a discussion of the effect of self-induction is sufficient for an elementary grasp of the subject of commutation.

The self-induction of an armature coil is due partly to the flux set up about the coil in the teeth and partly to that set up about its end connections. In addition there will be mutual induction between adjacent coils both in the slots and at the end connections.

The time in which the reversal of current in a short-circuited coil takes place is very short, so that *the rate of change of current* as the coil is commutated is very high, especially when the machine is carrying a heavy load. Owing to the self-induction this rapid reversal of current sets up a voltage of self-induction which tends to maintain the current in its original direction. When this voltage of self-induction is high it is difficult to obtain reversal of current in the time allowed, and therefore sparkless commutation, and because of this fact armature coils must be made with low self-inductance. A low coefficient of self-induction is obtained by winding but few turns per coil, by placing the coils in open slots, and by limiting the axial length of the armature. This requires a commutator with a large number of segments, since with but few turns per coil there will be many coils.

14. Rate of Current Change during Commutation. The time during which a coil is short-circuited by the brush depends upon the peripheral speed of the commutator, the thickness of the brush, and the thickness of the insulation between commutator bars, the latter never greater than 15 per cent of the thickness of a commutator bar.

If the insulation between bars is infinitesimally small, a coil will be short-circuited for just the time it takes for the insulation between the bars, to which the coil is connected, to travel across the brush. This may be seen from Fig. 6–11a. With appreciable thickness of insulation, the coil will be short-circuited for the time it takes a point on the commutator to travel a distance equal to the thickness of the brush minus the thickness of insulation between two bars. This may be seen from Fig. 6–11b and c.

Suppose that a commutator $9\frac{5}{8}$ inches in diameter is making 1200 rpm or 20 rps, that the brush thickness is $\frac{5}{8}$ inch, and that the thickness of

insulation between bars is 0.0315 inch. The peripheral speed of the commutator is $\pi \times 9.625 \times 20 = 604.8$ inches per second. The time of commutation is $(0.625 - 0.0315)/604.8 = 0.00098$ second. If the current per path is 120 amperes, the current must change from $+120$ to -120 amperes, so that the rate of change of the current is $240/0.00098 = 244{,}900$ amperes per second. To keep the voltage of self-induction low, the above armature has only one turn per coil.

It appears that the time of commutation may be extended by using thicker brushes. But if very thick brushes are used, commutation will begin before the coil has moved out from under a main pole and the coil will still be generating voltage. Brush width is therefore somewhat of a compromise.

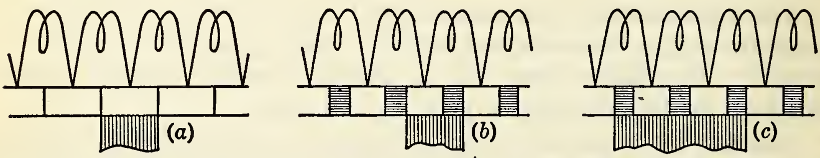


FIG. 6–11. The time of commutation depends upon the peripheral speed of the commutator, the width of the brush, and the thickness of insulation between commutator bars.

15. Methods for Obtaining Sparkless Commutation. In the earliest machines copper brushes were used and these gave very poor commutation, besides causing excessive wear of the commutator. The introduction of carbon brushes brought about a great improvement in commutation. It was stated (page 184) that there was a definite contact resistance between a carbon brush and a copper commutator, and it is the *variation of this contact resistance* between the brush and the two segments to which a coil is attached that tends to make the current reverse. Commutation which depends upon this resistance effect to produce good commutation is sometimes called *resistance commutation.* It is used in all machines as an assisting method except in those requiring very low brush-contact resistance, such as electroplating generators (page 184).

Practically all modern machines use commutating poles which generate within the short-circuited coil a voltage equal and opposite to that of self-induction. This method of improving commutation is sometimes called *emf commutation.* Before the introduction of commutating poles, emf commutation was effected by shifting the machine brushes, as will be explained later.

16. Resistance Commutation. The idea of resistance commutation may be understood by studying the variation of the contact resistance as the commutator bars move under a brush. In Fig. 6–12 is shown a coil B which is about to be commutated, as well as a few coils on either side of it.

Position 1 shows coil B before it begins to be commutated, when all the current, i, flowing through the left side of the winding has to go through coil B and lead c to reach the brush and the external circuit. When the brush is in position 2, this current from the left side of the armature has *two paths* by which to reach the external circuit; through coil B, lead c, and commutator segment g as before, or else not through coil B at all but directly through lead a and commutator segment f. The division of the current between these two paths depends upon their relative resistances, and in the case of carbon brushes *practically all the resistance of either path is in the brush contact.* In position 2, three-quarters of the brush is represented as in contact with segment g, and one-quarter in contact with segment f. If brush-contact resistance is assumed as varying inversely as area of contact, we should expect the current flowing from segment g to the brush to be three times that flowing from segment f to the brush. All the current, i, from the right side of the armature must flow down lead d, so that the current from the left side must divide, one-half, $i/2$, passing directly to the brush through segment f, and the other half, $i/2$, continuing, as before, through coil B and lead c and flowing to the brush through segment g.

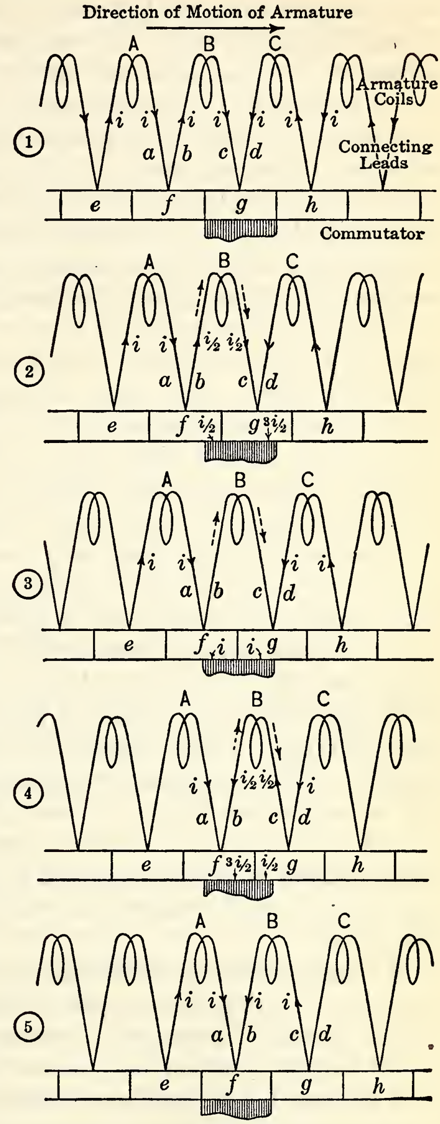


FIG. 6–12. A detailed study of the conditions which must exist in any one coil, during the time it is short-circuited by the brush. The current must die down to zero and build up to an equal value in the opposite direction during the short-circuit period.

It will be seen that, as the armature continues to move, the contact area between the brush and segment f increases and that between the brush and segment g decreases.

In position 3 they are represented as equal, and it would be expected that all the current from the left side of the armature would enter the brush through segment f, and that from the right side through segment g. Accordingly, there would be no current through coil B.

In position 4, the contact area between the brush and segment f is represented as three times that between the brush and segment g. If, therefore, the current flowing to the brush from segment f is to be three times that from segment g, it is necessary that one-half the current, $i/2$, from the right side of the armature flow to the brush through segment g, and the other half, $i/2$, flow through coil B and down lead b, uniting with the current, i, from the left side in segment f.

As the armature continues to move, the resistance of the path through segment g becomes higher as the area of brush contact on this segment becomes smaller, so that more and more of the current from the right side of the armature winding comes through leads d and c, through coil B, and then through b and f to the brush. Finally, in position 5, the brush and segment separate, and all the current from the right side of the armature must pass through coil B to get to the brush.

Thus we see that the variation of brush-contact resistance first tends to stop the original current through B (positions 1 to 3) and then tends to build it up in the opposite direction (positions 3 to 5).

17. Current during Commutation. If this resistance commutation is to work successfully, the current through B must have reversed and built up to the same value of current as that flowing in coil C, *before segment g leaves the brush.* If this condition is not satisfied, there will be more or less sparking. The variation of current in coil B, as it is being commutated, may be well shown by a curve, as in Fig. 6–13. If the current is 10 amperes in each side of the armature, and the time during which the coil is short-circuited is 0.001 second, and if the current in B completely reverses during this time, the current curve has the form shown by the full lines in Fig. 6–13.

As was previously mentioned, an armature coil possesses some self-induction. During the first part of the commutation of coil B, when the current through it decreases, a counter emf must be generated because of its self-induction, which has the same direction as the decreasing current, since the counter emf tends to oppose the decrease. This counter emf is represented in Fig. 6–12 by the dotted arrows. When the current through B increases in the opposite direction, the counter emf of self-induction acts to oppose the increase of current. Its direction must then be opposite to the growing current, or the same as it was when the current was decreasing. In other words, *the direction of the counter emf of self-induction is always the same during commutation, provided the current always changes in the same direction.*

The effect of the counter emf of self-induction must then be to *delay the reversal* of the current in the coil being commutated. In position 3 of Fig. 6–12, for example, some current will still be flowing through coil B from left to right; it will not have reached zero as was supposed during the discussion of that figure.

Now suppose that the effect of varying the brush-contact resistance is not great enough to reverse the current in B completely during 0.001 second. The current curve then might have the form shown by the dotted curve in Fig. 6–13. In this case, at the time when segment g leaves the brush, the current in B is only 2 amperes (negative); it should be 10 amperes (negative). But if the current flowing in the right-hand part of the arma-

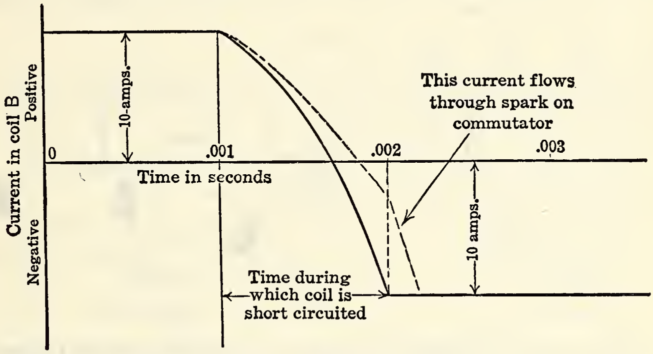


FIG. 6–13. If, during the short-circuit period, here shown as 0.001 second, the current follows the full-line curve, the commutation will be satisfactory; no sparking at the brushes will occur. If the current follows the dotted-line curve, sparking will occur at the trailing tip of the brush, and the current will persist through a spark as indicated in the diagram.

ture cannot reach the external circuit through segment g, *it must go through coil B*, and this means that, at the instant the brush and segment g separate, the current in B must suddenly change from 2 amperes to 10 amperes. The rate of change of current in B is very large at this instant, and so a large counter emf of self-induction is set up in B.

18. Cause of Sparking. The current from C has then two possible paths: it may force its way through B against the high counter emf in B, or it may form an arc over the mica insulation and get to the brush without going through coil B. This condition is indicated in Fig. 6–14 as well as in Fig. 6–13. The formation of this small arc depends upon the high counter emf of coil B, and this in turn depends upon the rapid change in current through B when segment g leaves the brush. This excessively rapid change of current is necessary because during the time of short circuit the resistance variation has not been sufficient completely to reverse the current in B (dotted line, Fig. 6–13).

The practical result of sparking at the commutator is to cause wearing of the receding edges of the commutator bars and the trailing brush tips. This is represented in exaggerated form in Fig. 6–15. It will be seen that this wear of the trailing brush tip materially decreases the area of brush contact and so tends to produce overheating, due to excessive current density, as well as to the sparking itself; it also amounts practically to a slight shift of the brushes.

19. Condition for Sparkless Commutation. If the current in B was completely reversed during the short-circuit interval (as in the full line of Fig. 6–13), there would be no change of current in B, as segment g left the brush, and hence no counter emf of self-induction to overcome. In this

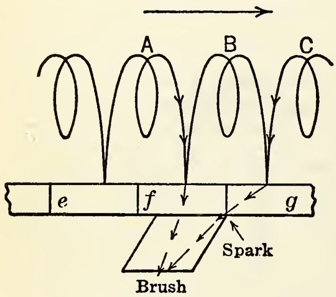


FIG. 6–14.

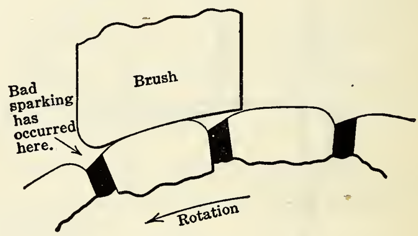


FIG. 6–15.

FIG. 6–14. If the current through coil B has not completely reversed when the short circuit is removed by the brush, a spark will occur between the brush and the receding commutator bar g.

FIG. 6–15. Bad sparking at the brush causes the commutator and brush to assume this shape. Really the surface of the commutator should be a smooth cylinder, and the brush should be in intimate contact with this cylinder over its whole cross-section.

case, there would be no tendency for the current from C to arc over the commutator, as segment g left the brush, and there would be no sparking. In fact, *commutation is always sparkless when the current in the coil undergoing commutation is completely reversed during the time the coil is short-circuited by the brush.*

20. EMF Commutation. Emf commutation is used on nearly all modern machines. With high commutator speeds the resistance variation is never sufficient completely to reverse the current in the coil short-circuited. If, as was mentioned before, some means was employed to generate, in the short-circuited coil, an emf equal and opposite to the counter emf of self-induction, resistance commutation could be depended upon to accomplish the desired reversal of the current. This generated emf, which we shall call the *commutation emf*, may be induced by the pole fringe, or by separate poles intended especially for this purpose, called *commutating poles*. If the pole fringe is employed, *the brushes are advanced* on the commutator so

that the short-circuited coil lies in the edge of the magnetic field under the leading pole tips, as shown in Fig. 6–16. (In case the machine is a motor and not a generator, this *brush shift must be backward,* not forward as in Fig. 6–16.)

In Fig. 6–12 commutation for a generator was considered, and it was shown that the direction of the counter emf of self-induction was opposite to the final direction of the current in the short-circuited coil B, that is, opposite to the direction of the emf generated by the coils to the right of coil B which are moving under the pole. So by moving the brushes forward a proper amount, as in Fig. 6–16, causing the short-circuited coil B to move in the pole fringe, we can generate a voltage in the coil during commutation, which, if equal and opposite to the counter emf of self-induction, should cause the current in the short-circuited coil to reverse according to the full-line curve of Fig. 6-13.

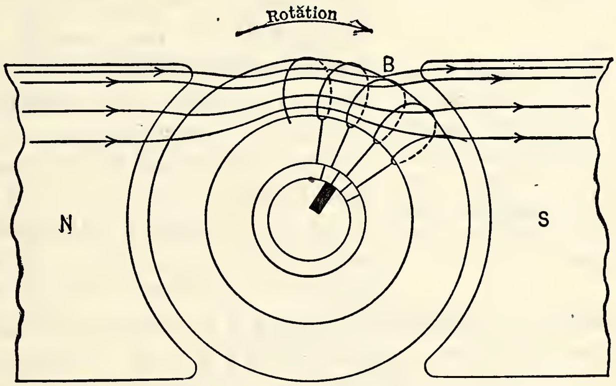


FIG. 6–16. By advancing the brushes of a generator, the short-circuited coil may be caused to lie in the pole fringe and so generate a commutation emf which will be opposite to the counter emf of self-induction.

21. Commutation from the Standpoint of Component Currents. The action of the commutation emf in assisting the current to reverse in the short-circuited coil may well be analyzed by considering that the current in the short-circuited coil is the resultant of several component currents. Let us consider that each of the component voltages present sets up a current independently and that we can combine these to obtain the resultant current. This method is different from the method of analysis just discussed, in which we considered the resultant of the component voltages producing a resultant current in the circuit.

Suppose that in Fig. 6–16, with the brushes shifted as indicated, no current is drawn from the generator. With no current to be commutated, no counter emf of self-induction will exist in the short-circuited coil. Therefore, the commutation emf, or voltage induced

in coil B while moving within the pole fringe, will tend to set up a current, which, as indicated in Fig. 6–17, flows through the coil B, down lead b, through f, the brush, segment g, and up c. As the coil B, during the period of commutation, is continually moving into a stronger and stronger field, this current increases in magnitude as shown in Fig. 6–18 by the line DE. At the end of the short-circuit period, i.e., when the brush and bar g (Fig. 6–17) separate, this current is ruptured, resulting in a spark.

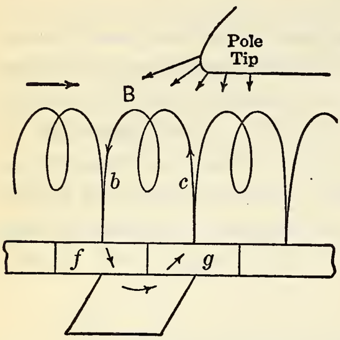


Fig. 6–17. If we imagine an armature rotating, carrying no load current, with the brush so advanced that the short-circuited coil lies in the pole fringe, a current will flow through the coil, commutator bars, and brush, as shown here.

If current is now taken from the generator with its brushes shifted forward, as in Fig. 6–16, we may consider the actual current as the resultant of two components, the sum of which at any instant represents the actual current flowing at that time. One component will be the decaying current, shown in Fig. 6–13, and the other, the current produced in the short-circuited coil by the emf induced in it by the pole fringe, as was shown in Fig. 6–18.

Let us consider, for example, that coil B is carrying 10 amperes and is approaching commutation, as shown at C, Fig. 6–19. It is supposed that resistance commutation is not sufficient and that the current in B at the end of the short-circuit period is shown at A, Fig. 6–19, whereas it should be at B. The current produced in the short-circuited coil by the induced emf is shown by the line DE, and the total current in the short-circuited coil is the sum of the two component currents, CA and DE, shown by CFB. This is evidently the current required for sparkless commutation.

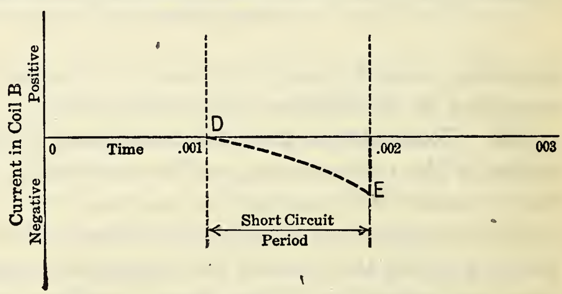


Fig. 6–18. The form of the short-circuit current set up in the coil, as indicated in Fig. 6–17, will be about as shown in the heavy dashed curve above.

If the load on the machine changes so that the current per path is 20 amperes instead of 10 amperes, the emf required for sparkless commutation is much greater than before. If the short-circuited coil again moves through the same field as before, the current in B during short circuit fol-

lows the curve $C'G$. The resistance variation, as explained above, gives the current $C'A'$, and the addition of the short-circuit current DE (the same as before because the emf producing it is the same) gives the current $C'G$. At the end of the short-circuit period the current in coil B should be at B' (negative), and it really is at G (positive).

22. Necessity of Shifting the Brushes with Load Variation. If the brush is advanced more, so that coil B is in a denser field than before, a greater emf is induced in it while it is short-circuited and the short-circuit

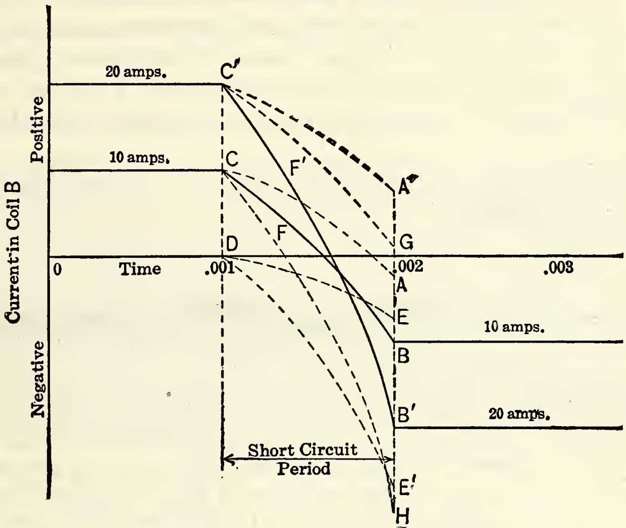


FIG. 6–19. The position of the coil in the pole fringe, which gives successful commutation for one load, will not be suitable for another. The strength of field in which the short-circuited coil lies should vary in proportion to the load carried by the machine.

current becomes DE'. Now DE' added to $C'A'$ gives the current $C'F'B'$, which is just right for sparkless commutation.

If the load on the generator again changes, so that the current per path becomes 10 amperes as before, with the brushes shifted to give sparkless commutation for a current per path of 20 amperes, the current in B, during short circuit (so far as resistance commutation is concerned), follows the curve CA. The addition of the current DE', produced by the commutation emf, gives the current CH, or a final value of about 30 amperes. But when the short-circuit period is over, the current in the coil must be 10 amperes; it is evident that this sudden change of current from 30 to 10 amperes will produce sparking at the commutator, so that the brushes must again be shifted backward.

When the conditions are such that the counter emf of self-induction is not opposed, or only partially neutralized, by a commutation emf, the com-

mutation is frequently referred to as *under-commutation*; such conditions are indicated in Fig. 6–19 when the current in the short-circuited coil follows curves CA or $C'G$. When the value of the commutation emf is greater than that required to neutralize the counter emf of self-induction, so that the current in the short-circuited coil "overshoots" the required final value, the conditions may be referred to as *over-commutation.* This condition was indicated in Fig. 6–19 by the curve CH.

In Fig. 6–20 are given three oscillograms of current in one coil of a generator, under full-load, light-load, and no-load conditions. The brushes were advanced sufficiently to produce sparkless commutation at full load, and left in this position for the other loads. It will be seen that the emf induced in the short-circuited coil is just right completely to reverse the current at full load, but that at light load the current "overshoots" and

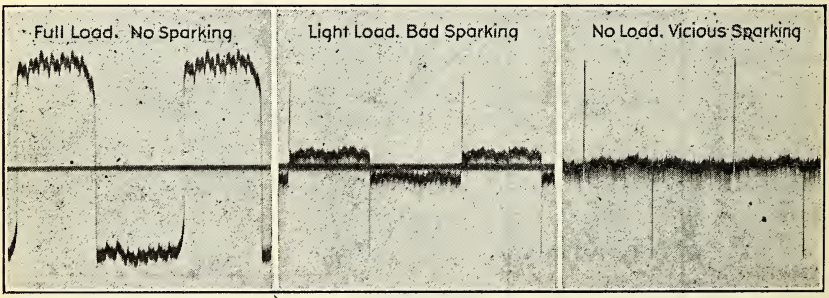


FIG. 6–20. An oscillogram of the current in one coil of a bipolar generator without commutating poles; the brush position was advanced sufficiently to give sparkless commutation at full load and left in that position. It will be seen that the emf induced in the short-circuited coil gave a current just sufficient to commutate full load properly, but that this commutation current was too great for light load and, of course, correspondingly worse for no load.

at no load there is a large current (almost equal to full-load current) in the coil while it is short-circuited by the brush. For light load and no load vicious sparking occurred at the brush contacts.

It follows, then, that by means of *brush shifting,* sparkless commutation may be obtained; but the amount of shift required varies with the load, so that, if brush shifting is used, the operator has to change the position of the brushes (by moving the brush-holder yoke) as the load changes. The amount of brush shift necessary depends upon the design of the machine. A well-designed slow-speed machine without commutating poles operates fairly well with no brush shift at all. The brushes are placed somewhere between the proper no-load position and proper full-load position and are left there no matter what the load may be.

23. Use of Commutating Poles. When the brushes of a machine are shifted, the short-circuited coil is moved to find its proper commutating

flux in the pole fringe. When commutating poles are used, brush shifting is unnecessary, the proper commutating flux being brought to the short-circuited coil. Small poles, wound with a *series winding*, are placed midway between the main poles, as shown for a generator in Fig. 6–21, and the flux produced by them acts in the same way as does the pole fringe just discussed. The brushes short-circuit the coil lying directly under the commutating pole and are left fixed in this position; the brushes on a commutating-pole machine must never be shifted after correct adjustment has once been made.

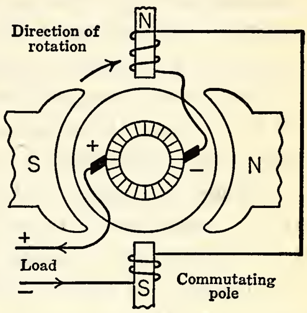


Fig. 6–21. Commutating poles are used in practically all modern d-c machines. The windings of these poles, being in series with the armature, automatically adjust the strength of the commutating field in proportion to the load. The polarity of the commutating poles, as is here shown, is correct for a generator.

To determine the proper polarity for a commutating pole, it is only necessary to remember that, in a generator without such poles, the brushes are shifted forward to place the short-circuited coil in the fringe of the flux from the next pole. As the commutating pole is to produce the same result, its polarity must be the same as that of the next pole, in the direction the armature is moving. (As explained later, in a motor the brushes are shifted backwards as load comes on; therefore the polarity of a commutating pole is the same as that of the main pole just preceding it, in the direction of rotation.)

As these poles are equipped with a series winding the *strength of field produced by them* (and hence the magnitude of the emf induced in the short-circuited coil) is proportional to the load (they are operated at low flux density and are never permitted to become saturated); the magnitude of the short-circuit current will then be proportional to the load, and, as was shown in the discussion of Fig. 6–19, this is the necessary condition for maintaining "black" commutation as the load varies. The use of commutating poles (sometimes called *interpoles*) has become almost universal on d-c motors and generators, especially on *adjustable-speed motors* in which the speed variation is obtained by field weakening.

The field frame of a machine equipped with commutating poles is shown in Fig. 5–9, page 166. The armature has been removed to permit a clear view of the field construction. The commutating poles are made very narrow; generally they reach over only two teeth (and the included slot) of the armature core.

When full-pitch armature windings are used, only one commutating pole per pair of main poles is required, the entire commutating emf per coil *being induced in one coil-side.* This is done however, only in the

smaller machines; larger machines usually have as many commutating poles as main poles.

24. Armature Reaction. When the armature of a d-c machine is carrying current, the armature acts like an electromagnet. The current flowing in the turns of the armature winding sets up a mmf independent of the main field windings *which alters both the distribution and the magnitude of the flux which would be produced by the field windings alone*; this action of the mmf set up by the armature windings upon the field is called *armature reaction.* It may be a distorting action only, in which case the resultant magnetic field of the machine is not directly strengthened or weakened by the armature reaction, but is merely twisted out of its normal position. In other cases the armature mmf may not only distort the main field but may either demagnetize (weaken) it or magnetize (strengthen) it.

When the main field is both twisted out of its normal position and either strengthened or weakened, the armature mmf may be resolved into two components: a *distorting* or *cross-magnetizing* component which causes the twisting of the main field; and a *magnetizing* or *demagnetizing* component, as the case may be, which alters the magnitude of the main field.

25. Armature MMF. In Fig. 6–22 is shown a winding with twelve coils arranged for simplicity as a single-layer winding. The back pitch is eleven which makes it a short-pitch winding. The slots, front connections of the coils, and the commutator have been omitted. It is assumed that the armature is carrying current set up either by voltage generated within the armature as it rotates in the main field or by some outside source of voltage, in which case the armature may be stationary, or, if it is rotating, the main field is not considered for the time being. It is assumed further that the current in the conductors under the right-hand pole is flowing into the paper, and in the conductors under the left-hand pole it is flowing out of the paper. It is to be noted that the armature when rotating always presents the same aspect to the field; whatever conductors are moving under a given pole will always be carrying current in the same direction.

Current flowing through the turns of the coil occupying position 1–12 will produce a mmf over the plane of the coil and perpendicular to it, which may be represented by the heavy vector A. Similarly, the ampere-turns of coil 3–14 will produce an mmf represented by vector B, and those of coils 11–22 and 17–4 by the vectors C and D.

It will be seen that coils 1–12 and 3–14 occupy the same relative position with respect to the horizontal and that the same is true as regards coils 11–22 and 17–4. In fact, if all the coils were drawn, they all could be paired off in the same way. The resultant mmf of each such pair of coils (1–12 and 3–14, etc.), and therefore the resultant mmf of the entire armature, will lie in the brush axis. The brush axis is fixed by the position of the brushes; if the brushes are in the no-load neutral position, the

armature mmf is at right angles to the field mmf. In a drum winding the actual position of the brushes themselves is 90° from the vertical of Figs. 6–22 and 6–23, but this is due to the twist of the front connections of the coils. In any case the position of the brushes is fixed so that the coils undergoing commutation, and thus connected to the brushes, are moving in the interpolar space.

Since the armature always presents the same aspect to the field so far as conductor currents are concerned, it might be considered, for purposes of analysis, as though the armature were wound as in Fig. 6–23, i.e., as though the coil-sides were now paired up as shown, 7 with 8, 6 with 9, 2 with 13, etc. The mmf of each such artificial coil lies in the vertical and

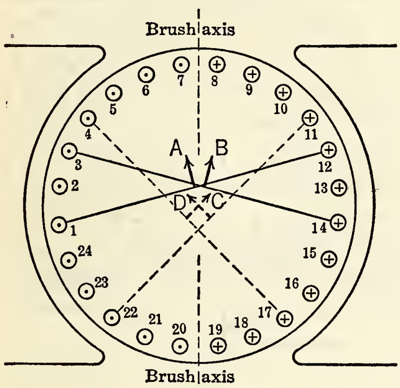


Fig. 6–22.

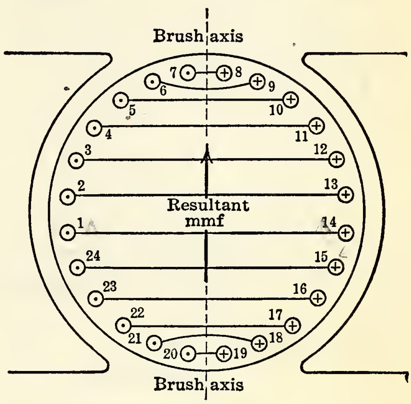


Fig. 6–23.

Fig. 6–22. A single-layer drum winding. When current flows each coil sets up a mmf, but the resultant of all the coils lies along the brush axis.

Fig. 6–23. Since the armature of a d-c machine always presents the same current aspect to the field, it may be supposed that the armature of Fig. 6–22 is wound with artificial coils as shown here. The resultant mmf of all coils lies in the brush axis as before.

the resultant armature mmf again lies in the brush axis as before. This conventional pairing off of coil-sides to obtain a mmf is fixed by the directions of the currents in the coil-sides; there would be no object in pairing coil-sides 10 and 17; the mmf of such a coil would be zero since the currents in both sides of the coil are in opposition.

26. Distortion of Field by Armature MMF. In Fig. 6–24 is shown the field produced by the main-field coils alone. The strength of the field depends upon the excitation due to the field winding and upon the shape and materials of the magnetic circuit. If the air gap is of uniform length, the flux density within it will also be uniform except at the pole tips where fringing occurs.

As the armature rotates, voltage will be generated in the conductors

which will be directed out of the paper under the north pole and into the paper under the south pole. Since the field is symmetrical horizontally, the conductors will generate no voltage in the middle of the interpolar spaces and the brushes must be so set that commutation will take place at that instant. The no-load neutral axis is therefore perpendicular to the field mmf OF, as shown at the bottom of Fig. 6–24.

In Fig. 6–25 the main-field excitation has been removed and current is being passed through the armature from an external source in the same direction it would have if the generator were supplying current to a load.

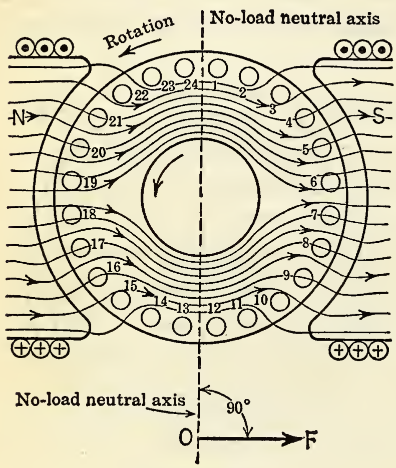


FIG. 6–24.

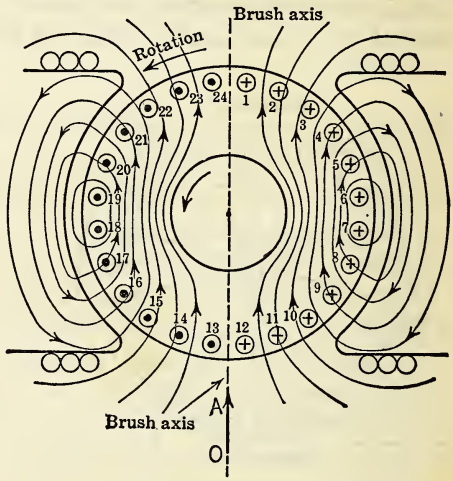


FIG. 6–25.

FIG. 6–24. When only the field windings of a generator carry current, a symmetrical field is set up. The flux density in the air gap is uniform except at the pole tips, where fringing occurs.

FIG. 6–25. When only the armature carries current, a field is set up which is in line with the brush axis. If the brushes are set on the no-load neutral, the armature mmf is perpendicular to the main-field axis.

Currents under the left-hand pole are out of the paper (head end of an arrow) and into the paper under the right-hand pole (tail end of a feathered arrow).

The mmf of the armature, OA, may be considered as the resultant of the mmf's of the artificial coils 24–1, 23–2, etc., as was developed in Fig. 6–23. The resultant mmf lies along the brush axis and is perpendicular to the axis of the main field. The distribution of the field set up by the armature is approximately as shown in Fig. 6–25.

When the generator is supplying current, the armature and field mmf's exist simultaneously, and the resultant field distribution will be the resultant of these two mmf's. It may be considered that the resultant distribu-

tion is approximately the combination of the two fields shown separately in Figs. 6–24 and 6–25, as indicated in Fig. 6–26. That the main field is twisted *in the direction of rotation of the armature,* as shown in Fig. 6–26, may be determined by noting, in Figs. 6–24 and 6–25, that under the upper north and lower south pole tips the main and armature fields are in opposite directions, so that a weaker field in these regions may be expected than when the armature is not carrying current. Under the lower north and upper south pole tips, the two fields are acting in the same direction, so that a stronger field should exist under these pole tips than in Fig. 6–24.

In the case of a motor, the armature currents of Fig. 6–25 for the field polarity and the direction of rotation assumed would have to be in the opposite direction to that shown. As a result, the twist of the main field is in the direction opposite to that of rotation of the armature. (See Fig. 7–3, page 314.)

27. Distortion Dependent upon Load and Field MMF. As the field distortion is produced by the reaction of the armature mmf upon the field mmf, it will depend upon the relative values of the two mmf's. With constant field mmf the amount of distortion will vary with the armature mmf or the amount of current the armature is carrying. At no load there is no current flowing in the armature, and hence no armature reaction. The distortion is a maximum when the machine is carrying full load or overload.

With a given load on the armature the distortion will be greater with a weak main field than with a strong field. This is the reason that, in a well-designed machine, the field ampere-turns are greater than the full-load armature ampere-turns, and it accounts for the fact that the air gap in d-c machines is longer than a mere mechanical clearance.

28. Effect of Distortion on Commutating Plane or Neutral Axis. The most important effect of armature reaction is the shift that it produces in the commutating plane. If no armature reaction were present, the coil which is short-circuited by the brushes should lie in the no-load neutral axis of Fig. 6–24. In Fig. 6–26, because of the distortion produced by armature reaction, conductors 24 and 1, 12 and 13 are moving in a field and so are generating voltage. These voltages, moreover, are in the *wrong direction* to produce sparkless commutation; they are in the same direction as the counter emf's of self-induction of the short-circuited coils.

The brushes must therefore be shifted ahead from the no-load neutral or brush axis, as shown in Fig. 6–24, to a load neutral, at which position the coil undergoing commutation is again not cutting any flux. If the necessity of shifting the brushes, so that the short-circuited coil is under the pole fringe for emf commutation, is considered, it will be seen that the brushes must be advanced still farther to some axis BB', as in Fig. 6–27. This shift of the brushes, however, moves the brush axis ahead and shifts the load neutral still farther ahead. The brushes could never reach

the load neutral position if the iron poles did not limit the amount by which the flux can be twisted; the flux can never be twisted beyond the trailing edge of the pole. The brushes can, in an actual machine, be set into, and even ahead, of the load neutral axis.

29. Effect of Brush Shift. This change in brush position changes somewhat the current distribution in the armature conductors. Thus, after the brushes have been moved through the angle α, conductors 10, 11, and 12 in Fig. 6–27 which before the shift carried current into the paper now

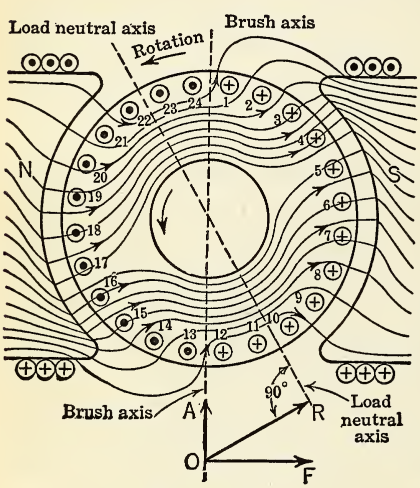


FIG. 6–26.

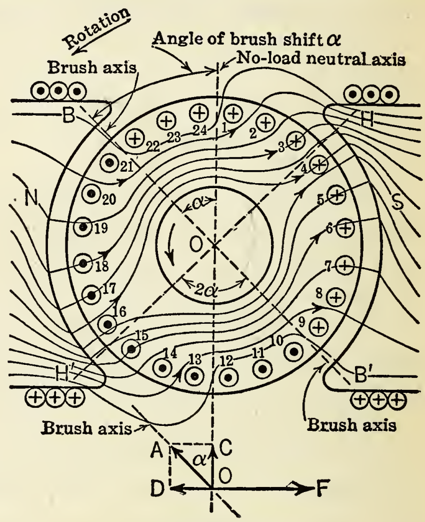


FIG. 6–27.

FIG. 6–26. When the armature and field ampere-turns of a generator are acting together, the resultant field produced is an asymmetrical one. It becomes twisted in the direction of rotation of the armature.

FIG. 6–27. If the brushes are shifted forward from the no-load neutral for purposes of commutation, the armature ampere-turns both weaken and distort the main field, and may be divided into demagnetizing and distorting ampere-turns. The angle of shift has been exaggerated here, for emphasis; in an actual machine it would be very much less than the angle shown here.

carry current directed out of the paper. The direction of the current in conductors 22, 23, and 24 has also reversed.

The direction of the armature mmf, OA, is still along the brush axis, BB' as before, but is now also inclined to the vertical by the angle α.

30. Demagnetizing Turns and Cross-magnetizing Turns. In Fig. 6–26 the armature mmf could be considered as the resultant of the mmf's of the artificial coils 24–1, 23–2, 22–3, etc., the resultant lying in the vertical axis. In Fig. 6–27, the armature mmf, now lying along the brush axis BB', may be considered as the resultant of the mmf's of the artificial coils 21–22, 20–23, etc.

The resultant armature mmf, OA, of Fig. 6–27 may be resolved into a vertical component OC and a horizontal component OD. It is evident that the turns producing the vertical component OC will produce distortion of the field, as was the case in Fig. 6–26. These turns are therefore called the *cross-magnetizing* or *distorting* turns. The turns producing the horizontal component OD are called the *demagnetizing* turns as their mmf is directly opposed to that of the main field.

The demagnetizing turns lie in the angle 2α formed by the brush axis BB' and another axis, HH', which is at an angle α on the other side of the no-load neutral axis from BB'. The turns may be considered as formed by the artificial coils 22–15, 23–14, $\cdots$ 2–11, and 3–10. That their mmf is

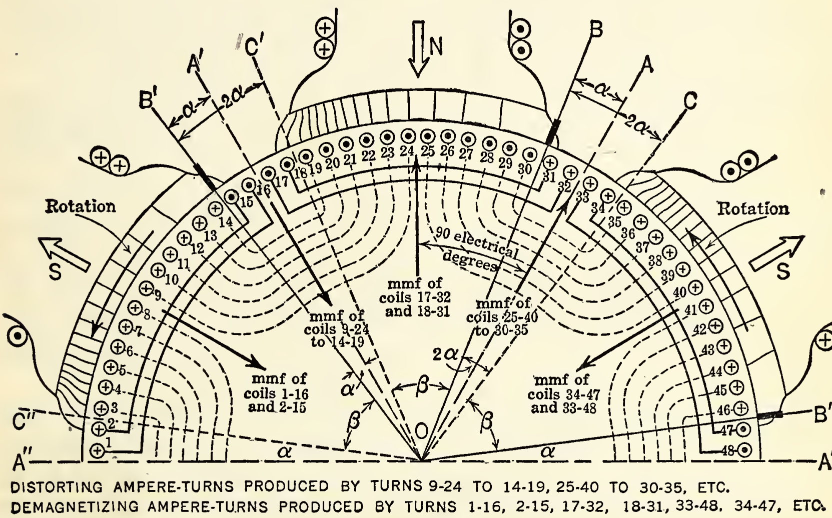


FIG. 6–28. Showing the distribution of cross-magnetizing and demagnetizing ampere-turns in a multipolar generator.

opposed to that of the field is shown by the fact that their currents are in the opposite direction to those of the field turns.

The cross-magnetizing turns lie in the angles BOH' and HOB' formed by the artificial coils 21–4, 20–5 $\cdots$ 17–8 and 16–9.

Shifting of the brushes therefore produces both distorting and demagnetizing effects upon the main flux of the machine so that the field distribution might appear as in Fig. 6–27.

It is to be noted in the previous figures that the number of coils on the armature was made far less than would be used on a commercial machine, and that the number of flux lines is not representative of commercial densities. This was done to obtain a simpler figure. As a result the field distortion is excessive and, in order to show the commutated coil moving

in the pole fringe, in Fig. 6–27, the angle of brush shift had to be made much greater than would be the case in a well-designed machine.

31. Armature Reaction in a Multipolar Generator. The effect of armature reaction in a multipolar generator is shown in Fig. 6–28. The no-load neutrals are along the radii OA, OA', etc., and the full-load positions of the brushes are along the lines OB, OB', etc., the brushes having been shifted forward by the angle α. The angle between adjacent pole centers or between lines OA and OA' represents 180 electrical degrees, and the distance between two successive south poles is 360 electrical degrees.

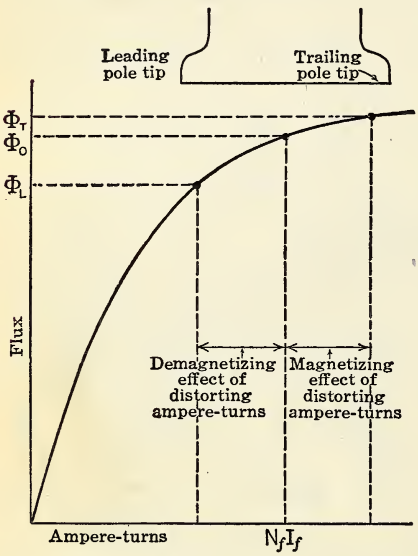


Fig. 6–29. The effect of the distorting armature ampere-turns is to magnetize the trailing pole tip of a generator and demagnetize the leading pole tip. Because of the form of the magnetization curve, the flux under the leading pole tip is decreased more than that under the trailing pole tip is increased.

The inductors lying in the angle 2α, between OC and OB and between OC' and OB', etc., namely, 15, 16, 17, 18, and 31, 32, 33, 34, etc., may be paired up to form artificial turns as 1–16, 2–15, 17–32, 18–31, 33–48, 34–47, etc., as shown by the solid lines, to constitute the demagnetizing ampere-turns. The mmf's of these artificial turns are in each case directly opposite to those of the poles under which they lie, thereby weakening the field.

The inductors lying in the angle $\beta = 180° - 2\alpha$ electrical degrees, namely, 3 to 14, 19 to 30, 35 to 46, etc., constitute the cross-magnetizing or distorting ampere-turns if paired up as 9–24, 10–23, $\cdots$ 13–20, 14–19, 25–40, 26–39, $\cdots$ 29–36, 30–35, etc., as shown by the dotted lines. Their mmf's center along OA, OA', etc., 90 electrical degrees from the demagnetizing mmf's. The important fact is that these distorting ampere-turns tend to magnetize the adjacent trailing pole tips and to demagnetize the adjacent leading pole tips, accounting for the field distortion, as study of Fig. 6–28 will reveal.

Owing to the fact that the trailing pole tip becomes saturated, the flux under it will not increase as much as the flux under the leading pole tip is reduced. This may be seen from Fig. 6–29, in which the magnetization curve is shown. The no-load flux, Φ_0, is produced by the field ampere-turns, N_fI_f, the flux being uniformly distributed. Under the leading pole

tip the flux is reduced to Φ_L by the demagnetizing effect of the distorting armature ampere-turns. The magnetizing effect of the distorting ampere-turns increases the flux under the trailing pole tip to Φ_T. The overall effect of the distorting ampere-turns is to cause some reduction in the total flux per pole.

32. Calculation of Demagnetizing and Distorting Ampere-turns. The total ampere-turns upon an armature with a full-pitch winding will be equal to the total number of turns multiplied by the current per conductor. With Z total conductors, there will be $Z/2$ turns. If the total current carried by the armature is I_a amperes, and there are m paths on the armature, the current per conductor will be I_a/m amperes and the total number of ampere-turns will be $ZI_a/2m$. With p poles on the machine, the ampere-turns per *pair of poles*, $p/2$, will be ZI_a/mp.

The total ampere-turns are divided into demagnetizing and distorting ampere-turns in the ratio $2\alpha/\beta$. When α and β are measured in electrical degrees, since 360 electrical degrees $= 4\alpha + 2\beta$, for any machine,

elec

$$\text{Demagnetizing ampere-turns} = \frac{4\alpha}{360} \times \frac{ZI_a}{mp} = \frac{\alpha ZI_a}{90mp} \tag{4}$$

Cross

$$\text{Distorting ampere-turns} = \frac{2\beta}{360} \times \frac{ZI_a}{mp} = \frac{\beta ZI_a}{180mp} \tag{5}$$

The number of electrical degrees in a machine of p poles will be $360 \times p/2$, and the angles α and β (in electrical degrees) will be $p/2$ as large as if they were expressed in mechanical or space degrees as α' and β'. Thus $\alpha = \alpha' p/2$ or $\alpha' = 2\alpha/p$ and $\beta' = 2\beta/p$. With the angle of brush shift expressed in mechanical or space degrees, the last expressions become

mech

$$\text{Demagnetizing ampere-turns} = \frac{\alpha' ZI_a}{180m} \tag{6}$$

$\frac{ZI}{2P}$ Cross

$$\text{Distorting ampere-turns} = \frac{\beta' ZI_a}{360m} \tag{7}$$

The above expressions are for an armature with full-pitch multiple-circuit windings. The effect of using fractional-pitch windings is to reduce somewhat the values that would be obtained from the expressions as given. The same is true in the case of wave windings where the back pitch is a little less than the distance between pole centers.

33. Reduction of Field Distortion. The amount of distortion of the main field caused by armature reaction depends upon the ratio of armature mmf to field mmf, the degree of saturation of the pole tips, the length of the air gap, and other factors.

A most important method of preventing excessive field distortion is to make the field ampere-turns greater than the armature ampere-turns at all loads. In a machine of given output, speed, and voltage, the armature ampere-turns are more or less fixed, the current per conductor determined by the rated current of the machine, and the turns determined by the number needed to generate the voltage.

If the air gap of a d-c machine were made with only a mechanical clearance, the number of field ampere-turns required to set up the required flux would be much less than the armature ampere-turns. If the field ampere-turns are to be greater than the armature ampere-turns, the reluctance of the magnetic circuit must be increased if the flux is to be the same, and this is done by increasing the length of the air gap.

An additional scheme to prevent field distortion depends upon saturating the pole tips. If, without any distortion at all, the trailing pole tips of a generator are saturated, no more flux can crowd into them, even if the distorting mmf is large. Laminated pole pieces are sometimes constructed with the laminations having but one pole tip. These are stacked so that the first lamination extends into the leading pole tip and the next into the trailing pole tip, etc., so that there are in each pole tip only half as many laminations as there are in the pole itself. Such a construction tends to give saturated trailing pole tips in a generator and consequently a field that is not easily distorted; such a field is said to be a stiff field.

Another scheme, used with machines intended to be operated only in one direction, is to make the air gap slightly longer under the trailing pole tips of generators than under the leading pole tips.

34. Flux-distribution Curves. The distribution of the flux at the surface of an armature may conveniently be indicated by *flux-distribution curves,* in which the surface of the armature is represented as a horizontal plane, as though the surface had been peeled off the armature and laid out flat. In Fig. 6–30, the flux distribution for a generator at no load is indicated. Any abscissa, OX, represents the distance of a point on the armature (measured around the circumference) from a point midway between a north and a south pole, and the ordinate, XY, is proportional to the flux density in the air gap at the point considered. The flux leaves the armature under a south pole, indicated in the curve as positive, and enters it under a north pole, indicated as negative. Figure 6–30 thus represents the same conditions as Fig. 6–24.

The flux conditions existing when current flows only through the armature of a generator (no field current), with its brushes on the no-load neutral axis, are shown in Fig. 6–31. The armature winding is represented as a single concentrated coil, with the direction of the currents the same as would be generated if the armature of Fig. 6–30 were supplying current. The hollows in the curve over the brushes are caused by the interpolar

spaces, as may be seen from Fig. 6–25; although the flux due to armature

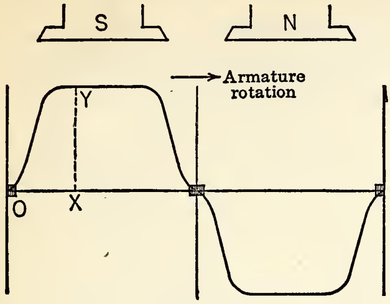


Fig. 6–30.

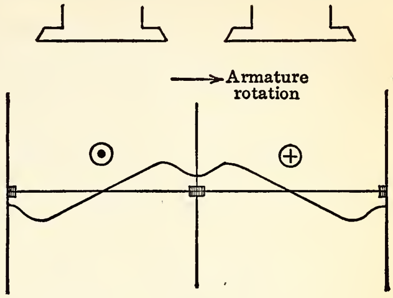


Fig. 6–31.

Fig. 6–30. Indicating the field distribution due to the field ampere-turns acting alone.

Fig. 6–31. The field distribution which would be produced by the armature ampere-turns acting alone. The dip in the curve at the brush positions is caused by the comparatively high reluctance for the flux in this part of the magnetic circuit, there being a very long air gap opposite this part of the machine.

mmf is actually very small at these points, the armature mmf is actually a maximum in these regions. The high reluctance of the magnetic circuit for the flux set up by the armature in these positions, due to the flux paths lying largely in air, accounts for the low values of flux density.

If, with the field excited, the armature carries current, the brushes still being left on the geometrical neutral, a flux distribution as shown in Fig. 6–32 results, being obtained by adding the curves of Figs. 6–30 and 6–31. It is to be remembered here that problems of this kind must generally be solved by superimposing mmf's, getting the resultant mmf and then the flux produced by this mmf. Fluxes can be superimposed, and the resultant thus obtained, only *when the relation between flux and mmf is linear*, i.e., permeability is constant. Since the permeability is not exactly constant, this procedure leads to some error, but indicates the distribution fairly well. It will be seen that the resultant flux obtained by adding the two component fluxes

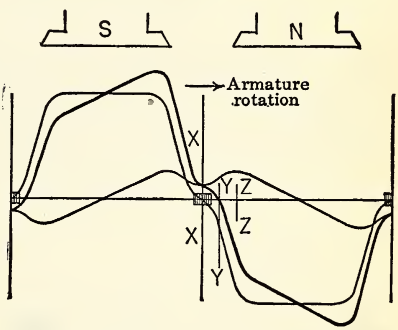


Fig. 6–32. The flux distribution produced by the simultaneous action of field ampere-turns and armature ampere-turns. (This resultant curve has been obtained by merely adding the two component curves, a procedure which is permissible only when the permeability of the path is independent of the flux density, and leads to some error in this case.) The position of zero field is shifted ahead from XX to YY by the effect of armature reaction.

is distorted in the direction of rotation, that the neutral point has shifted from the position XX to the position YY. In order to generate a commutation emf of the proper direction, the brushes would have to be shifted still further in the direction of rotation, to a position ZZ, beyond YY. This shift would have the effect of shifting the component wave

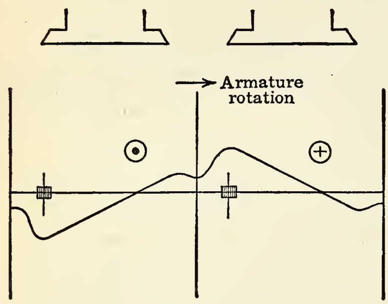


FIG. 6–33.

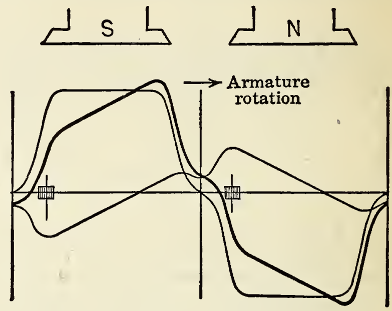


FIG. 6–34.

FIG. 6–33. When the brushes are shifted to position ZZ of Fig. 6–32, the wave representing the armature flux is shifted and altered as is shown here.

FIG. 6–34. The combination of the fluxes of Figs. 6–30 and 6–33, with the brushes shifted forward to obtain good commutation.

due to the armature mmf, Fig. 6–31, until it takes the shape and position of Fig. 6–33. When added to the field flux, Fig. 6–30, the resultant flux-distribution curve, Fig. 6–34, is obtained. This curve, when compared with Fig. 6–32, shows clearly the demagnetizing and distorting action of armature reaction when the brushes are shifted.

35. Experimental Determination of Flux-distribution Curves. It will be noted that in Fig. 6–32, if any ordinate represents the flux density in the air gap, the voltage generated by a conductor on the armature at the corresponding point will also be proportional to the ordinate, provided the armature is rotating at constant speed. In one scheme for experimentally determining flux distribution, two narrow brushes, insulated from each other and separated by a distance equal to one commutator bar, are mounted in a suitable rigging, so that they may be placed in any position with respect to the main brushes, as indicated at bb, in Fig. 6–35. A low-reading voltmeter (with a small condenser across it to steady the readings) is connected across the brushes; its reading will be proportional to the average flux density over the distance on the

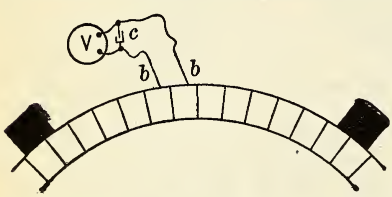


FIG. 6–35. One scheme for getting the field distribution by a pair of pilot brushes, spaced from each other by the width of one commutator bar.

armature surface corresponding to the distance between the pilot brushes. By taking a series of voltmeter readings for successive positions of the pilot brushes, and plotting them against the distance of the pilot brushes from one main brush, a flux-distribution curve is obtained.

A set of flux-distribution curves taken by means of two such pilot brushes is shown in Fig. 6–36. The machine tested was operated as a generator at no load and at full load, and also as a motor at full load. In

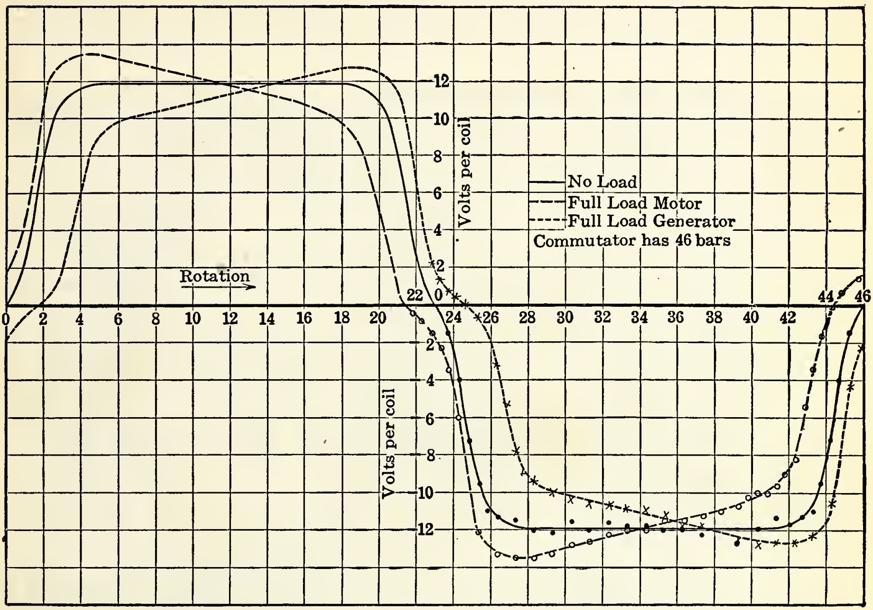


FIG. 6–36. Actual field distribution of a bipolar machine obtained by the double pilot brush scheme of Fig. 6–35. The field had a uniform distribution when the machine was not loaded, but when loaded the field was distorted owing to armature reaction. In the case of the generator the field is distorted in the direction of rotation and in the case of the motor, against the direction of rotation. The average voltage of a coil is considerably higher when acting as a motor because of the reversed effect of the *IR* drop in the coil. Note the shift in the commutating plane as the machine changes from generator to motor.

each run the speed and the field current were the same, and the armature current was the same in both load runs; the distortion of the field is evident.

Flux-distribution curves can be determined more exactly, by means of the oscillograph, by inserting a single fine wire into one of the slots and connecting it to suitable slip rings at either end of the armature. The voltage wave generated by this test wire will be a measure of the flux distribution of the machine. The flux-distribution curves of Figs. 6–3, 6–41, 6–42, 7–26, 7–27, etc., were obtained by this method.

This latter scheme, *using only one conductor*, does give the flux distribution actually existing in the machine, whereas the method using two pilot brushes does not. The brushes, *bb*, of Fig. 6–35, evidently connect *across one coil* of the armature; the coil has two sides, one under one pole and the other under an adjacent pole. If the coil has full pitch, the voltage on brushes *bb* will result from the average density of flux in two corresponding points under adjacent poles, and if the coil is not full-pitch the curve of flux density obtained in the test is likely to be misleading. It represents the average flux density at two differently placed points, under adjacent poles.

36. Compensation of Armature Reaction. The magnetizing effect of the armature coils may be neutralized by a compensating or pole-face

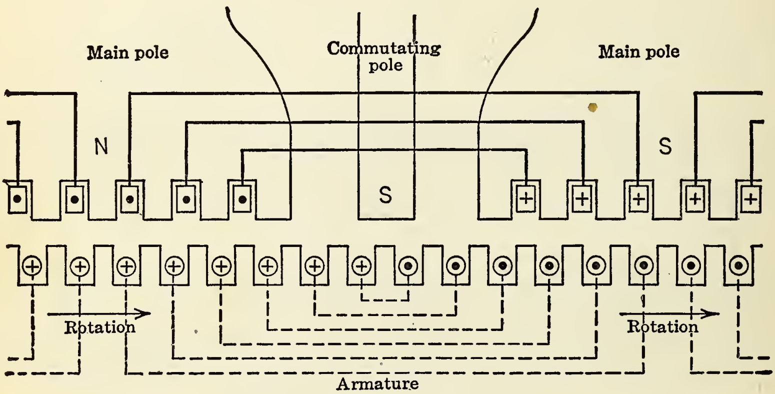


Fig. 6–37. Compensating or pole-face winding used to neutralize the distorting or cross-magnetizing ampere-turns of the armature. The pole-face winding is shown in heavy full lines and is made to have the same number of ampere-turns as the armature, since the brushes are set on the no-load neutral. The armature mmf is set up as though the armature conductors were joined as shown by broken lines.

winding. This consists of a winding imbedded in slots in the pole faces, the winding having exactly as many ampere-turns as there are on the armature. These conductors in the pole faces are put in series with the armature, so that they carry the same current as the armature; but the connection is made so that every conductor in the compensating winding carries current in the opposite direction to an adjacent conductor on the armature. Figure 6–37 shows how these conductors are placed in the pole face, and the relative direction of the current in the armature and the compensating winding. The compensating winding is not used in the average machine because it makes a machine costly to build; it is only in high-speed, high-capacity, high-voltage generators or heavy-duty motors, such as rolling-mill motors, that it is found necessary to supplement the commutating

poles with the compensating winding. The field frame of a large rolling-mill motor showing the compensating winding is pictured in Fig. 6–38.

37. Effect of Armature Reaction on Commutating Poles. The commutating poles must be designed with turns enough so that the armature distorting mmf is neutralized *under the face of the commutating pole* and, in addition, the proper amount of flux required for sparkless commutation is forced into the armature core around the coil being commutated.

It must be realized that the armature mmf is distributed over the armature periphery, whereas the commutating-pole mmf is concentrated.

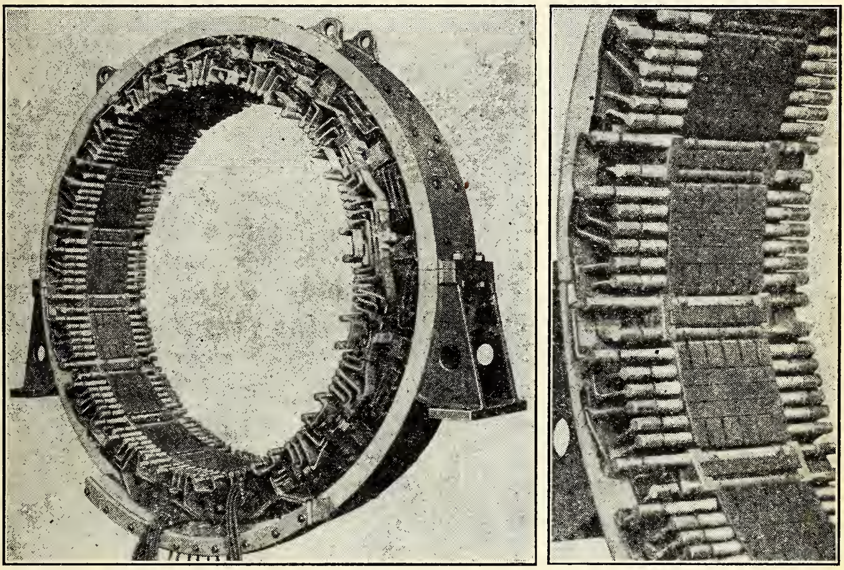


FIG. 6–38. Frame of a heavy-duty rolling-mill motor equipped with a compensating, or pole-face, winding in addition to commutating poles. The right-hand figure shows an enlarged view of the pole-faces. *Courtesy of the Allis-Chalmers Manufacturing Co.*

While the latter mmf will be greater than the armature mmf under the commutating pole, the armature mmf is still active on either side of the commutating pole. Thus the commutating poles do not prevent the flux from crowding into the tips of the main poles, as may be seen in the magnetic circuit of a two-pole generator equipped with commutating poles, as shown in Fig. 6–39. In fact the distortion is even made worse by the presence of the commutating-pole flux which in a generator passes to the adjoining trailing pole tip. The commutating pole is hence *not added to a machine to neutralize armature reaction.*

In the average machine, field distortion, unless excessive, is not objectionable so long as the proper field for emf commutation at the two points

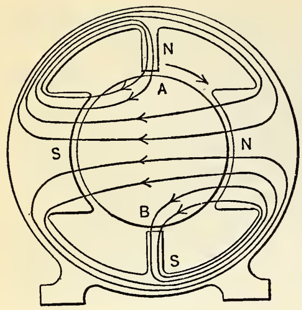


Fig. 6–39. Approximate distribution of field flux in a bipolar, commutating-pole generator.

A and B of Fig. 6–39 is obtained. In heavy-duty machines, where large shifts in the load are encountered, flux distortion is prevented by compensating the armature mmf by means of a pole-face winding as previously described.

38. Flux-distribution Curve with Commutating Poles. A theoretical flux-distribution curve for a generator equipped with commutating poles may be constructed in the same way as was done for Fig. 6–32. In Fig. 6–40, curves F and A represent the field and armature fluxes separately, and C the commutating-pole flux which would exist at full load. When combined, at full load, these fluxes form the resultant flux curve, R. It will be noted that there still remains sufficient flux of the proper polarity at the geometrical neutral for emf commutation.

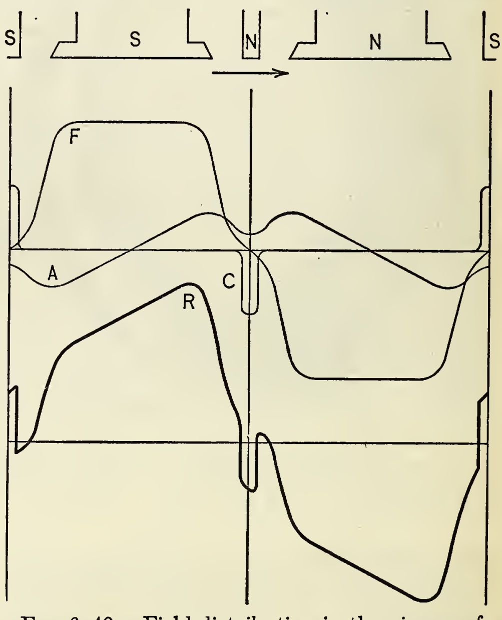


Fig. 6–40. Field distribution in the air gap of a commutating-pole generator carrying load. The commutating pole, besides neutralizing the effect of the armature mmf at the brush position, provides a proper flux density for black commutation. This curve is a theoretical one, the actual field distribution not showing the effect of the commutating poles as definitely as it is shown here.

In Fig. 6–41 is given an oscillogram of the flux-distribution curve of a small generator equipped with commutating poles, operated at no load; Fig. 6–42 represents the flux distribution in the same machine at half load and full load. It will be seen that the main field is twisted in the direction of rotation, the amount of twist being greater with heavier loads. The commutating poles prevent the building up of flux *in the commutating plane* due to armature reaction, and, in addition, give the small amount of flux required for commutation.

Some flux-distribution curves for a commutating-pole motor are given in Figs. 7–26 to 7–28, pages 346 and 347.

39. Field Excitation. Since the poles of a d-c generator or motor must be continuously excited in the

same direction, direct current must be passed through the field coils. This field current for a d-c generator may be supplied from the armature of the machine itself, in which case it is said to be *self-excited*; or the field current may be furnished from some other d-c electric circuit, in which case the machine is said to be *separately* excited. Practically all d-c generators are self-excited.

40. Field Connections and Windings. In Fig. 6–43 are shown the various ways in which the field windings of a self-excited generator may be connected to its armature. In part *a* of the figure, the field is connected

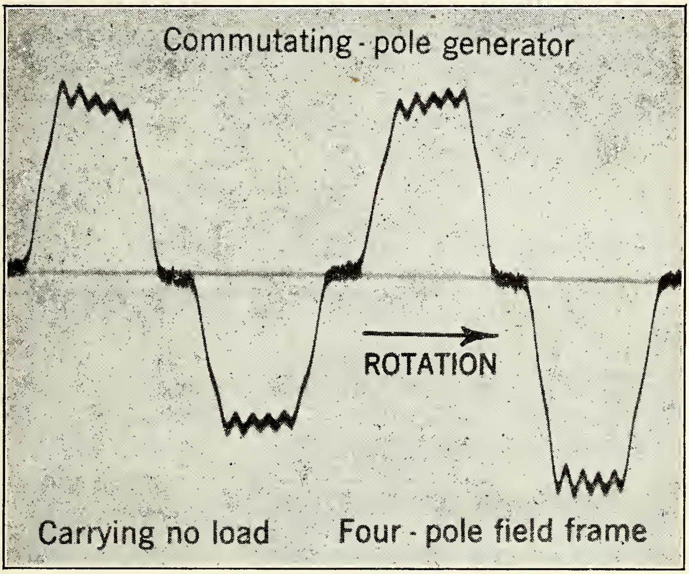


Fig. 6–41. Flux distribution of a commutating-pole generator when carrying no load, obtained by the oscillograph. The fact that the air gap under the four poles is not exactly the same is shown by the different heights of the voltage waves.

directly across the armature and hence has impressed upon it the full armature voltage; it is known as a *shunt* connection, the field winding is spoken of as a *shunt field*, and the machine itself as a *shunt generator*.

As the required number of ampere-turns of a field winding may be supplied by using either a large number of turns and a small current, or a few turns with large current, a shunt winding having the armature voltage impressed upon it will have a large number of turns. As the copper loss will be given by either I^2R or EI, where E is the voltage impressed, I is the field current, and R the winding resistance, the loss will be reduced by using a small current with high resistance and hence many turns.

In Fig. 6–43*b*, the field winding is in series with the armature and

hence, as represented, carries the entire armature current. This arrangement is known as a *series connection*, the winding as a *series field*, and the machine as a *series generator*. Since a series field must carry a large current it will have but few turns of large-sized conductor.

In most d-c generators, the field coil is wound in two parts. The larger part of the coil is made up of comparatively small wire and forms the shunt field, and the remaining part of the coil is wound of a few turns of large wire or copper ribbon, and forms the series winding of the generator. Such

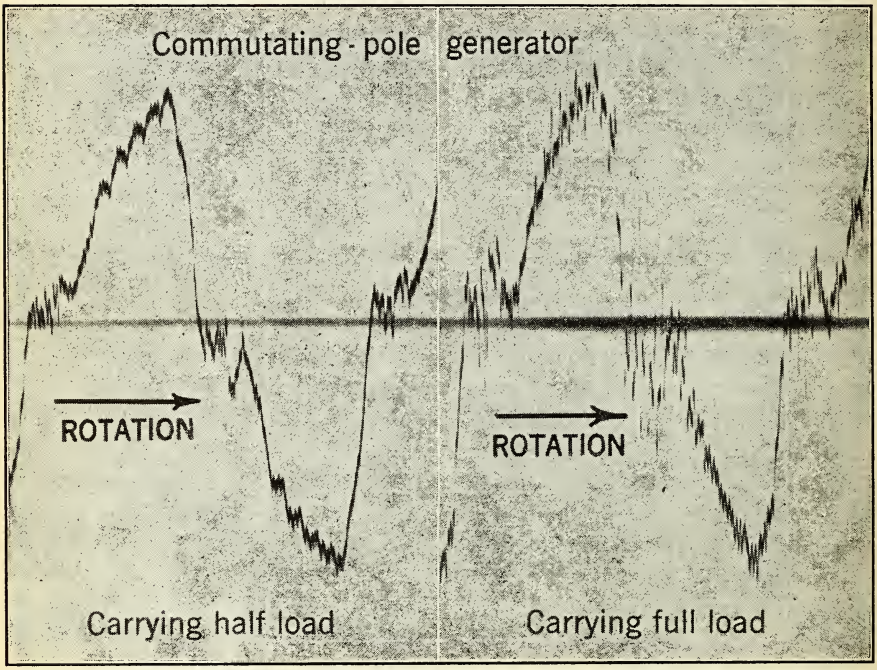


Fig. 6–42. Flux distribution of the same generator as was used in obtaining Fig. 6–41, when the machine was carrying half load and full load, as noted on the two plates. The terminal voltage was the same for all three curves and the scale of all the films was the same. It is seen, therefore, that the maximum voltage per bar is greater at full load than at half load, or no load, although the terminal voltage is the same for all three cases.

a combination of a shunt and a series winding is known as a *compound field* and the machine as a *compound generator*. As shown in Fig. 6–43*c*, the shunt field may be connected directly across the armature as in full lines, or it may be connected across both the armature and series field, as shown by the dotted lines in the same figure. The first connection is called a *short shunt* and is generally used in compound generators; the second connection (in dotted lines) is called a *long shunt* and is generally used in compound motors. Coils for compound-wound generators are

shown in Figs. 5–36 and 5–38. It will be seen that the terminals of the series field are heavily constructed to carry a large current safely.

41. Field Rheostats. In series with the shunt field of a generator is generally placed a variable resistance (Fig. 6–44) made of some high-

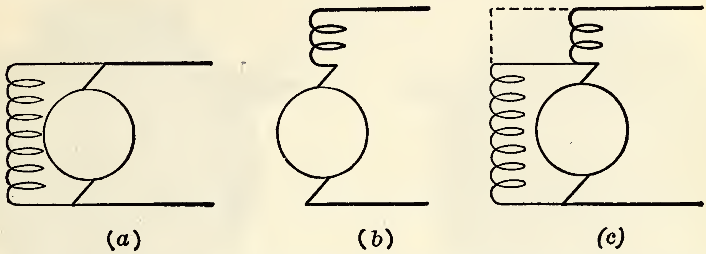


FIG. 6–43. Various connection schemes for magnetizing the field of a self-excited generator. The circles represent the armature; the coils represent the fields.

resistance material imbedded in enamel, porcelain, or other heat-resisting material or, in the case of large rheostats, merely supported in air. The amount of resistance can be varied by a movable shoe (carried on an arm that may be rotated) which makes contact with any one of many taps into

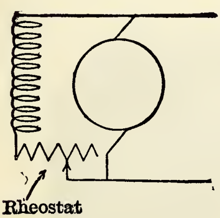


FIG. 6–44.

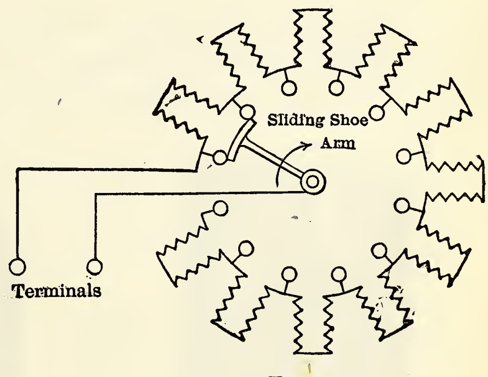


FIG. 6–45.

FIG. 6–44. A shunt-wound generator is always equipped with a variable resistance, called the field rheostat, in series with the field winding.

FIG. 6–45. Connections of a field rheostat. The wire of the first steps of the rheostat can generally carry safely about twice as much current as can that used for the last steps.

the resistance. This adjustable resistance is called a *field rheostat.* A diagram of the connections of the movable arm and the taps into the resistance is shown in Fig. 6–45. Figure 6–46 shows the external appearance of a small rheostat, the connections of which are given in Fig. 6–45. The pressed steel plate serves as a mechanical support for the enamel and

resistance wire, and also serves to radiate the heat generated in the resistance wire. Rheostats for larger generators are shown in Fig. 6–47.

The size of wire used in a field rheostat is "tapered"; the wire is much

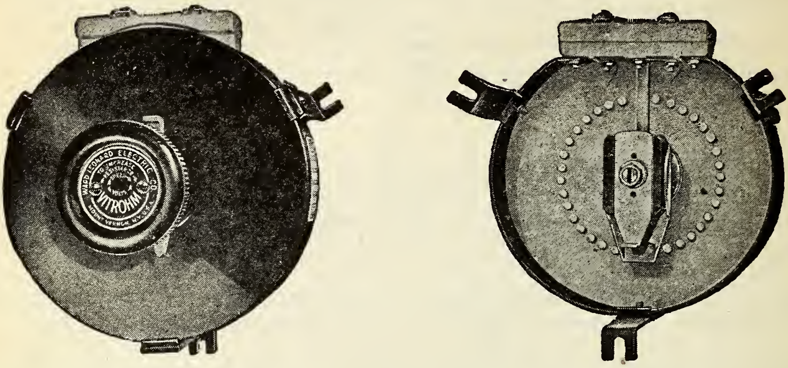


Fig. 6–46. Front and rear view of a modern field rheostat for a small generator. The resistance wires of the rheostat are completely imbedded in a vitreous enamel; the contact buttons project through the enamel, are ground off smooth, and thus serve to permit connection of the wire with the sliding shoe. *Courtesy of the Ward Leonard Electric Co.*

larger on one end than on the other. If the rheostat is connected in series with a shunt field (as in Fig. 6–44) and the amount of resistance in the rheostat is varied, the current through the rheostat and field winding will

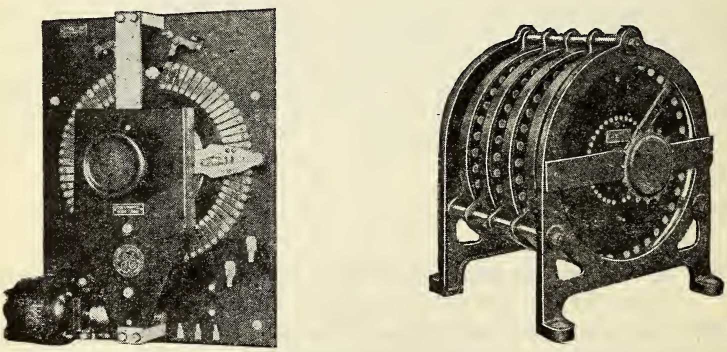


Fig. 6–47. Field rheostats for large generators. The one on the left is motor-driven; the one on the right is operated by chain and sprockets from a suitable handle on the generator panel of the switchboard. *Courtesy of the General Electric Co.*

vary. As the rheostat is "cut in" (i.e., as its resistance is raised) the current decreases; for this reason the rheostat wire is made larger on the first steps than on the last steps. Intermediate steps will be made of

intermediate sizes of wire. The amount of taper is generally 2 : 1, i.e., the rheostat can safely carry twice as much current on one end than on the other.

42. Rating a Rheostat. A certain field rheostat might be rated on its name plate: resistance 32 ohms, maximum current 8 amperes, minimum current 4 amperes. This rating signifies that the total resistance of the rheostat is 32 ohms, that it will safely carry 4 amperes *through all its resistance,* and that the *first step* will safely carry 8 amperes. Such a rheostat would be suitable for a field having 30 ohms resistance, connected to a 240-volt supply. When the rheostat is "all in" the current would be slightly less than 4 amperes, and when only the first step of the rheostat was left in the circuit the current would not be greater than 8 amperes.

43. Relation of Rheostat to Field Resistance. If this rheostat were used to regulate the field of a small generator (say one having 200 ohms resistance in the shunt field) it would not have much effect. The change in field-circuit resistance from the "all in" to the "all out" position of the rheostat would be 232 to 200 ohms, and quite probably this amount of change would be insufficient. The field rheostat to be of use with any generator must be properly designed for the field circuit of that machine. A rheostat suitable for one field may not be at all suitable for another. In general, the rheostat should have a resistance equal to about 80 to 100 per cent of the resistance of the field, and the current capacity of the first few steps should be equal to the rated voltage of the machine divided by the resistance of the shunt-field winding.

44. Shunting the Series Field. As it is difficult to design the series field of a compound generator with exactly the correct number of turns, these fields are therefore always "overdesigned." By this is meant that the coil is wound with more turns than are really necessary to give the required number of series-field ampere-turns, when all the generator output current is flowing through them.

The action of a series field is weakened by putting a *shunt* across the terminals of the series field. This shunt, or *diverter,* generally made of ribbon-shaped conductor, serves to by-pass part of the load current. The division of load current between the series field and the diverter depends upon their relative resistances. By adjusting the resistance of the diverter, the number of ampere-turns in the series field (for any given load) may be changed as desired. After the diverter has once been adjusted it is not necessary to change it, and therefore it is not made adjustable; in this respect, it is different from the shunt-field rheostat.

45. Characteristic Curves of Generators. If the load of a generator is increased (speed, the setting of the shunt-field rheostat, etc., being kept constant), the terminal voltage of the generator tends to decrease because of the effects of armature resistance and armature reaction. The curve

showing how the terminal voltage changes with increase in load is called the *external characteristic* of the generator. If the terminal voltage is kept constant by increasing the field current (thus increasing the generated emf) as the load increases, and a curve is plotted to show how the field current varies with the load, the curve is called the *armature characteristic* (or field compounding curve) of the generator. Many other curves of similar nature may be constructed, and they are all grouped under the general name *characteristic curves.* By inspection of these curves it may easily be seen how one quantity varies with respect to another, the rest of the variables involved in the operation of the machine being maintained constant.

The most important curves of a generator are the *external characteristic,* the *efficiency,* and the *magnetization curves.* The first two are plotted with terminal voltage and efficiency respectively, as ordinates, and load current as abscissae in both cases.

The efficiency curve of a generator will not be taken up here, as Chapter VIII will treat this subject in detail.

46. Magnetization Curve. The magnetization curve is plotted between terminal voltage and field current, the machine being operated at no load and normal speed while the data for the curve are being obtained, the field being generally separately excited. When there is no load on a generator, the terminal voltage and generated emf are the same. The generated emf is directly proportional to the flux through the armature, if the speed is held constant; the curve plotted between no-load terminal voltage and field current shows, therefore, the relation between the field current and the flux which this field current produces—hence its name of magnetization curve.

If the reluctance of the complete magnetic circuit of the machine were constant, no matter how much the flux might be, the magnetization curve would be a straight line. The reluctance of the air gap, which constitutes the principal reluctance of the magnetic circuit, is independent of flux density; the reluctance of the iron part of the path, however, increases as the flux density increases, especially at the higher densities. Hence the magnetization curve of a machine is nearly a straight line, but tends to bend over slightly at the higher values of the field current. The amount of bending shows how nearly the iron part of the magnetic circuit is saturated. In a well-designed machine, normal voltage is obtained with that value of field current which gives a voltage somewhere just below the *knee* of the magnetization curve, as it is called; this point is shown at C in Fig. 6–48. It will be seen that for an increase in field current over the normal field, OB, the increase in flux and hence in generated emf becomes smaller and smaller. This condition indicates that the magnetic circuit is becoming saturated.

47. Residual Magnetism. When the field current is zero, the generated emf is not zero but has some small value, perhaps 3 per cent of the normal voltage; this is due to the *residual magnetism* of the machine. After the field of a generator has once been magnetized, the iron pole pieces and yoke stay magnetized to a slight extent, and this magnetism which stays in the frame after the magnetizing current has been reduced to zero is called the residual magnetism of the machine. The amount of this magnetism depends upon the quality of iron used in the magnetic circuit; if all the iron of the magnetic path, poles, and yoke, as well as that of the armature core, was soft and well annealed, there would be practically no residual magnetism.

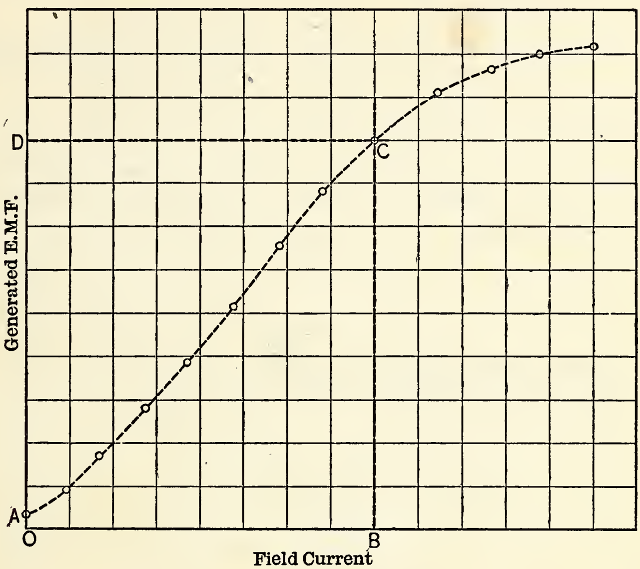


Fig. 6–48. Magnetization curve of a generator; the normal operating voltage is just below the knee of the curve.

48. Separately Excited Generator. The terminal voltage of any machine can be obtained for any load, by subtracting the armature IR drop from the generated voltage at that same load. It has been shown that the armature of a generator exerts a demagnetizing effect on the main field; thus, even though the field current of a generator is maintained constant, as the machine is loaded the *generated* voltage falls, because of the field weakening due to the armature reaction.

Consider a generator with its field separately excited, as in Fig. 6–49, the field resistance, R, being adjusted to give the machine a field current OA in Fig. 6–50. From the magnetization curve, the machine will generate a no-load voltage equal to $AD = OB$. As load is added to the gener-

ator (its field current and speed being maintained constant), its generated voltage falls, on account of armature reaction; this relation between generated voltage and load current is represented by the curve *BC*.

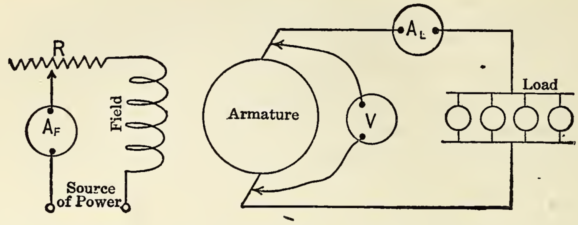


Fig. 6–49. Connection scheme for obtaining the external characteristic or the armature characteristic of a separately excited generator.

If the armature *IR* drop corresponding to each load current is subtracted from the curve of generated voltage, the curve *BG* results; this represents the relation between terminal voltage and load current, and is the external characteristic of the separately excited generator.

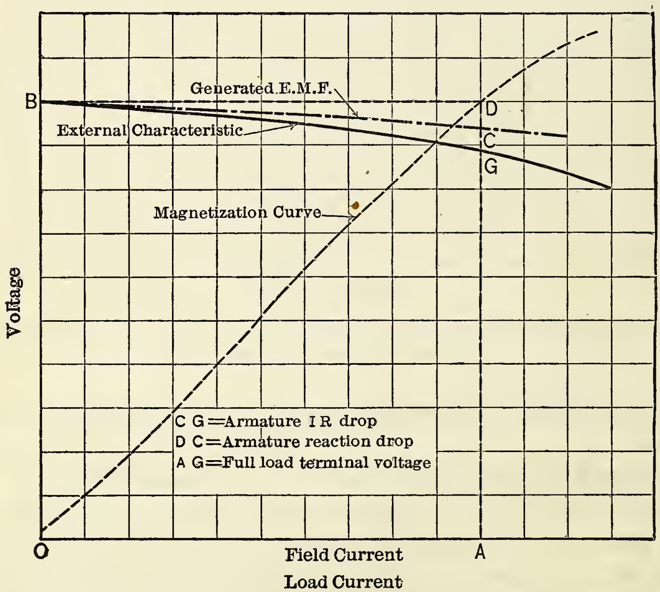


Fig. 6–50. External characteristic, and curve of generated voltage, of a separately excited generator.

The terminal voltage of a separately excited generator with constant field current, therefore, falls as its load is increased. If constant voltage with increasing load is desired, it is accomplished by adjustment of the field rheostat, permitting sufficient increase in the field current to bring

up the generated voltage. If the generated voltage is maintained greater than the terminal voltage, by an amount equal to the armature *IR* drop, the terminal voltage remains constant.

49. Series Generator. In the series generator, field and armature are in series and carry the same current, as represented in Fig. 6–51. The mmf of the field is directly proportional to the load current, and so the generated voltage is nearly proportional to the load. Where a load current, as *OA* in Fig. 6–52, is flowing, we should expect the generated voltage to be equal to *AD*. The demagnetizing effect of the armature reaction reduces the generated voltage to the value *AC*; subtracting the armature and series-field *IR* drops gives the terminal voltage corresponding to a load current of *OA*, as *AG*.

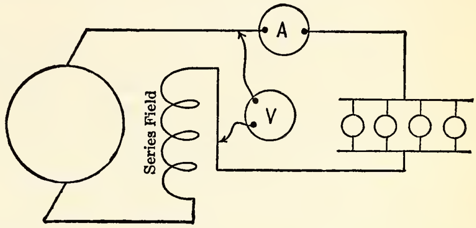


Fig. 6–51. Connection scheme for obtaining the external characteristic of a series generator.

The demagnetizing effect of the armature really neutralizes a part

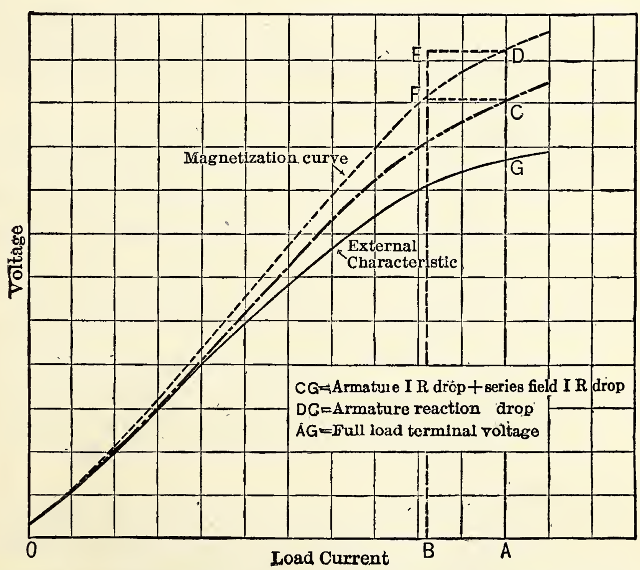


Fig. 6–52. External characteristic, and curve of generated voltage, of a series generator.

of the field mmf. If the strength of this armature mmf is taken as equal to *AB*, the *effective* current in the series field, so far as its magnetizing effect is concerned, is really *OB* and not *OA*. The current *OB* gives a voltage on the magnetization curve of *BF*, and *AC* is equal to *BF*. There-

fore, C is one point on the curve of generated emf, and other points can be found in a similar manner. The armature demagnetizing effect (AB) is of course proportional to the load.

The series generator, having a different terminal voltage for each load, finds no application as such. Its only use at present is as a series booster, as will be described later.

50. Self-excitation. Shunt and compound generators are self-excited machines; both have shunt fields connected across the armature, which therefore supplies the shunt-field current. When such a machine is brought up to rated speed with the field circuit open, the residual magnetism generates a very small emf in the armature. *As the shunt field is connected across the armature, this small emf produces a small current through the shunt field of the machine,* and, even though this current is small, it increases somewhat the field strength of the machine. As a result, the emf generated in the armature increases and hence more current flows in the shunt-field circuit.

51. "Building Up." This action and reaction between the armature and shunt field is called the "building up" of the generator; it continues until the generator is operating at normal voltage. It is evident that if there were no residual magnetism in the field this "building-up" operation could not take place; it would be necessary to excite the fields from some outside source every time the generator was started, and this would complicate the operation of a generating station.

The successful building up of a given self-excited generator, and the polarity with which it builds up, depend on four conditions: the polarity and magnitude of the residual magnetism, the resistance of the field circuit, the order of the connections of the field winding to the armature terminals, and the direction of rotation.

In Fig. 6–53*a*, a shunt-wound generator is represented as rotating clockwise, its residual magnetism being such as to make its upper pole a north pole. As soon as the armature starts to rotate, current flows as a result of the small induced voltage as indicated, flowing out from the right-hand brush and through the field coils as shown; its direction is evidently such as to assist the residual magnetism, and the machine will tend to build up with polarity as indicated.

If the residual magnetism had been reversed, as in Fig. 6–53*b*, the upper pole being now south, the current in the armature would also be reversed, and the machine would tend to build up, *but with polarity opposite to that in Fig.* 6–53*a*.

It may happen that the connection of the field to the armature is such that what small current is sent through the field circuit by the voltage due to residual magnetism tends to decrease, instead of increase, the field magnetism. In this case the machine cannot build up as may be seen

from Fig. 6–53*c*. By reversing the order of connection of the field to the armature, or, as it is generally stated, reversing the field, we come back to the connections of Fig. 6–53*a*, which are satisfactory. However, the machine may also be made to build up by reversing the direction of rotation from clockwise in Fig. 6–53*c* to counter-clockwise, as in Fig. 6–53*d*.

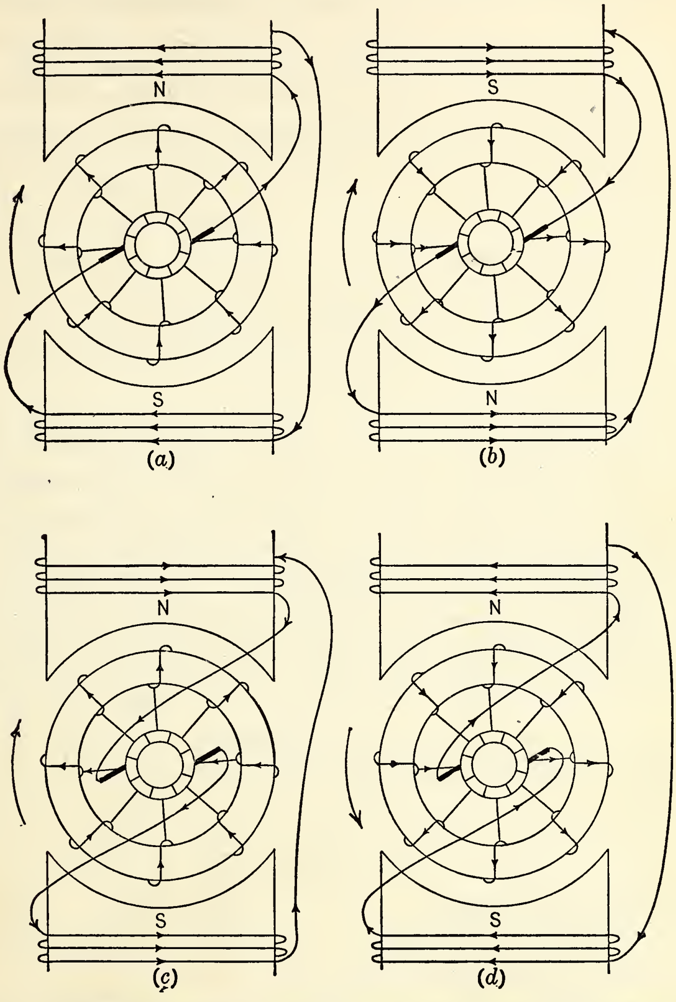


FIG. 6–53. Conditions which may possibly be encountered when a shunt-wound generator is supposedly connected properly for "building up." The polarities indicated on the poles are those of the residual magnetism, not those set up by the current in the windings.

If a shunt generator, by some chance, loses its residual magnetism (the jarring it receives during shipment might possibly effect such a result)

it is necessary to connect its field circuit to some source of electric power, and so re-establish the residual magnetism.

If the residual magnetism of a generator should become reversed for any reason, it will build up with reversed polarity if the field is properly connected to the armature terminals. This condition of reversed polarity may be permissible in the laboratory; but if it happened on a railway generator, for example, the station meters would all deflect backward, and the polarity of trolley and ground would be reversed. In practice, it is therefore necessary to reverse the residual magnetism by supplying current in the right direction to the field coils from some outside source.

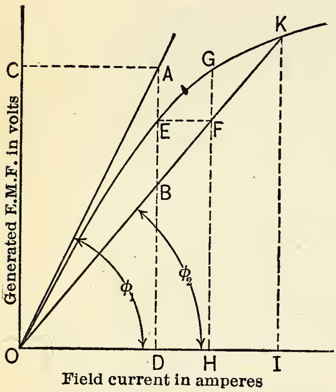


FIG. 6–54. This diagram shows, graphically, the conditions required for a shunt-wound generator to excite itself. The tangent of the angle between the straight line and the horizontal, equal to the resistance of the field circuit (including the field rheostat), must be sufficiently low to make the straight line lie below the straight part of the magnetization curve.

If a generator having residual magnetism refuses to build up, the difficulty probably lies with the connections of the field to the armature. If these are reversed the machine should build up. Other reasons why a generator will not build up are an open circuit somewhere in the field circuit, too much resistance in the field rheostat, wrong direction of rotation, or a commutator so dirty as to cause abnormal brush-contact resistance.

52. Why a Generator Builds Up. The current which flows through a shunt field is

$$I_f = \frac{E}{R_f + R_r} \tag{8}$$

where E = impressed voltage;
R_f = resistance of field coils;
R_r = resistance of field rheostat.

Now suppose the magnetization curve of the machine is as given in Fig. 6–54. If a line AO is drawn through the origin, making an angle ϕ_1 with the horizontal, it is evident that, since

$$\tan \phi_1 = \frac{E}{I_f} = R_f + R_r \tag{9}$$

it must represent some definite value of field resistance. (The same scale is supposedly used for both volts and amperes. If not, a proper change

must be introduced in the value of $\tan \phi_1$.) Any other line, say KO, making a smaller angle, ϕ_2, represents some other smaller value of field resistance; if a field current OD is desired through this resistance, a voltage BD must be impressed upon it; for a current OH, the impressed voltage must be HF, etc.

With the resistance of the field circuit equal to $\tan \phi_1$, it is apparent that at no value of the field current (except possibly at very low values) does the armature generate enough voltage to force through the field circuit enough current to excite the field sufficiently to produce in the armature the voltage required. Hence, with this value for the field-circuit resistance, the generator could not build up.

But suppose that the field rheostat is cut down, so that the resistance of the field circuit becomes equal to $\tan \phi_2$. Now when the generator has a field current equal to OD, it generates a voltage DE; but to force the current OD through the field circuit requires only the voltage DB. Hence, the voltage DE forces through the field circuit a current OH, which in turn makes the armature generate the voltage HG, which again increases the field current. This process continues until the point K is reached, where the generated voltage is just sufficient to force through the field circuit the current OI.

Therefore, in starting a shunt generator, it is necessary to adjust the resistance in the field rheostat properly. To obtain voltages higher than IK, more of the field rheostat must be cut out.

53. Shunt Generator. The shunt-wound generator has an external characteristic that falls off more rapidly than does that of the separately excited machine, because in the shunt generator there are *three effects* tending to make the terminal voltage fall as the load is increased. Armature reaction and the armature resistance drop are primary factors, and, *in addition, the shunt-field current falls as the load is increased because of the lowered terminal voltage.* The shunt-field current is evidently proportional to the terminal voltage, as may be seen from Fig. 6–55, and so decreases with load increase. The decrease in shunt-field current is the result of the effects of armature reaction and armature IR drop and would not occur if the terminal voltage had not fallen; its

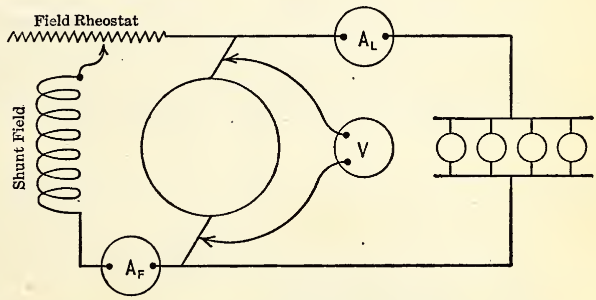


Fig. 6–55. Connections for obtaining the external characteristic of a self-excited, shunt-wound generator.

effect is to decrease further the generated voltage. The external characteristic of a shunt generator is shown in Fig. 6–56.

If the resistance of the external circuit of a shunt generator is continually decreased, the external characteristic doubles back because of the decreased field current. From B to F (Fig. 6–56) the machine is stable, and this is the working part of the external characteristic. The part of the curve from G to H is generally difficult to obtain, as the machine does not definitely maintain any special load on that part of the curve.

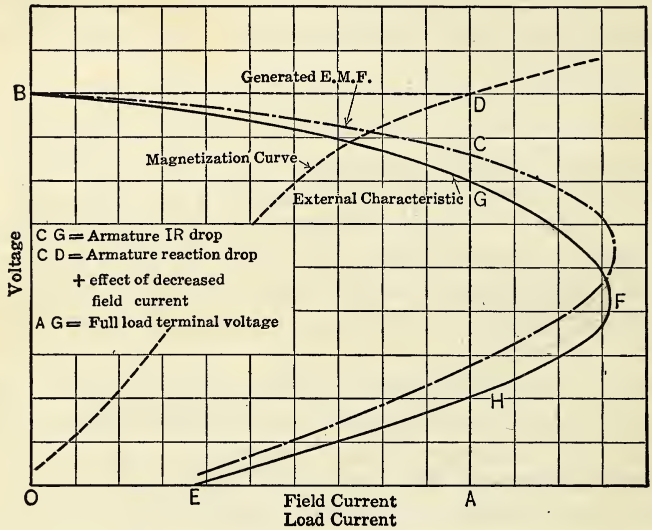


FIG. 6–56. External characteristic, and curve of generated voltage, of a shunt-wound generator. Rated full-load current is shown by OA and rated full-load voltage by AG.

That the external characteristic of a shunt generator bends back on itself, may be seen if we consider the equations

$$E_t = E_g - I_a R_a \tag{10}$$

and

$$I_l = \frac{E_t}{R_e} \tag{11}$$

where E_t = terminal voltage;
E_g = generated voltage;
I_a = armature current;
R_a = armature resistance;
I_l = load or external current;
R_e = resistance of load or external circuit.

Both R_e and E_t are decreasing quantities throughout the determination of the external characteristic; the operator causes R_e to decrease as he

chooses, by adding load to the machine, but the rate of decrease of E_t is fixed by Eq. (10), the magnitude of armature reaction, and the shape of the magnetization curve. Whether the external current, I_l, increases or decreases, depends upon the relative rates of decrease of R_e and E_t. At first, as R_e is decreased, E_t does not decrease relatively as much, and the load current increases. The relative rate of decrease of E_t, however, increases and, when the machine reaches the steeper portion of its magnetization curve, E_t decreases relatively faster than R_e, and I_l begins to decrease.

Even when the external resistance is reduced to zero by the application of a short circuit to the terminals, there will be some current circulating through the armature, as shown at E, Fig. 6–56. Since the external resistance is zero, the terminal voltage also is zero. The field coils can produce no magnetic effect because they have no current, but there is some residual magnetism to give the armature emf, and this produces the short-circuit current.

The above analysis is for machines having no commutating poles. Machines equipped with commutating poles, even if the brushes are set directly under the center of these poles, show a certain compounding action which makes the external characteristic droop less rapidly than that shown in Fig. 6–56.

Because of its poor regulation, the shunt generator is used but very little in practice. It is well adapted for charging storage batteries, because of its drooping characteristic; as the batteries become charged their voltage rises, and the charging current is thereby reduced. Such installations are rare, however.

54. Graphical Construction of the External Characteristic of a Shunt Generator. When load is added to a shunt generator, the shunt-field circuit resistance remaining constant, the terminal voltage falls for three reasons: armature I_aR_a drop, armature reaction, and decreased field current.

In Fig. 6–57 the magnetization curve of the machine has been plotted between field ampere-turns and voltage, and it is assumed that at no load the machine voltage has been adjusted to give a voltage E_0. The required field ampere-turns are OF_0. At this point, while the load current is zero, the shunt-field current will be flowing through the armature, so that at no load

$$I_a = I_s = \frac{E_0}{R_s} \tag{12}$$

where I_a is the armature current, I_s the shunt-field current, and R_s is the shunt-field circuit resistance. If the no-load shunt-field current is E_0A_0, then point A_0 represents the no-load armature current.

Since the field-circuit resistance will be constant, the field current at any terminal voltage will be the horizontal intercept between the vertical axis and a line OA_0, making an angle α with the vertical, where $\cot\alpha = E_0/I_s = R_s$.

Assume that sufficient load was added to the generator to bring its terminal voltage down to the value E_1. Since E_1 volts are applied to the field, the field ampere-turns must be OF_1. Now with load on the armature there will be some demagnetizing effect due to armature reaction, F_1D_1, which reduces the effective field ampere-turns to OD_1 and the voltage generated within the armature to E_g'.

The difference between the generated voltage E_g' and the terminal voltage E_1, i.e., the intercept $E_g'H'$, must represent the drop in the armature circuit (armature drop plus brush drop).

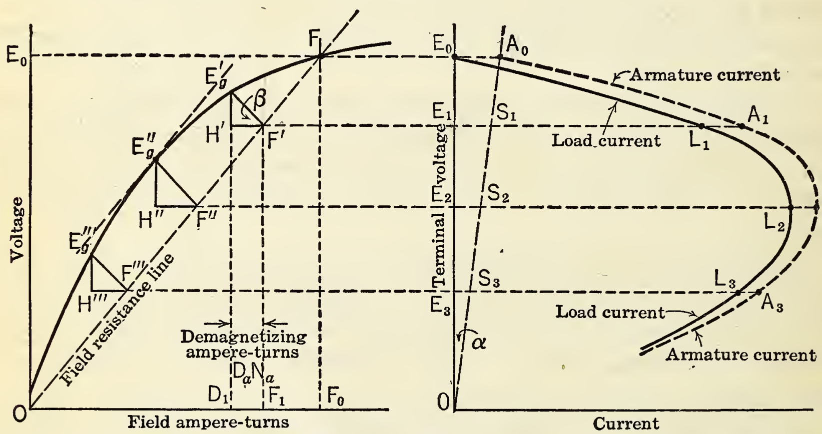


FIG. 6–57. Graphical construction of the external characteristic of a shunt generator. The change in size of the triangle E_gHF shows why the load current first increases and then decreases. The shape of the triangle E_gHF is determined by the ratio of the armature-circuit I_aR_a drop to the demagnetizing effect of the armature.

In the triangle $E_g'H'F'$ with side $E_g'H'$ representing the armature-circuit drop, and side $H'F'$ representing the demagnetizing armature ampere-turns, D_aN_a, the angle β, between $H'F'$ and $E_g'F'$ will be such that

$$\tan\beta = \frac{E_g'H'}{H'F'} = \frac{I_aR_a}{D_aN_a} \tag{13}$$

If it is assumed that both armature-circuit drop and demagnetizing ampere-turns may be determined, and also that both quantities will vary directly with the armature current, it follows that the angle β will be the same at all loads, and that the triangle $E_g'H'F'$ will be similar for all loads. The shape of the triangle is determined by the ratio of the I_aR_a drop to the demagnetizing ampere-turns.

The armature current for the load assumed as causing the terminal voltage E_1 may be determined by dividing the I_aR_a drop $= E_g'H'$ by the armature-circuit resistance, and plotting the result as the horizontal distance E_1A_1. The corresponding field current being E_1S_1, if this value is subtracted from E_1A_1, the armature current, the intercept E_1L_1 is the corresponding load current and L_1 is a point on the external characteristic.

By similar construction more points on the external characteristic may be determined. The method is open to two errors which cause inaccuracies. The first error is caused by the fact that the armature resistance is affected by temperature and that the brush drop does not vary directly with the armature current. The second error is caused by the difficulty in determining the demagnetizing effect of the armature, particularly in machines with commutating poles.

The analysis, however, shows clearly why the load current of a shunt generator, without change in field-circuit resistance, first increases to a maximum and then decreases. Since the armature current is proportional to the side $E_g'F'$ of the triangle $E_g'H'F'$, the armature current will be greatest when the intercept $E_g'F'$ has its greatest value, as $E_g''F''$, determined by drawing a tangent to the magnetization curve parallel to the field resistance line, OF.

55. Regulation. The difference between the no-load terminal voltage and full-load terminal voltage (field rheostat position fixed and speed constant), expressed in percentage of the *full-load voltage*, is called the *regulation* of a generator. Thus, a machine having no-load voltage of 121 and full-load voltage of 110 would have a regulation of 10 per cent.

56. Armature Characteristic of a Shunt Generator. If constant terminal voltage is desired from a shunt generator, with increasing load, it is necessary to increase the shunt-field current by adjustment of the shunt-field rheostat. This manipulation compensates for the armature IR drop and the effect of armature reaction, and results in an increasing field current with increase of load; the curve expressing their relation is called the armature characteristic. Such a curve is shown in Fig. 6–58; it is usually concave upward, because the increase in flux, per unit increase of the shunt-

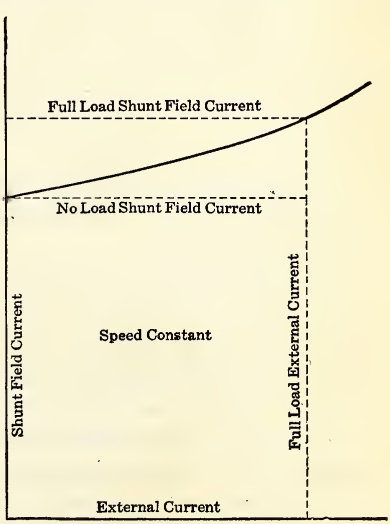


Fig. 6–58. Armature characteristic, or field compounding curve, of a shunt-wound generator.

field current, decreases as the magnetic circuit approaches saturation, as shown by the magnetization curve.

The increase in the field ampere-turns required to compensate for the armature IR drop and armature reaction may be determined from this curve, being equal to the increase in shunt-field current from no load to full load, multiplied by the number of shunt-field turns; the latter is, of course, constant.

57. Compound Generator. A very simple way of causing the necessary increase in the field ampere-turns, to compensate for armature IR drop and armature reaction, is to employ the external current (which causes the terminal voltage to fall) by passing it through the series-field winding, which consists of several turns of large wire and is therefore of low resistance. A machine in which this has been done is called a compound generator; its connections are indicated in Fig. 6–59. The shunt field provides

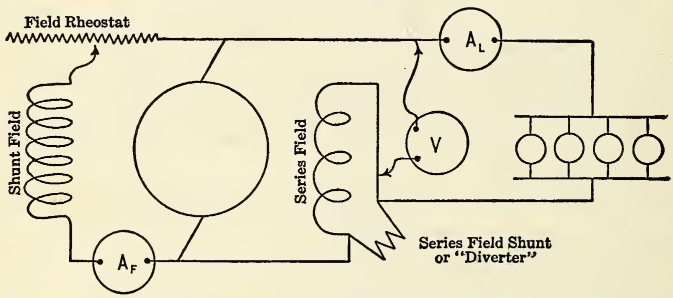


Fig. 6–59. Connections for obtaining the external characteristic of a compound-wound generator.

the correct no-load flux of the machine, while the series field, carrying all, or a fixed percentage, of the external current, increases the flux by a predetermined amount.

If the series field provides more flux than is required to compensate for all the losses of voltage, the terminal voltage will rise with increase of load and the generator is said to be *over-compounded.* If the series field provides just enough increase in flux to make the terminal voltage at full load the same as at no load, the machine is said to be *flat-compounded.* If there are not enough turns in the series field to keep the terminal voltage from dropping as the load increases, the machine is said to be *under-compounded.*

In an over-compounded generator, the shunt-field current increases somewhat with increase of load, because of the increase in terminal voltage brought about by the series-field winding. This is the reason for using the short shunt connection of the shunt field in a compound generator. Even with a flat-compounded generator, the armature voltage, and hence the shunt-field voltage, will be greater than the terminal voltage by the IR drop in the series field.

In Fig. 6–60, the external, or compound, characteristic of an over-compounded generator is shown by the curve *BG*. The curve of generated voltage, *BC*, is seen to rise as the load increases.

58. Calculation of Series-field Turns. The number of series-field turns required to convert a given shunt generator into a compound generator may be determined from the armature characteristic of the shunt generator.

Consider that the armature characteristic of Fig. 6–58 is that for a 7.5-kw shunt generator the voltage of which was maintained constant at 110 volts from no load to full load. The shunt-field current at no load was 4 amperes, and, with a full-load current of 7500/110 = 68.2 amperes, it

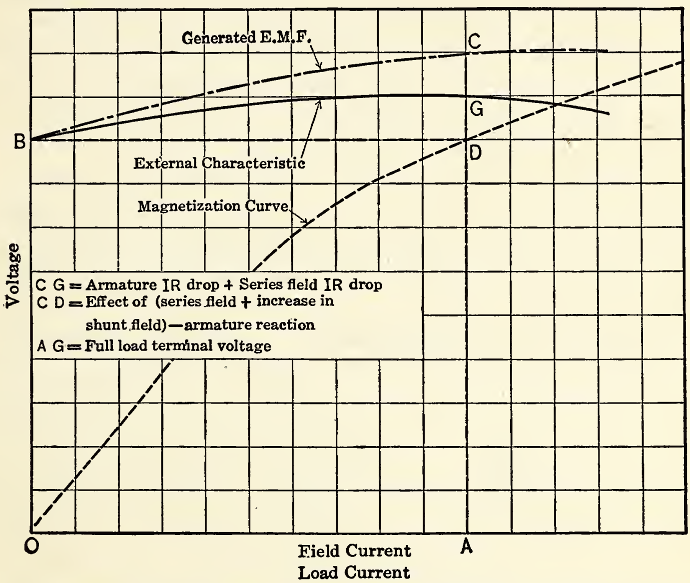


Fig. 6–60. External characteristic, and curve of generated voltage, of a compound-wound generator.

was 4.7 amperes. If there are 1500 turns per pole in the shunt-field winding, the increase in field ampere-turns was $(4.7 - 4.0) \times 1500 = 1050$. Neglecting the *IR* drop in the series field, the minimum number of series turns necessary to convert the machine into a compound generator is therefore 1050/68.2 = 15.4 per pole. More turns per pole than this would be added, say up to 25, and a diverter adjusted across the series field. Its resistance would be such as to allow 1050/25, or 42.0, amperes to pass through the field and divert 26.2 amperes. Considering that there will be a small *IR* drop in the series field to be compensated, the resistance of the diverter would have to be slightly higher than this value so as to make the series-field current somewhat greater than 42 amperes.

If the armature characteristic as a shunt generator of Fig. 6–58 had been determined with the voltage increasing uniformly with load increase, from 110 volts at no load, to 125 volts with a full-load current of 68.2 amperes, the calculation would be altered by the fact that, when the machine is converted into an over-compounded generator, the shunt-field current will automatically increase with load.

Suppose the shunt-field current to have increased from 4.0 amperes at no load to 5.35 amperes at full load of 68.2 amperes, increasing the shunt-field ampere-turns by $1.35 \times 1500 = 2025$. As a compound generator, the no-load shunt-field current will be 4.0 amperes, and the shunt-field circuit resistance $110/4 = 27.5$ ohms. At full load (again neglecting the IR drop in the series field) the voltage impressed on the field circuit will have risen to 125 volts, so that the shunt-field current becomes $125/27.5 = 4.55$ amperes, increasing the ampere-turns provided by the shunt field when operating as a compound generator from 4×1500 to 4.55×1500, or an increase of $0.55 \times 1500 = 825$ per pole.

The series field to be added in this case need therefore furnish only $2025 - 825 = 1200$ ampere-turns per pole, which can be done with a minimum of $1200/68.2 = 17.6$ turns per pole. The kilowatt rating of the generator as used in the second case is slightly increased, being $125 \times 68.2/1000 = 8.53$ kw.

59. Use of the Compound Generator. Practically all machines used in lighting or railway service are over-compounded. In such work the characteristic to be obtained is *a constant voltage at the load,* not at the generator. The life and efficiency of an incandescent lamp are both greatly affected if the voltage of the line to which it is connected goes either above or below the voltage at which the lamp is rated. Now, if the terminal voltage of the generator should remain constant, the voltage of points on the distributing system must fall as the load is increased because of increased IR drop in the wires of the distributing system.

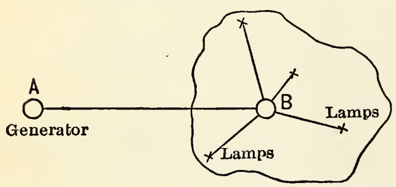


FIG. 6–61. In electrical distribution systems it is customary to keep the voltage constant at some distant feeding point, such as point B of the diagram. This requires an increase in the terminal voltage of the generator proportional to the load current, to offset the IR drop in the feeder between A and B.

In Fig. 6–61 this point is illustrated. A group of lamps is connected to a center of distribution B; the generator is located at A. If the voltage at A remains constant as the load increases, it is evident that the voltage at B must fall because it is always equal to the voltage at A minus the drop in the line AB. But, if the increase in voltage at A, from no load to full load, is just equal to the IR drop in AB when full-load current is

flowing, the voltage at B will be the same at full load as it is at no load. At intermediate loads, the voltage at B will be somewhat above normal; the external characteristic of a generator is always more or less curved (because of the form of the magnetization curve) so that if the machine is properly compounded at full load it is always somewhat over-compounded at half load. This effect is not great enough to cause any trouble on the system.

The amount of over-compounding used in practice depends upon the service for which the machines are intended. For isolated plants it may be as low as 3 per cent or as high as 10 per cent. Under-compounded generators are never used in practice.

Compound generators find their greatest application in the isolated plants of factories and office buildings, where steam heating is a factor. In such cases the generators are often driven by slow-speed reciprocating engines, which expand the steam to a few pounds above atmospheric pressure and exhaust into the heating system. In the summer such plants may use diesel-driven generators, or, if cheap cooling water is available, operate the engines condensing.

Isolated plants are economical only if the combined cost of electric power and heating is less than the cost of heating plus the cost of purchased power. In the large northeastern cities, where cheap power is usually not available, they are very numerous.

60. The Three-wire Generator. Practically all d-c installations supplying lighting and power circuits are arranged as a three-wire system as described on page 114, and as shown in Fig. 6–62. This requires either two separate 115-volt generators or some special arrangement that will provide the connection for the third wire if a 230-volt generator is used. This is done practically by means of a balancer set (described in Chapter VII) or a three-wire generator.

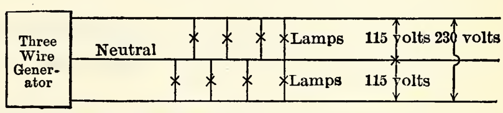


FIG. 6–62. A three-wire network connected to a three-wire generator. The neutral wire generally carries a much smaller current than either of the two outside wires.

The three-wire generator is an ordinary machine with a special feature for connecting the neutral to the armature windings. This is done by connecting an inductance coil between two diametral taps on the armature winding and connecting the neutral wire to the center of the coil, as shown in Fig. 6–63.

The inductance coil consists merely of turns of insulated wire wound on a laminated iron core. By connecting the neutral to the middle of the winding, it is always maintained midway in potential between the brushes, to which the two outer wires of the system are connected. There will be

only a very small current flowing through the coil from the armature because the voltage impressed across it by the armature is alternating, i.e., reversing its polarity, as described on page 228. This changing voltage causes a current to flow which likewise changes in direction. This changing current produces a counter emf due to the self-inductance of the coil, purposely made very high, which opposes the applied voltage and keeps the actual current flowing very small. The d-c current flowing in from the neutral, since it is unchanging, will not cause this effect to appear and will flow freely. The inductance coil may be built into the armature spider, as shown in Fig. 6–63*a*, and its middle point connected to a slip ring *A*. (In this figure the commutator has been omitted for simplicity, two brushes being shown making contact directly upon the armature winding.) In

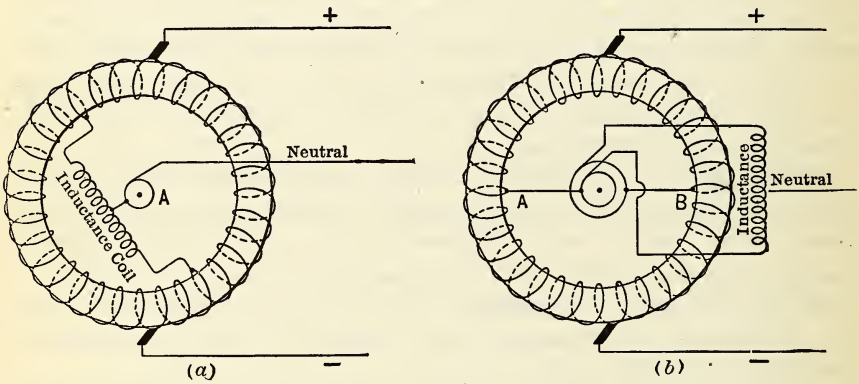


Fig. 6–63. Two schemes for connecting the neutral wire to the armature winding of a generator.

the scheme shown in Fig. 6–63*b*, the two diametral taps, *A* and *B*, connect to two slip rings, and so through brushes and leads to an inductance coil outside the machine.

61. Boosters. A booster is a generator the armature of which is connected in series with a circuit, its generated voltage being added to or subtracted from that of the circuit to increase or decrease the voltage of the line; the former is the more general usage. Boosters may be driven by any type of prime mover; a motor taking its power from the line in which the booster is connected is generally most convenient.

Boosters are used for the control of the charge and discharge of storage batteries used in connection with generators in isolated plants. In such installations the batteries are operated in parallel with the generators, taking the peak loads, but generally charging during the day. When the load is light, as during the night, the generators are shut down, and the battery takes the entire load. The boosters used may have their fields

wound shunt, series, compound, or differential; in the last case the series field opposes the shunt field.

Series boosters are used for compensating for the IR drop in long, heavily loaded feeders. In Fig. 6–64 a station is shown supplying a number of short feeders to distribution centers near by, together with one long feeder. If it is required that the voltage at the end of this long feeder be maintained constant, this may be conveniently done by means of a series booster connected as in Fig. 6–65.

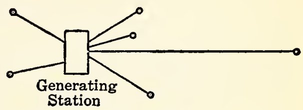


Fig. 6–64. When a long feeder is supplied from the same bus-bars as are the short ones, it becomes necessary to use a booster to offset the high IR drop in the long feeder.

The field current of the booster is evidently the load current of the feeder. Since the IR drop in the feeder varies directly as the load current, by designing the booster to operate on the straight portion of its magnetization curve, thereby making its generated voltage proportional to its field current, the IR drop in the feeder may be exactly compensated.

Obviously, the polarity of the booster, i.e., whether it adds its voltage to that of the feeder, or subtracts it therefrom, depends upon the field connections. By reversing the field of a booster which is raising the voltage of a feeder, the polarity of its armature will be reversed and it will subtract its voltage from, or *crush*, the voltage of the feeder.

In Fig. 6–65, the booster is shown as driven by a shunt motor, connected on the station side of the booster; if the voltage on the station side of the booster is substantially constant, the booster will be driven at practically constant speed.

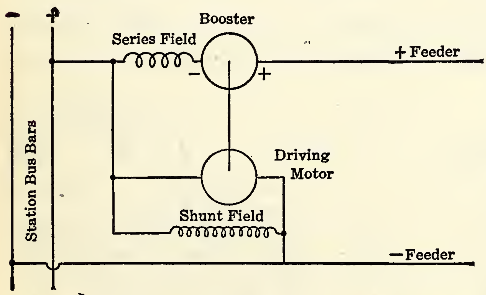


Fig. 6–65. Connections of a booster and its driving motor.

Another common use of the series booster is as a *negative booster*. In railway work, where the current is brought back to the power house by means of the track and the surrounding earth, there is danger of the current leaking from the track and following gas or water mains, as illustrated in Fig. 6–66. This leakage current causes electrolytic action, which results in corrosion of the pipes. In order to prevent leakage of current to water and gas mains, insulated *negative feeders* are placed in parallel with the track, to assist in bringing the return current back to the power house. Such negative feeders, alone, naturally reduce the resistance of the return circuit, but their equivalent resistance may be further

reduced by placing a series booster in series with them, as shown in Fig. 6–67. With a properly designed booster so connected, most of the return current will flow back to the station over the feeder.

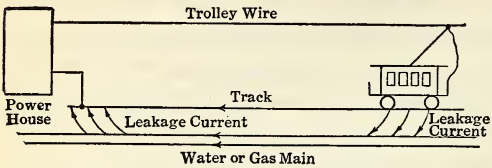


Fig. 6–66. In some electrical networks, especially railway systems using rail return, stray currents may cause serious damage to water and gas mains by electrolytic action.

62. Constant-current Variable-speed Generators. Generators designed to charge the batteries and furnish power for lamps, etc., for train-lighting, automobiles, etc., are required to operate at widely varying speeds. For such generators special forms of field excitation are employed, all depending upon a storage battery connected in parallel to maintain constant voltage, and upon armature reaction within the machine to maintain practically constant current over a wide speed range.

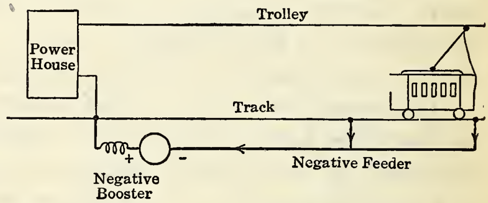


Fig. 6–67. The use of a negative feeder and booster as shown here will generally eliminate trouble from electrolysis.

63. Third-brush Generators. A type of variable-speed generator used on automobiles is shown in Fig. 6–68. The battery is connected to the regular brushes through a potential relay, but the field is connected from the positive brush to a third brush which is less than 180° ahead of the positive brush.

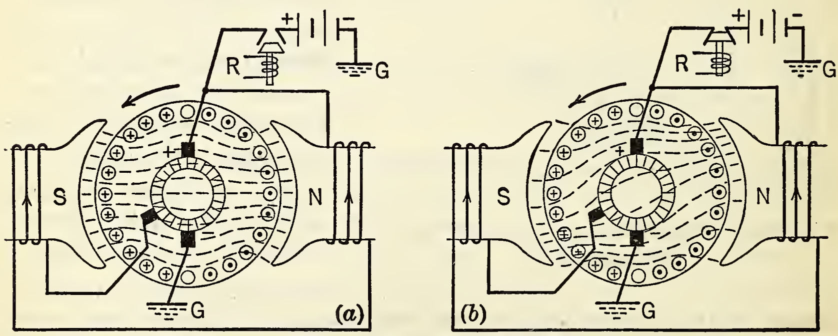


Fig. 6–68. A third-brush, variable-speed generator, as used on automobiles. It delivers a practically constant current to the battery and the load, as the speed varies through wide limits.

As the generator speed picks up from zero, the generator voltage builds up in the usual way from residual magnetism. So long as the voltage of

the generator is less than that of the battery, the battery would discharge through the generator and, to prevent this, the relay, R, is placed between the generator and the battery.

As shown in Fig. 6–69, the cutout relay has two windings. One is a potential winding, P, which closes the contacts, C, when the voltage of the generator has built up to a predetermined voltage slightly higher than the voltage of the battery. The second winding, A, is a series or current winding; its function is to open the circuit whenever the voltage of the generator drops below that of the battery and the latter begins to discharge into the generator. While the generator is furnishing current, both windings hold the cutout relay closed; when the battery discharges into the generator the reversed current through coil A demagnetizes the relay and the spring separates the contacts, breaking the circuit.

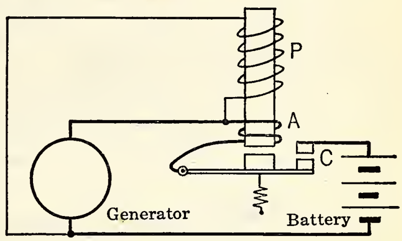


FIG. 6–69. The "cutout" relay of an automobile equipment. The battery is not connected to the generator until the generated voltage is greater than that of the battery. If the voltage of the generator falls below that of the battery, the reversed current flowing through coil A demagnetizes the relay, and so allows the spring to open the contact C.

So long as the generator carries no current, its flux distribution in the air gap is uniform, as is shown in Fig. 6–68a, the voltage impressed upon the field being that generated in the inductors lying between the positive and third brushes. When the generator supplies current, armature reaction twists the field in the direction of rotation, as is shown in Fig. 6–68b, the amount of twist depending directly upon the magnitude of current flowing in the armature. The effect of the twisted field is to transfer more flux into the region between the negative and third brushes so that less flux is available for producing voltage between the positive and third brushes. This results in a decreased field current.

The peculiar action of this third-brush excitation results in a nearly constant current delivered to the battery and load circuits of the generator, even though the speed varies through wide limits. The current for fixed load-circuit resistance usually increases a little with increase of speed and then decreases as the speed is raised still further.

It must be recognized that the battery voltage controls the voltage of the system, for if the generator voltage tends to rise above that of the battery a charging current results, producing armature reaction and armature IR drop in the generator. The generator terminal voltage cannot rise above the terminal voltage of the battery and the latter cannot rise above its own generated voltage unless a charging current flows. It is

important, therefore, that the generator never become disconnected from the battery. Should this happen, the voltage of a 6-volt generator may rise to as much as 50 volts, burning out the lamps if connected, the generator field coils, and probably the potential winding of the cutout relay. The same results may occur if the generator is connected to a completely discharged battery.

64. Rosenberg Generator. A type of constant-current, variable-speed generator, known as the Rosenberg generator, is much used in modified form for train-lighting. Its regulation depends upon the fact that the armature sets up a strong mmf at right angles to the field poles, and this "cross-field" becomes the source of the load current.

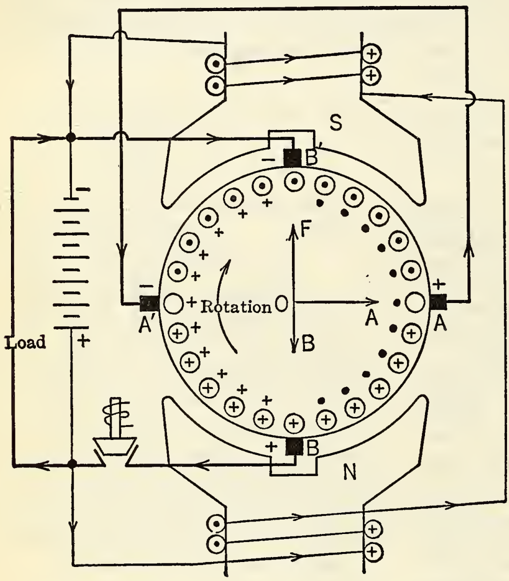


FIG. 6–70. The Rosenberg generator. It depends for its regulation upon the establishment, by the armature, of a cross-field.

As shown in Fig. 6–70, in bipolar form, the machine has poles with pole shoes larger than normal and an armature with two sets of brushes. These are shown as bearing upon the periphery of the armature, the commutator being omitted for simplicity. Brushes BB' are connected to the load and battery through a cutout relay (Fig. 6–69) and brushes AA' are short-circuited upon themselves.

The field winding is connected directly across the storage battery, causing a north pole at the bottom and a south pole at the top. The mmf of the field poles is represented by the vector OF, and this will be directly proportional to the terminal voltage of the battery.

With clockwise rotation of the armature in the flux of the field poles, the armature conductors will generate voltage which acts into the paper under the north pole and out of the paper in the upper half of the armature, as is shown by crosses and dots *within* the conductors. The voltage thus generated causes a current to flow from brush A to brush A' over the short circuit. The short-circuit current sets up a mmf, OA, within the armature, at right angles to the pole axis, as in any normal machine.

The flux actually set up within the armature will, of course, be due to

the resultant of all the mmf's present. It is convenient, however, to assume that each mmf will set up its own component of flux in order to study the action of this machine. Accordingly, the short-circuit mmf, OA, sets up what is commonly called a "cross-flux," acting here from left to right within the armature. This field finds a return path of comparatively low reluctance through the pole shoes which are made especially large to cause the cross-flux to be very strong.

Now the armature conductors, cutting the cross-flux, will generate further voltages which act into the paper on the left of the vertical pole axis and out of the paper on the right, as is represented by dots and crosses *outside* the armature conductors. These voltages cause the lower brush, B, to become positive and the upper brush, B', to become negative. When the load voltage, across brushes BB', becomes higher than the battery terminal voltage, the relay will close, and the generator will deliver current to the battery and load. The load current from brushes BB' produces another mmf within the armature, represented by vector OB, which we shall call the load mmf. It is seen to be directly opposite to the field mmf, OF.

Up to the time that the generator reaches its critical speed, the relay is open and the battery supplies current to the load and also excites the field. While the field mmf is nearly constant, the voltage across the short-circuited brushes, AA', and the value of the short-circuited current from brush A to A' will increase with the speed. Since the strength of the cross-field depends upon the value of the short-circuit current, it will also increase with speed and therefore increase the voltage across the load brushes BB'.

Once the relay has closed and the load current flows, the load mmf, OB, directly opposes the field mmf, OF. An increase in load increases the load mmf, OB, and so reduces the main-field flux, causing a reduction in the short-circuit current, the cross-field, and so in the load generated voltage. Further increase in speed tends to increase the short-circuit current, the cross-field, the load generated voltage, and the load current. But an increase in the load current increases the load mmf and causes a reduction of the field flux. The machine is thus self-regulating.

In general the machine will assume load up to its capacity, at which limit the load mmf, OB, becomes nearly equal to the field mmf, OF. Beyond this limit the machine will not deliver more current at any speed above the critical value.

If the direction of rotation is reversed to become counter-clockwise, the direction of the main field remaining the same, the direction of the short-circuit current reverses (now flowing from A' to A), as well as the direction of the cross-field, OA. The armature conductors, now cutting the reversed cross-field with reversed direction of rotation, leave the polarity of the load brushes B and B' the same as before.

This type of generator differs from the normal machine in that the field set up by the poles is quite weak and of secondary importance with respect to the cross-field. Because the field is weak in the vicinity of the load brushes, BB' (made more so by the pole-face slot), commutation of these brushes offers no serious difficulty.

65. Welding Generators. Most electric welding is performed by passing current in the form of an arc from a metal pencil or electrode to the material to be welded, the end of the pencil melting and depositing to form the weld. For successful welding by direct current, a constant current is desirable and, in order to hold the current constant at any set value, a constant-current, variable-voltage generator is required. This generator should also provide a higher voltage to strike or start the arc. The simplest type of a satisfactory welding generator is a differentially-wound compound generator with a coil of high self-inductance connected across the series field as a diverter, as in Fig. 6–71. The series-field mmf is in opposition to that of the shunt field and at full load is about one-third the strength of the shunt field. The self-inductance of the diverter is greater than the self-inductance of the series field; it may be as great as forty times as much. The current distribution between the series field and the diverter is different when the current is changing from when it is steady.

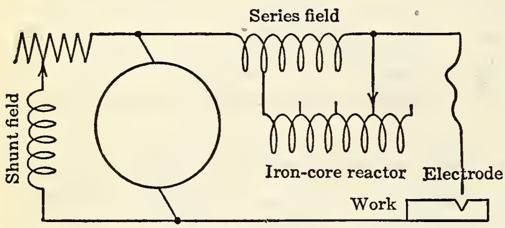


FIG. 6–71. A differentially-wound generator used for electric welding. The reactor is an iron-core coil of greater self-inductance than the series field.

With this generator the voltage at no load is built up to a desired value. Now when the arc is struck, by bringing the welding electrode into contact with the work, there will be a large value of current, much higher than is desired. Because of the higher self-induction of the diverter, most of this current flows through the series field, rapidly reducing the voltage. With the proper no-load voltage and the proper number of turns of the coil connected across the field, the generator regulates to hold the current constant. If the welding current increases, most of the increase flows through the field reducing the voltage. If the welding current decreases, most of the decrease occurs in the series field, raising the voltage.

The disadvantage of the machine lies in the fact that, since different welding operations require different values of current, the no-load voltage requires special adjustment and the number of turns of the coil may require changing.

A type of welding generator in which it is possible to pre-set the machine for a definite value of current (made by the Westinghouse Electric

and Manufacturing Company) is shown in bipolar form in Fig. 6–72. It is a modified Rosenberg generator with series excitation.

The modification in the type of Rosenberg generator, shown in Fig. 6–72, consists of two iron plates, *PP*, whose position with respect to the armature is adjusted by means of a handwheel. The welding current is taken from the brushes *BB'*, a lead being taken from each brush, passed around a small commutating pole in the main-pole faces, and then to the series field.

The magnetic circuits of this machine being rather involved it is more convenient to consider it from the viewpoint of flux components. At no load a small amount of residual magnetism exists in the field structure which sets up a small flux Φ between the poles. The armature cutting this flux sets up a certain current i in the short circuit between brushes *AA'*, and this short-circuit current sets up the flux Φ_A, which in turn induces a no-load voltage across brushes *BB'*. This no-load voltage is not of very high value.

As soon as the arc is struck a current *I* flows through the series fields of the machine and also through the armature from brush *B* to *B'*. Current flowing through the series fields tends to increase the flux Φ, the short-circuit flux Φ_A, and therefore the generated voltage between brushes *BB'*. Current through the armature from brush *B* to *B'* sets up the flux Φ_B, which acts in opposition to flux Φ.

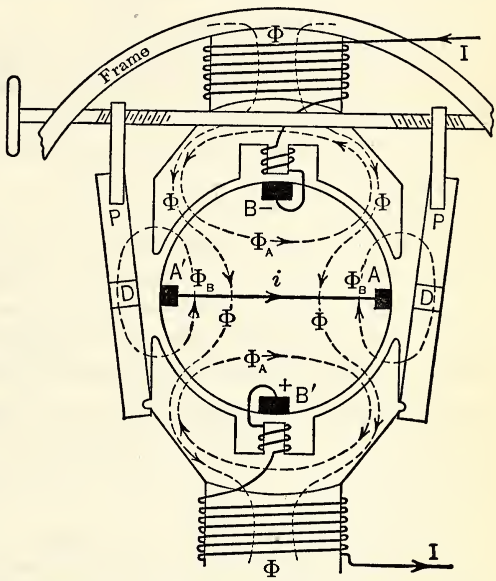


Fig. 6–72. A welding generator of the cross-field type. (Manufactured by Westinghouse Electric and Manufacturing Co.)

The current regulation of the machine depends upon the position of the plates *PP*. Moving them closer to the armature increases the flux Φ_B and also decreases flux Φ, since the plates also form a leakage path for the latter. The greater the flux Φ_B the lower will be the current for which the machine regulates. Once the machine has been calibrated, the current for which the machine regulates may be indicated by a dial and pointer working in conjunction with the handwheel.

In this operation there is a tendency for flux Φ_B to change more rapidly than flux Φ, and, to offset this tendency, correctly proportioned copper damping rings, *DD*, are clamped around the plates.

66. Operation of D-C Generators in Parallel. A d-c generating station has ordinarily several generators, all of which, or only one of which, may be operated to supply the station output. The factors that influence the number and sizes of the generators in a station are reliability and efficiency.

From the standpoint of reliability a station should be capable of carrying safely its maximum load with its largest unit out of service, so that repairs can be made if it should fail. Thus several smaller-sized generators are preferred to one large machine.

To obtain efficient operation of the plant, no generator should ever be operated for long periods at light load. While the full-load efficiency of a 500-kw generator might be 93 per cent and that of a 50-kw generator 90 per cent, if the larger machine were operated with an output of only 50 kw its efficiency would be about 70 per cent.

The choice of the number and sizes of the machines in a station necessary to insure reliable operation is often governed by the availability and cost of "break-down" or "stand-by" service. If service from a power company is available, the isolated plant may contract for a definite number of kilowatts of power, so as to supply all or a portion of its requirements in case of failure of its own equipment. There is usually a charge per kilowatt per month on the part of the power company for its readiness to serve, in addition to a charge for whatever energy is used. Some plants, rather than operate their own generators during the night, will purchase energy during that period.

The choice of the sizes and numbers of machines is arrived at after a study of the probable power demand on the station. If "break-down" service is not available, the station must have sufficient machines to carry the load efficiently at all times with provision for failure. If "break-down" service is available, the number of machines may be somewhat reduced.

67. Parallel Connection of Generators. In a d-c station the generators are connected in parallel by means of *bus-bars*. These are heavy copper bars, usually placed behind or above the switchboard, to which the various generators and load feeders are connected. The positive bus connects to the positive terminals of all machines in operation, and all negative terminals are connected to the negative bus. Such a connection is called a *parallel connection* of generators; all generators send their power into the one set of bus-bars, and the load, supplied by various feeders, is taken from them, as shown in Fig. 6–73.

When two generators are operating in parallel they are connected to the same bus-bars, and therefore their terminal voltages, being the bus voltage, will be the same. The sum of the output currents of both ma-

chines will be equal to the load current, which will be determined by the resistance of the load and the bus voltage.

For a shunt generator the relation between the generated voltage, E_g, and the terminal voltage, E_t, is given by the equation

$$E_t = E_g - I_a R_a \tag{14}$$

where I_a is the armature current and R_a is the armature resistance. If the terminal voltage is fixed, the armature current is proportional to the

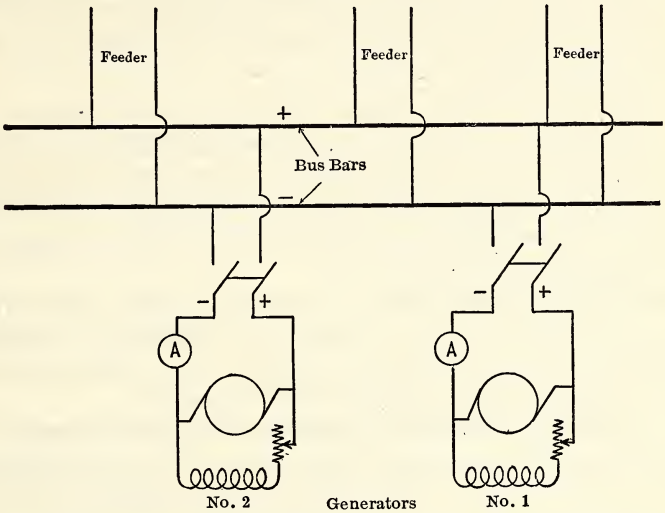


FIG. 6–73. Connection of shunt generators for parallel operation on common bus-bars.

$I_a R_a$ drop, the amount that the generated voltage is greater than the terminal voltage.

For two shunt generators operating in parallel the general equation is

$$E_g' - I'R' = E_t' = E_B = E_t'' = E_g'' - I''R'' \tag{15}$$

where E_B is the bus voltage.

68. Connecting a Generator to the Bus-bars. To connect a generator properly to the bus-bars to which another generator is already supplying current, the conditions should be such that, when its switches are closed, the incoming machine will neither furnish nor receive current. The incoming generator is first brought up to rated speed and its voltage built up to equal that of the bus-bars. Furthermore, the polarity of the incoming machine must be such that its positive terminal will be connected to the positive bus and its negative terminal to the negative bus.

Unless the voltage of the incoming machine has been built up to a value close to the bus voltage, the incoming generator will draw current from the bus-bars, the magnitude of which depends upon the difference

between its generated voltage (which is its terminal voltage since it is supplying no load current) and the bus voltage. If this flow of current is heavy it may well cause protective devices to operate and shut down the whole system.

If the polarity of the incoming machine is opposite to that of the bus-bars, the incoming generator will also draw a very heavy current. In the local circuit composed of the two generators and the bus-bars between their connections, the two machine voltages act to force current around the circuit in the same direction, the current being limited only by the two armature resistances and that of the bus-bars. In effect we have a short circuit at double voltage.

If the polarity of the incoming machine is correct and its voltage exactly equal to the bus voltage, then the voltages of the two machines in the local circuit are in opposition and no current will flow in the local circuit, i.e., the incoming machine neither furnishes nor receives current. This is also seen from Eq. (15), for if E_t'' and E_g'' for the incoming machine are equal it cannot carry current.

To cause the incoming machine to assume load it is evident from Eq. (15) that it is necessary to raise its generated voltage above the bus voltage. With the generators driven at constant speed, this is done by decreasing the field resistance, causing more field current.

69. Load Division with Shunt Generators Operated in Parallel. With the operation of two shunt generators governed by the conditions of Eq. (15), the load division between them as the load changes, if no shunt-field readjustment is made, is determined by the shape of their external characteristic curves, as shown in Fig. 6–74. Assume that both machines have been adjusted to the same voltage (117.5 volts) with no load on the bus-bars, so that both generators are furnishing no current. If the total load now increases to 160 amperes, the terminal voltage falls to 110 volts, and generator 1 will furnish 60 amperes and No. 2 will furnish 100 amperes.

70. Equalization of the Load. If it were desired to equalize the load between the two machines, always driven at their rated speeds, it might be done either by strengthening the field of No. 1 (cutting out some of its field resistance), or by decreasing the generated voltage of No. 2 (by increasing its field-circuit resistance). Reference to Eq. (15) will show that raising E_g' will result in an increase in E_B, an increase in I', and a decrease in I''. A decrease in E_g'' will lower E_B and cause the same changes in I' and I''.

The effect of strengthening the field of generator 1 is to raise its whole characteristic. Thus if the field of No. 1 is increased until both machines share equally a load of 200 amperes at 110 volts, the full-line characteristic of No. 1 becomes the dotted characteristic 1′.

The division of the load between two shunt generators and the bus-

bar voltage is therefore under the control of the operator. The division of load may be controlled by manipulating either field current; the bus voltage is raised or lowered by adjusting both field currents.

If, after the adjustment of equal load division of 100 amperes on each machine at 110 volts, the total load is decreased without further field adjustment, the machines will share the load according to characteristics 1′ and 2. As the load decreases the terminal voltage will rise, with generator 1′ carrying the major portion. When the load has fallen to 62 amperes, No. 2 will carry no current; all the load is supplied by No. 1′ at 117.5 volts. If the load is further decreased the bus voltage will rise above the generated voltage of No. 2, the flow of current through its armature will be reversed,

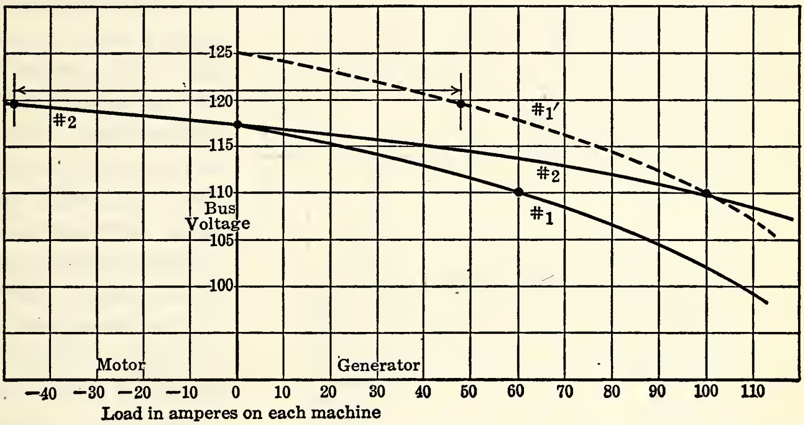


Fig. 6–74. If the external characteristics of two shunt generators operating in parallel are not identical, the two machines will not maintain an equal division of the load, as this is varied.

and it will receive power from No. 1′, operating as a motor. When the load drops to zero, No. 2 will be drawing 48 amperes as a motor from No. 1′, at 119.5 volts, as indicated in Fig. 6–74.

If two equal generators are to be operated successfully in parallel, they should have identical external characteristics. The factors affecting the external characteristics are armature-circuit I_aR_a drop, armature reaction, magnetic saturation, and brush position. If the machines are alike in these respects, they will have identical characteristics and the load once equally divided will remain so for all loads.

Because shunt generators have drooping external characteristics, the parallel operation of shunt generators is stable, even if one machine should, for some reason, operate as a motor.

71. Generators of Different Capacities. If two machines of different capacities are to be operated in parallel, their two characteristics should be *similar* in shape, and the no-load and full-load voltages of the two ma-

chines must be the same. If No. 1 generates 110 volts at no load and 105 volts when supplying rated current of 100 amperes, No. 2 should generate 110 volts at no load and 105 volts when furnishing its rated current of 50 amperes. With such characteristics, No. 2 will always carry one-half as much current as No. 1.

72. Disconnecting a Shunt Generator from the Bus-bars. If it is desired to disconnect one generator from the bus-bars, its load is first reduced to zero by increasing the resistance of its field rheostat, until its generated voltage is equal to the bus voltage. At the same time the field rheostats of the other generators must be changed so as to strengthen their fields, or the bus voltage will fall as the other generators take up the load being dropped by the generator being disconnected. It is obvious from Eq. (15) that otherwise only by a decrease in bus voltage can the additional load be carried by the generators left in service. When the generated and terminal voltages of a generator are equal, its output current must be zero. The switches connecting the generator to the bus-bars may then be opened, and the generator shut down.

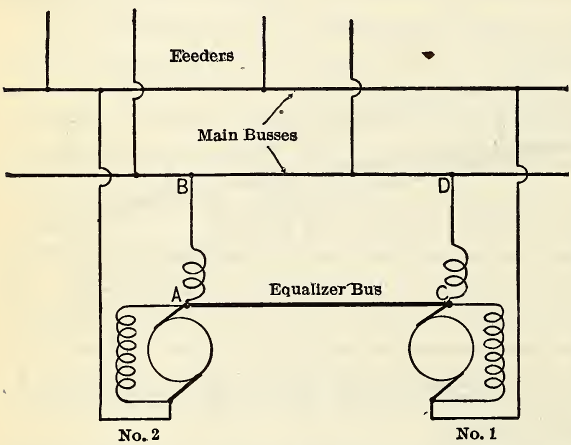


FIG. 6–75. When compound-wound generators are operated in parallel, it is necessary to provide an extra bus-bar, called the "equalizer bus," connected as shown here.

73. Parallel Operation of Compound Generator. The parallel operation of compound generators, if they have rising external characteristics, is unstable, unless a connection known as the equalizer connection is made. An equalizer bus connects the machines at the points where the series fields of the machines are connected to the armatures, as shown in Fig. 6–75. The equalizer connection stabilizes the parallel operation of compound generators having rising external characteristics and also tends to equalize the load between them.

74. Load Division with Compound Generators. The load division between two compound generators operating in parallel will be governed by the shape of their compound or external characteristics. If two machines have characteristics which are different as shown in Fig. 6–76, one machine will carry more load than the other. Thus, as shown, with a bus voltage

of 115 volts generator 1 will supply 50 amperes (point E) while No. 2 will supply 80 amperes (point D). For equal division of load the generators should have identical compound characteristic curves.

75. Action without the Equalizer Bus. The unstable action of compound generators operated in parallel without an equalizer connection is due to the action of the series fields.

Suppose that the two generators have the characteristics shown in Fig. 6-76 and that the load current is 130 amperes; generator 1 will supply 50 amperes (point E) and No. 2, 80 amperes (point D). Suppose something happens to change the load momentarily on No. 1 to 63 amperes (a sluggish governor on the prime mover of generator 2 could bring this about). The increased current through the series field of No. 1 tends to raise its voltage, say to the point F. As the whole load is 130 amperes, machine 2 will now carry only 67 amperes, and it is to be expected that No. 2 will now operate at point G of its external characteristic. The terminal voltage of generator 1 would now be higher than that of No. 2. But, as they are connected to the same bus-bars, this is really an impossible condition.

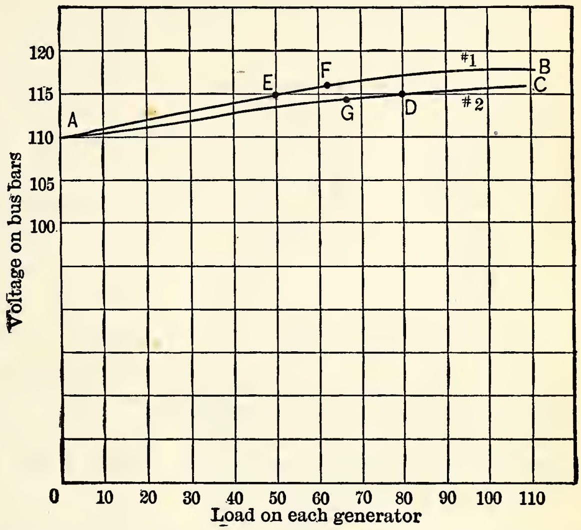


Fig. 6–76. Without an equalizer bus the operation of over-compounded generators in parallel is unstable. If one starts to increase its share of the load, this action continues until it has taken all the load and is, in addition, running the other as a motor.

Actually what happens is a shift in the load from one machine to the other. A small increase in load on one machine causes an increased series-field current, an increase in its voltage, and a tendency for the machine to take more load. A decrease in load on the other machine results in a decreased series-field current, and voltage, and a tendency for the machine to give up its load.

The swing in the load from one machine to the other may continue until the machine which has the stronger total field forces current through the other, running it as a motor, as is assumed in Fig. 6–77. The armature

and series-field current of generator 1 flows from the negative to the positive bus, while the shunt-field current flows from the positive to the negative brush, as indicated by arrows. The machine being a compound generator, the series-field mmf augments that of the shunt field.

Machine 2 being a motor, its armature and series-field current, as well as its shunt-field current, flow from the positive to the negative bus. In passing from generator to motor, the shunt-field current of No. 2 has remained in the same direction, but the direction of its series-field current has reversed; *its series field now tends to oppose the shunt field.* The greater the current taken by No. 2 becomes, the stronger its series field and the weaker its resultant field, until the net field is zero; at this time machine 2, having no field flux and generating no voltage, is practically a short circuit for the generator 1.

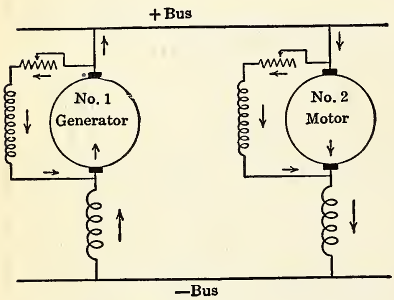


FIG. 6–77. When one of two compound generators, connected in parallel without an equalizer bus, operates as a motor, its armature and series-field current reverses, making it a differential motor.

76. Action of the Equalizer Bus. The equalizer prevents the unstable action described by maintaining the IR drop over the two series fields equal at all loads. As the resistance of the equalizer is made practically zero, as is that of the main bus-bars, it is evident that the drop in voltage from A to B (Fig. 6–75) must be equal to that from C to D. Evidently the drop in voltage from A to B (or C to D) is equal to the drop in the series field proper, plus the drop in the connecting cables.

The equalizer thus connects the two series fields in parallel, and if the two series fields have the same resistance the load current will divide equally no matter what the current division between the armatures may be. If the armature current of generator 2 were 200 amperes and that of No. 1 were 100 amperes, 50 amperes would flow over the equalizer from A to C, giving a current of 150 amperes in each series field.

The equalizer thus has a tendency to equalize the load. For if the load becomes unbalanced some of the armature current of the machine carrying the greater load flows through the series field of the weaker machine and raises its voltage. It was noticed that, without the equalizer connection, when generator 1 increased its load and thereby its voltage (Fig. 6–76) the voltage of No. 2 decreased, and that this was the cause of the unstable operation. When the equalizer bus is used, however, if No. 1 increases its load, the voltage of No. 2 increases also, thereby causing No. 2 to continue to carry a share of the increased load.

When there is an equalizer connection and one machine is deliberately caused to operate as a motor, the action is still stable. In Fig. 6–75, if machine 1 has its field weakened so much that it becomes a motor, current will flow from A to C over the equalizer rather than over the path A to B, and D to C, since the latter path through the series fields is of comparatively higher resistance. Machine 1 thus operates as a shunt motor.

The equalizer bus would not be needed if the external characteristics of the two compound generators were drooping, i.e., the generators were under-compounded. In practice, however, most compound generators are

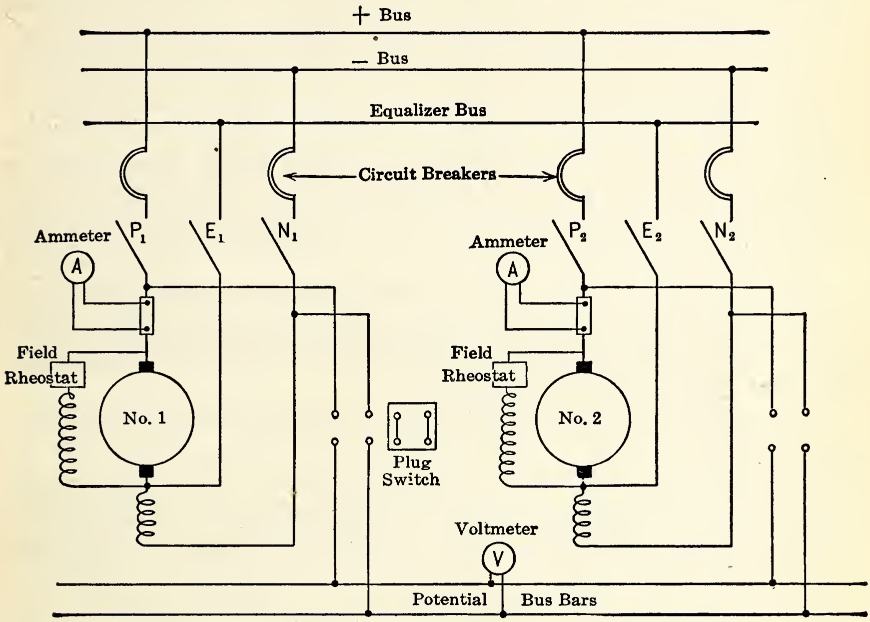


FIG. 6–78. Proper layout of machines and apparatus for compound generators in parallel operation.

over-compounded; hence they have rising external characteristics and require an equalizer bus if they are to operate successfully in parallel.

77. Compound Generators of Different Capacities. If two compound generators of unequal capacity are to operate well in parallel, dividing the load according to their capacities, the two machines must be adjusted to give the same no-load and full-load voltages, and *the resistances of the two series-field circuits must be inversely proportional to their capacities.* This last condition evidently is fulfilled if the full-load IR drop is the same for each series-field circuit.

78. Switching of Compound Generators. In Fig. 6–78, detail connections for two compound generators, intended for parallel operation, are

given. The connections for two shunt generators would be the same, except for the series fields and equalizer connections.

When it is necessary to connect an additional generator (say No. 1) to the bus-bars, it is first brought up to rated speed, and the main circuit-breakers closed. Switches E_1 and N_1 are then closed, passing some of the load current from the other generators through the series field of the incoming machine. The voltage of No. 1 is then adjusted to be the same as that of the bus-bars, and switch P_1 closed.

It is not necessary that switches E_1 and N_1 be closed before the voltage of No. 1 is adjusted; if the voltage is first adjusted, it will be found that further adjustment will be necessary after switches E_1 and N_1 are closed, owing to the current that will flow in the series field of No. 1, since it will then be in parallel with other series fields and will carry a part of the load current. Whatever order of switching is followed, however, it is necessary that the switch P_1 be the last closed, the voltage of the machine having been properly adjusted to equal that of the bus-bars. The machine is then made to assume its proper share of the load, by strengthening its shunt field.

To disconnect a generator from the bus-bars, its load is first reduced to zero by increasing the resistance of its shunt-field circuit, until its generated voltage is equal to the bus-bar voltage. Switch P_1 is *first* opened, after which the other switches and the circuit-breakers may be opened in any order, and the machine shut down.

It is evident, when a generator is to be connected in parallel with others already operating, that it must come in with proper polarity; i.e., its positive and negative terminals must be connected to the positive and negative bus-bars, respectively. The voltmeter connections of Fig. 6–78, being arranged symmetrically, take care of the fulfillment of this condition. If a generator, by any chance, has had its residual magnetism reversed and has built up with reversed polarity, the operator would recognize the fact at once; the voltmeter, which he would require in adjusting the voltage of the machine, *would be reading backward*, or indicating a reversed voltage on its terminals.

To meet the difficulty arising when a machine intended for parallel operation builds up with reversed polarity, the obvious thing to do is to properly restore its residual magnetism. With compound generators, this will ordinarily be done if, with considerable load on the system, switches E_1 and N_1 (Fig. 6–78) are closed, exciting the series field. Should this fail, the shunt field may be separately excited. An easy way of doing this is by lifting all the positive or negative brushes to disconnect the low-resistance armature and closing *all* switches and breakers of the machine for an instant, connecting the shunt field across the line. The brushes of the machine must not, however, be dropped until at least switch P_1 is again opened.

PROBLEMS

6–1. Assuming that each coil generates an emf wave of the form of a sine wave, find the variation in line emf, in percentage of the maximum line voltage, of

(*a*) A 2-coil winding with a 2-part commutator.
(*b*) A 4-coil winding with a 4-part commutator.
(*c*) An 8-coil winding with an 8-part commutator.

6–2. An armature winding having 288 conductors has a 2-path winding. The field has 4 poles, and the flux per pole is 750,000 lines. The armature turns 1800 rpm. What is the generated voltage?

6–3. A 6-pole, 720-rpm generator has a multiple-circuit winding with 432 conductors. The flux per pole is 11.1×10^6 lines. What voltage is generated?

6–4. An 8-pole, 720-rpm generator has a multiple-circuit winding with 272 conductors. What voltage is generated if the flux per pole is 8×10^6 lines?

6–5. An armature winding requires 800 feet of No. 10 wire. There are 6 paths in parallel with the winding scheme used. What is the resistance of the armature at 20 C? At 75 C?

6–6. A bipolar generator has 36 coils of 12 turns each of No. 12 wire. The active length per turn is 14 inches and the total length per turn is 26 inches. The pole face covers 65 per cent of the armature periphery and the average flux density under the poles is 8500 lines per sq cm. Peripheral speed is 3600 feet per minute, current density in the armature conductors is 400 cir mils per ampere. Brush-contact drop is 2 volts. Calculate generated voltage, armature resistance, and terminal voltage.

6–7. A small bipolar machine has 480 armature conductors and 650,000 lines of flux per pole. The wire used in the armature winding is No. 14, and the machine turns 2400 rpm. How much current can it safely carry if the current density in the armature conductors is 425 cir mils per ampere? What is its generated voltage? What is the terminal voltage at 80 C, the total length of wire per turn being 16 inches and the brush-contact drop being 2 volts?

6–8. A 6-pole, 750-rpm generator with a multiple-circuit winding has 81 slots with 4 conductors per slot. Each conductor is 86,000 cir mils in cross-section. The total length per turn is 60 inches, of which 20 inches are active in cutting flux. The armature is 24 inches in diameter. Flux density under the pole, where 60 per cent of the conductors lie, is 40,000 lines per square inch; and in the pole fringe, where 10 per cent of the conductors lie, the average density is 20,000 lines per square inch. Find the generated emf, the armature resistance at 75 C, and the terminal voltage at full load, allowing 600 cir mils per ampere in the armature conductors and 2 volts for total brush drop.

6–9. A 6-pole generator has a multiple-circuit winding and rotates at 720 rpm. There are 216 coils of 1 turn each, the conductor being 0.15×0.50 inch in cross-section. The total length per turn is 125 inches, of which 35 inches are active. The armature diameter is 33 inches. Sixty per cent of the conductors lie under the poles where the density is 50,000 lines per square inch, and 10 per cent lie in the pole fringe in an average density of 30,000 lines per square inch. Find generated emf, armature resistance at 75 C, and the full-load terminal voltage, allowing 700 cir mils per ampere in the armature conductors, and 2 volts for brush drop.

6–10. An 8-pole generator has a multiple-circuit winding. There are 264 coils of 4 turns per coil, the conductor being 30,000 cir mils in cross-section. The total length

per turn is 44 inches, of which 18 inches are active in cutting flux. The armature is 40 inches in diameter and turns at 250 rpm. The flux density directly under the pole, where 65 per cent of the conductors lie, is 9500 lines per sq cm, and in the pole fringe, where 10 per cent of the conductors lie, is 4400 lines per sq cm. Find the generated emf, the armature resistance at 90 C, and the terminal voltage at full load, 90 C, allowing 2 volts for total brush drop and 700 cir mils per ampere in the armature conductors.

6–11. A 6-pole generator has 240 conductors arranged in a multiple-circuit winding, and the flux per pole is 8×10^6 lines. The conductor used is 0.165 square inch in cross-section, and the machine rotates at 900 rpm. What voltage is being generated? What current may the armature deliver if 725 cir mils per ampere are allowed in the armature conductors? If the total length of turn is 80 inches and brush drop is 2.5 volts, what is the armature resistance at 75 C? What is the full-load terminal voltage?

6–12. The series-wound armature of a 4-pole generator rotates at 600 rpm. There are 270 conductors, and the flux per pole is 5×10^6 lines. The conductor is 0.05 square inch in cross-section, and the length per turn is 57 inches. What is the full-load terminal voltage at 75 C, allowing 500 cir mils per ampere of armature conductor and 3 volts brush drop?

6–13. A 4-pole, 1500-rpm generator has 57 slots with 6 inductors per slot, arranged in a series winding. Each conductor has a cross-section of 45,000 cir mils. The total length per turn is 36 inches, of which 15 inches are active in cutting flux. The armature diameter is 15 inches. Under the poles, where 62.5 per cent of the conductors lie, the flux density is 50,000 lines per square inch. In the pole fringe, where 10 per cent of the conductors lie, the average flux density is 30,000 lines per square inch. If 600 cir mils are allowed per ampere of conductor current, and 2 volts for total brush drop, find (*a*) the generated voltage, (*b*) armature resistance at 90 C, and (*c*) the full-load terminal voltage.

6–14. A 4-pole, 1200-rpm generator has 41 slots with 6 conductors per slot, arranged in a series winding. Each conductor is of 63,000 cir mils cross-section. The total length per turn is 44 inches, of which 17.5 inches are active. The diameter of the armature is 14 inches. Of the conductors, 67.5 per cent lie in a flux density of 35,000 lines per square inch and 5 per cent lie in the pole fringe, where the average density is 20,000 lines per square inch. Allow 550 cir mils per ampere in the armature conductors and 2 volts total brush drop. Find generated voltage, armature resistance at 80 C, and the terminal voltage. (A wave winding may have a fractional number of coils per circuit so far as this problem is concerned.)

6–15. The commutator of a bipolar generator is 5 inches in diameter, has 72 bars, and a speed of 1800 rpm. The armature delivers a total current of 50 amperes, the brushes are 0.5 inch thick, and the thickness of the insulation between bars is negligible. What is the average rate of change of current during commutation?

6–16. The 264-bar commutator of an 8-pole generator with a multiple-circuit winding is 24 inches in diameter and its speed is 250 rpm. What is the average rate of change of current during commutation when the generator delivers 376 amperes, the brushes being $\frac{5}{8}$ inch thick, and the thickness of insulation between bars being negligible?

6–17. What is the average rate of current change during commutation in a coil of a 6-pole generator with 162 commutator bars? The armature speed is 750 rpm, the brushes are $\frac{7}{8}$ inch thick, the thickness of insulation between bars is 0.03 inch, and the diameter of the commutator is 18 inches. The armature has a multiple-circuit winding and full-load current is 800 amperes.

6–18. A 6-pole, 400-rpm generator with a multiple-circuit armature winding has a commutator 15 inches in diameter with 180 bars. The brushes are $\frac{5}{8}$ inch thick, and the insulation between bars is 0.025 inch thick. If the full-load armature current is 360

amperes, what is the average rate of change of current in an armature coil during commutation?

6–19. A commutator turning 1750 rpm has 88 bars and its radius is 3.5 inches. Brush width is $\frac{3}{8}$ inch. Thickness of insulation between bars is 0.025 inch. Self-inductance per coil is 0.16 millihenry. Machine has a 4-pole, multiple-circuit winding and is delivering 150 amperes to its load. What is the average emf of self-induction in the coil undergoing commutation?

6–20. A commutator 8 inches in diameter with 118 bars turns 1350 rpm. The generator has 4 poles with a series armature winding and delivers 120 amperes at full load. The self-inductance per armature coil is 0.18 millihenry, the brush thickness is 0.625 inch, and the insulation between bars is 0.025 inch thick. What is the average voltage of self-induction in a coil during commutation?

6–21. The rate of change of current in a full-pitch coil of 6 turns during commutation is 24,000 amperes per second, and there are as many commutating poles as main poles. The self-inductance of the coil is 0.0008 henry. The dimension of the commutating pole parallel to the shaft is 7.5 inches and the peripheral speed of the armature is 3600 feet per minute. (*a*) What is the voltage of self-induction during commutation? (*b*) How much voltage per conductor must be induced in the coil under each commutating pole? (*c*) What must be the flux density under the commutating pole?

6–22. What must be the flux density under the commutating poles if the rate of change of current in a full-pitch coil of 1 turn during commutation is 55×10^4 amperes per second and there are as many commutating poles as main poles? The self-inductance of the coil is 0.012 millihenry, the peripheral speed of the armature is 6800 feet per minute, and the width of the commutating pole (parallel to the shaft) is 15.5 inches.

6–23. A 5-kw, 250-volt, bipolar generator has 800 armature conductors arranged so that the coil-sides span 180 electrical degrees. (*a*) How many ampere-turns per pole are there when full-load armature current is flowing? If the brushes are shifted (*b*) 6 electrical degrees and (*c*) 12 electrical degrees, how many distorting and demagnetizing ampere-turns are there per pole?

6–24. A bipolar, 5-kw, 125-volt generator has 300 armature conductors, the coil-sides spanning 180 degrees. If the brushes are shifted (*a*) 5 degrees and (*b*) 10 degrees, how many demagnetizing and distorting ampere-turns are there per pole?

6–25. In a 4-pole, 25-kw, 250-volt generator there are 372 conductors on the armature arranged as a 4-path, full-pitch winding. If the brushes are shifted (*a*) 10 electrical degrees and (*b*) 20 electrical degrees, calculate the cross-magnetizing and demagnetizing ampere-turns per pair of poles. If the machine had had a 2-circuit armature with the same current density in the conductors, what would be the answers? What would be the new rating of the machine?

6–26. In an 8-pole, 100-kw, 250-volt generator there are 1600 armature conductors arranged as a multiple-circuit, full-pitch winding. (*a*) What is the current per conductor and how many total ampere-turns are there when the armature carries rated current? If the brushes are shifted (*b*) 10 electrical degrees and (*c*) 20 electrical degrees, how many demagnetizing and cross-magnetizing ampere-turns are there per pole? If the armature was rearranged as a series winding with the same current density in the conductors, what would be the answers? What would be the rating of the machine?

6–27. A 6-pole, 750-kw, 575-volt generator has 432 armature conductors arranged as a full-pitch, multiple-circuit winding. How many demagnetizing and distorting ampere-turns will there be per pole, if the brushes are shifted (*a*) 7.5 electrical degrees and (*b*) 15 electrical degrees? If the armature winding were rearranged as a two-circuit winding, what would be the answers and what would be the rating of the machine?

6–28. A 100-kw, 250-volt, 8-pole generator has an armature periphery of 360 cm.

The width of each pole face is 35 cm, and there 161 slots with two conductors in each, all conductors connected as a two-path winding. How many cross-magnetizing and demagnetizing ampere-turns are there per pair of poles, if the brushes are advanced to be opposite the pole tips? How many, if the same conductors are reconnected as an 8-path, full-pitch winding, the current density in the conductors to be the same as before?

6–29. To generate 20 volts, a certain 110-volt shunt generator requires (at rated speed) a field current of 0.172 ampere. If the shunt-field circuit has a resistance of 135 ohms, will the machine build up? If the speed were increased 15 per cent above normal, what would be the answer?

6–30. In a certain generator the resistance of the field circuit is 400 ohms. The magnetization curve shows that to generate 60 volts, the required field current is 0.160 ampere. (*a*) Will the machine build up? (*b*) If not, would it build up if the speed were raised 10 per cent? (*c*) With the speed as in (*a*), below what critical value must the field-circuit resistance be reduced, so that the machine will build up?

6–31. The magnetization curve of a generator shows that to generate 30 volts the required field current is 0.18 ampere. To what critical value must the field-circuit resistance be reduced to make the generator build up? If the field-circuit resistance were 200 ohms, by what percentage would the speed have to be raised to cause the generator to build up?

6–32. An automobile generator with 6 volts terminal voltage is charging its storage battery at the rate of 10 amperes. Battery resistance is 0.02 ohm, and generator resistance is 0.24 ohm. Two 6-volt tungsten head-lights, each of 20 watts rating (2400 C), are switched on to the circuit. What are the battery current and generator current the instant after the switch is closed? (Note that lamps are about 20 C when switched on. Consider ρ to remain constant to 2400 C.)

6–33. In designing a generator it was specified that there are to be 15,000 feet of No. 19 wire (at 20 C) in the shunt-field winding. The calculated magnetization curve of the machine shows that, to generate 34 volts, a field current of 0.1667 ampere is necessary, and to generate 110 volts, at no load, the required field current is 0.6667 ampere. (*a*) Was the machine properly designed? (*b*) If so, how much resistance may be added to the field, as a field rheostat, and still allow the machine to build up? (*c*) How much resistance would be needed in the field rheostat to adjust the no-load voltage to 110 volts? (*d*) If No. 22 wire had been used in error to wind the fields, what would have been the result?

6–34. The magnetization curve of a generator shows that to generate 63 volts, a field current of 0.200 ampere is necessary, and to generate 220 volts at no load, 0.800 ampere is necessary. The field is wound with 20,000 feet of No. 20 wire (at 20 C). How much resistance may be added to the field and still permit the generator to build up? How much added resistance will be needed at 220 volts? If No. 22 wire had been used to wind the field, would it have been satisfactory?

6–35. The shunt-field current of a certain 110-volt, 25-kw shunt generator has to be increased from 3.6 amperes to 4.1 amperes, to give flat compounding. There are 1300 turns on each field pole. If the machine is to be equipped with a series field for flat-compounding across the armature, how many turns per pole are required?

6–36. In a certain 100-kw, 125-volt shunt generator the field current must be increased from 17.8 to 22.0 amperes to give rated voltage across the armature from no load to full load. If there are 900 turns per pole in the shunt-field winding, how many series-field turns are necessary to obtain the same no-load and full-load voltages across the armature as a compound generator?

6–37. In the shunt field of a 100-kw, 250-volt shunt generator there are 1500 turns per pole. The shunt-field current at no load at 250 volts is 9.1 amperes, and at full

load at 250 volts it is 11.2 amperes. How many series-field turns are needed to convert this machine into a compound generator with 250 volts across the armature at no load and at full load?

6–38. A 25-kw. shunt generator, with 2250 turns per pole in the shunt-field winding, requires 1.21 amperes shunt-field current to generate 250 volts at no load and 1.77 amperes to furnish 275 volts across the armature with full-load current. If the machine is to be converted into a compound generator to furnish the same no-load and full-load voltages across the armature, what is the minimum number of series-field turns per pole required?

6–39. In a certain 50-kw shunt generator, a shunt-field current of 5.6 amperes is required to generate 110 volts at no load and 7.8 amperes to furnish 125 volts across the armature at full load. If there are 1200 turns per pole in the shunt-field winding, how many series turns per pole are required to convert the machine into a compound generator with the same no-load and full-load voltages across the armature?

6–40. At no load at 220 volts the shunt-field current of a 250-kw shunt generator is 16.1 amperes, and at full load at 260 volts it is 20.0 amperes. If there are 1500 turns per pole in the shunt field, how many series turns are needed to convert this machine into a compound generator to supply the same no-load and full-load armature voltages?

6–41. If the machine of problem 6–38 is wound with 25 series turns per pole, how much current must flow through them? If the resistance of the series field is 0.0042 ohm (hot), what must be the resistance of the diverter?

6–42. A certain 4-pole, 100-kw, 250-volt generator requires 3600 series-field ampere-turns per pole at full load. There are 15 turns per pole of 120,000 cir mils conductor in the series field, and the average length per turn is 40 inches. With the temperature of the series field at 90 C, what must be the resistance of the proper diverter?

6–43. The number of series ampere-turns per pole required at full load for a certain 4-pole compound generator is 1250. There are $4\frac{1}{2}$ turns per pole, of 100,000 circular mils cross-section, the average length of a turn being 34 inches. If the rating of the machine is 110 volts, 750 amperes, what must be the resistance of the series-field diverter, if the temperature of the field coils at full load is 95 C?

6–44. What would be the resistance of the proper diverter for a 4-pole, 50-kw, 250-volt compound generator if it requires 1800 series-field ampere-turns at full load? The average length of a turn is 30 inches, the conductor being 60,000 cir mils in cross-section, with 15 turns per pole. The temperature of the field is 90 C.

6–45. A series-field diverter takes 70 per cent of the total current and the series field 30 per cent at 15 C. If the diverter is of manganin, what will be the division at 90 C? Consider that the temperature-resistance coefficient of manganin is zero.

6–46. The diverter of a compound generator takes 50 per cent of the output current and the series field 50 per cent when the temperature of the field is 95 C. What will be the current division when the field is at 20 C? Consider that the diverter is of manganin and that its temperature-resistance coefficient is zero.

6–47. A flat-compounded generator running at 900 rpm develops a no-load voltage of 230 volts. If a load current of 1000 amperes is suddenly required, how much torque opposing rotation will the generator armature momentarily develop?

6–48. A 250-volt generator is flat-compounded at 1000 rpm. How will the compounding be affected if the no-load voltage were adjusted to be (*a*) 220 volts and (*b*) 280 volts, the speed being 1000 rpm? Explain.

6–49. A 250-volt generator is flat-compounded at 1000 rpm. If it were driven (*a*) at 1200 rpm and (*b*) at 800 rpm, how will the compounding be affected if the no-load voltage at each speed is adjusted to be 250 volts? Explain.

6–50. Two separately excited generators are to be operated in parallel. Machine *A*

has such excitation that at no load it generates 120 volts, and machine B is excited to generate 128 volts at no load. The armature resistance of each machine is 0.01 ohm. When thrown in parallel with no load on the bus-bars, what value of current will flow, and what will be the terminal or bus-bar voltage? If a load of 800 amperes is then put upon the bus-bars, what current will each machine supply and what will be the bus-bar voltage? (Disregard brush drop and any effects due to armature reaction.) If the load is increased to 1000 amperes, what will be the machine currents and the bus-bar voltage?

6–51. Before being connected in parallel, and with no load on the bus-bars, the field excitation of generator 1 is such as to generate 230 volts, and that of No. 2, to generate 260 volts. If the armature resistance of each machine is 0.06 ohm and brush drop is 2 volts, what value of current will flow and what will be the bus-bar voltage when the machines are connected in parallel? (Neglect any effects due to armature reaction.) If a load of 600 amperes is then put upon the system, how much current will each machine deliver and what will be the bus-bar voltage?

6–52. Two equally compounded generators of 50-kw and 150-kw rating are to be operated in parallel. The series-field circuit resistance of the 50-kw machine is 0.002 ohm. What must be the resistance of the series-field circuit of the other? If the actual resistance of the field winding is 0.0015 ohm, what must be the resistance of its diverter?

6–53. Two flat-compounded 275-volt generators of 500 and 750 kw are to be operated in parallel. If the resistance of the series-field circuit of the smaller machine is 0.00020 ohm, what must be that of the larger?

6–54. A 4-pole compound generator has a terminal voltage of 125 at no-load and is to have a terminal voltage of 130 when the load current is 100 amperes. The resistance of the armature is 0.040 ohm, of the series field 0.006 ohm, and of the shunt field 35 ohms. The shunt field has 600 turns per pole and is connected short shunt. The armature has a full-pitch, multiple-circuit winding with 144 conductors. The brushes are set ahead 10° (electrical). Calculate the series-field turns if the machine is driven at 1480 rpm at no load, and at 1440 rpm with the 100-ampere load on the generator. Assume that the flux is directly proportional to the field ampere-turns.

6–55. A 10-kw compound generator is to be driven by a gas engine which runs at 1480 rpm at no load and 1425 rpm with full load on the generator. The no-load voltage is 230 when the fields are connected long shunt and 1.3 amperes are in the field. If each pole has 780 turns in the shunt field, the armature resistance is 0.102 ohm, and the series field resistance is 0.057 ohm, calculate the percent change in flux required to have the full-load voltage equal to 230 volts. How many turns are necessary in the series field to supply this change?

CHAPTER VII

THE DIRECT-CURRENT MOTOR

1. Relation between Generator and Motor. As mentioned in Chapter V the d-c generator and the d-c motor are nearly identical in construction. The construction which is best for one is generally best for the other; a machine which runs well as a generator will generally operate satisfactorily as a motor.

Although the construction of the two is practically the same, they differ quite materially in operation. The generator is supplied with mechanical power by a prime mover, being driven at a speed as nearly constant as possible. The motor, supplied with electrical power, is itself a prime mover; its speed is generally not constant as the load is varied.

The function of a generator is to *generate voltage* by moving conductors through a magnetic field, while that of a motor is to produce a *turning effort*, or *torque*.

2. Torque Acting in a Generator. When a machine is operating as a generator, and its armature is carrying current, it, too, develops a torque (because the armature consists of conductors carrying current, in a magnetic field). This torque *opposes* the motion of the armature and must be overcome by the prime mover, in order to keep the armature of the generator operating at constant speed. The current in the armature flows *with* the emf generated in the moving armature. We may say, therefore, that so far as mechanical power is concerned the generator is absorbing energy, and so far as electrical power is concerned the generator is *giving out* energy.

3. Torque Acting in a Motor. In the case of a motor, the armature turns in the same direction as it is urged by the force acting on the armature conductors; the mechanical power must therefore be positive, or *output*. When the armature revolves it must generate an emf (conductors moving in a magnetic field generate a voltage), and the direction of this emf will be opposite to the direction of current flow; the emf generated in a motor armature is therefore called a *counter emf*. This counter emf opposes the flow of current in the armature; the applied voltage must overcome this counter emf to cause the current to flow. As the current flows against the armature emf, the electrical power of a motor must be negative, i.e., *input*.

4. Torque of a Motor. A motor develops torque, because on the periphery of its armature are placed conductors, through which current flows, and these conductors lying under the pole faces are in a magnetic field. These conductors are then acted upon by a force which tends to move them in a direction perpendicular to the magnetic field and to their length. In d-c machines the radial length of the air gap is made uniform under the entire face of the poles, so that the lines of force enter the armature radially. The force exerted by the conductors is therefore tangential to the periphery of the armature.

The fundamental expression used in calculating the torque is that of Eq. 5, page 29. It stated that, *if a conductor l centimeters in length lies in a uniform magnetic field of a density of B lines per square centimeter (direction of conductor being perpendicular to field) and carries a current of I amperes, the conductor is acted upon by a force which tends to move it in a direction perpendicular to its length and to the direction of the magnetic field; the magnitude of this force is given by the equation*

$$F = \frac{BlI}{10} \text{ dynes} \tag{1}$$

If we wish to express B in lines per square inch, l in inches, I in amperes, and F in pounds, since there are 2.205 pounds and 981,000 dynes in a kilogram, we have

$$F = 2.54l \times \frac{B}{2.54^2} \times \frac{I}{10} \times \frac{2.205}{981{,}000} = 0.885BlI \times 10^{-7} \text{ pound} \tag{2}$$

Suppose that a conductor 10 inches long lies in a field of 60,000 lines per square inch and carries a current of 100 amperes. The force on the conductor in pounds is evidently

$$F = 60{,}000 \times 100 \times 10 \times 0.885 \times 10^{-7} = 5.31 \text{ pounds}$$

If we desire the force in dynes, we have

$$F = \left(\frac{60{,}000}{2.54^2}\right) \times (10 \times 2.54) \times \frac{100}{10} = 2{,}360{,}000 \text{ dynes}$$

5. Direction of Torque the Same for All the Armature Conductors. In Fig. 7–1 is shown the section of a four-pole motor. The conductors marked with a cross are carrying current into the paper; those marked with a dot are carrying current out from the paper. The conductors under the north poles carry current into the paper and lie in flux which is directed toward the center of the armature; they therefore exert a torque which tends to rotate the armature in counter-clockwise direction. The conductors under the south poles carry current out from the paper, lie in a field which is directed radially out from the center, and so exert a torque which also

acts counter-clockwise on the armature. Thus all conductors act to produce torque in the same direction.

It is also to be seen that the torque of a d-c motor is a constant or steady one, not pulsating as in the case of steam, gasoline, or oil engines, so that a flywheel is not required with an electric motor.

6. Direction of Rotation. It is evident that the direction of rotation will be determined by the direction of the force on a conductor (see Fig. 7–1). This direction can be reversed by changing either (but not both) the direction of the field or of the armature current.

It is evident also that with a certain direction of rotation, and a certain direction of field, the current will flow in one direction in a motor and in the opposite direction in a generator.

All of this may be summarized by considering the four quantities: function, as a motor or generator (usually determined by the use of the machine); direction of rotation; direction of armature current; and direction of field. From the above it is evident that these quantities *may be changed only in pairs.* Thus if a shunt machine is to be reversed in direction of rotation (one change), it must have its field reversed (a second change) *or* have its armature current direction changed (a second change). This rule is quite useful when considering what changes take place when the machine is used in a changed application.

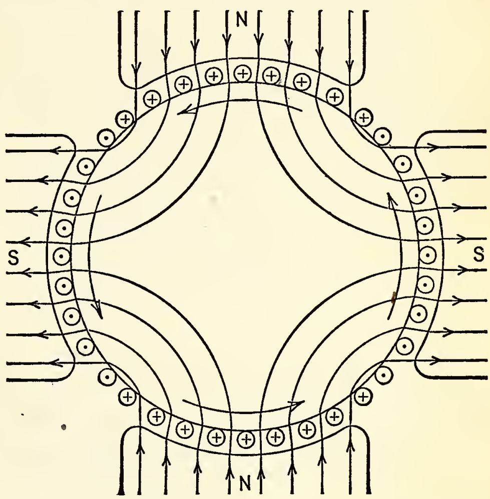


Fig. 7–1. All the conductors on the periphery of a motor give torque in the same direction. This is due to the fact that, by the scheme of connecting the conductors together, the currents in conductors under poles of opposite polarity flow in opposite directions.

7. Torque Calculation. To calculate the magnitude of this torque, or turning effort of a motor, we must know the number of conductors lying in the magnetic field, the active length of each conductor (i.e., length of conductor under the pole face), strength of field in the air gap where the conductors are situated, and the current carried by the conductors. By the use of Eq. (1) the force in dynes on one conductor may be calculated, and this value, multiplied by the number of conductors situated in the magnetic field, gives the total force acting on the periphery of the armature.

It is to be noticed that those conductors lying in the interpolar space

produce no turning effort; they are *inactive* conductors. When calculating the emf of a generator it was pointed out that these same conductors were inactive in the production of voltage in the armature; in fact, *that portion of the armature winding which generates an emf when the machine is operated as a generator serves to produce torque when the machine is operating as a motor.* In calculating the emf of a generator it was found advisable to divide the active conductors into two parts, those lying directly under the pole face, where the field has its maximum intensity, and those lying in the pole fringe, where the field is weaker and non-uniform. The same division of conductors is advisable when calculating the torque of a motor.

Suppose that it is desired to calculate the peripheral force on the armature of Fig. 7–1. Assume that the length of the pole face, parallel to the shaft, is 15 cm, that the conductors lie in a field of uniform density of 8000 lines per sq cm, that there are 200 conductors, of which 60 per cent lie under a pole face, and that the current in each conductor is 20 amperes.

The force per conductor is given by

$$F = \frac{BlI}{10} = \frac{8000 \times 15 \times 20}{10} = 24 \times 10^4 \text{ dynes} = 0.245 \text{ kg}$$

Since there are 200 × 60 per cent = 120 active conductors, the peripheral force on the armature is 0.245 × 120 = 29.4 kg; and, if the radius of the armature is 10 cm, the torque developed is 2.94 kilogram-meters.

8. Calculation of Horsepower of a Motor. If the peripheral speed of the armature is known, the output of the motor in foot-pounds per minute, or horsepower, is readily determined. If a body is exerting a force F, and moving in the direction in which this force is acting with a velocity v, then the rate of doing work is equal to Fv. Hence, if we multiply the peripheral pull on the armature by the velocity of the armature periphery, the product obtained is equal to the amount of power which the motor is giving.

Consider a lap-wound armature 4 feet in diameter, having 12 poles. The winding consists of 240 coils of 4 turns each and the length of the pole face is 10 inches. Sixty per cent of the conductors lie under the pole face where the flux density is 60,000 lines per square inch, and 15 per cent lie in the pole fringe where the average density is 35,000 lines per square inch. It is desired to know the horsepower the motor develops if the current flowing into the armature is 480 amperes, and the machine is rotating at 200 rpm.

There are two things to find—the peripheral pull in pounds and the peripheral velocity in feet per minute. The product of these two quantities divided by 33,000 (the number of foot-pounds per minute in 1 hp) will give the horsepower of the motor.

As the armature is lap-wound and the machine has 12 poles, the winding must have 12 paths. Therefore the *current per path* = 480/12 = 40

amperes, and this is the current in each conductor. The active length of each conductor is 10 inches. The total number of conductors = $240 \times 4 \times 2 = 1920$. Of these, 60 per cent (i.e., 1152) lie in a field of 60,000 lines per square inch and 15 per cent (i.e., 288) lie in a field of 35,000 lines per square inch.

The force is therefore equal to

$$0.885 \times 10^{-7} \times 40\,[(1152 \times 10 \times 60{,}000) + (288 \times 10 \times 35{,}000)] = 2800 \text{ pounds}$$

The peripheral speed = $4 \times \pi \times 200 = 2515$ feet per minute

The horsepower developed = $2515 \times 2800/33{,}000 = 213.4$

The general expression for the horsepower developed by a motor may be written

$$\boxed{\mathrm{HP} = \frac{2\pi FLn}{33{,}000} = \frac{FLn}{5250}} \tag{3}$$

where F is the force exerted at the periphery of the armature (or pulley) in pounds, L is the radius of the armature (or pulley) in feet, and n is the rpm.

The derivation of this equation is obvious when we consider that the product $2\pi Ln$ is the peripheral velocity of the armature (or pulley). The product FL is the torque exerted by the motor; representing this by T, we may rewrite Eq. (3) and obtain the useful equation

$$\mathrm{HP} = \frac{Tn}{5250} \tag{4}$$

9. Further Torque Calculations. The procedure in calculating the torque of a motor in terms of the force per active inductor (section 7) was based upon Eq. (2),

$$F = 0.885BlI \times 10^{-7}$$

where F = the peripheral force of a conductor in the field in pounds;
B = flux density in lines per square inch;
l = length of conductor lying in the field, in inches;
I = current per conductor, in amperes = I_a/m;
I_a = armature current;
m = number of parallel paths in the armature winding.

With Z total conductors upon the armature, of which a fraction, P, are active, the total active conductors are PZ and the total peripheral pull of the armature is

$$F_T = \frac{0.885BlI_aPZ \times 10^{-7}}{m} \text{ pounds} \tag{5}$$

With an armature diameter D, in feet, the torque is

$$T = \frac{0.885BlI_aPZD \times 10^{-7}}{2m} \text{ pound-feet} \tag{6}$$

The percentage of active inductors upon the armature also represents the percentage of the total armature periphery covered by the poles. The periphery of the armature is $12\pi D$ inches and the portion covered by the poles is $12\pi DP$ inches. Since the active length of a conductor, l, represents the length of a pole face parallel to the shaft, the total armature area covered by the poles is $12\pi DPl$ and this is also the area of all the pole faces.

With a flux density B lines per square inch, the total flux entering and leaving the armature is $12\pi DPlB$. With p poles and a total flux per pole, Φ, the total flux entering and leaving the armature is also $p\Phi$, so that

$$12\pi DPlB = p\Phi$$

The torque then is

$$T = \frac{0.885I_aZp\Phi \times 10^{-7}}{2\pi \times 12m} \tag{7}$$

$$= \frac{0.1174I_aZp\Phi \times 10^{-8}}{m} \text{ pound-feet} \tag{8}$$

It is to be noted that in the last expression the torque is not expressed in terms of the radius of the armature.

If all quantities are held constant while the armature radius is increased, the flux density, and therefore the peripheral force of each conductor, will decrease inversely, leaving the torque constant.

Since

$$\text{HP} = \frac{2\pi Tn}{33{,}000}$$

or, substituting the value for T from Eq. (7),

$$\text{HP} = \frac{2\pi \times 0.885I_aZp\Phi n \times 10^{-7}}{2\pi \times 12 \times 33{,}000m}$$

$$= \frac{0.2235I_aZp\Phi n \times 10^{-12}}{m} \tag{9}$$

10. Necessity of Commutator in a Motor. We saw in the case of the generator that the function of the commutator was to rectify the alternating emf's induced in the several coils so that the voltage impressed on the external circuit was uni-directional. It will be seen from Fig. 7-1 that, if in a motor the torque exerted by the conductors on the armature is to be always in the same direction, the current in any one conductor must be

reversed as the conductor passes from one pole to another. A d-c motor is supplied with uni-directional current, and the commutator must therefore change this uni-directional current so as to make it an alternating one in each conductor.

11. Commutation in a Motor. In Fig. 7–2, coil X of a motor armature winding is being commutated. The active portion of any coil of the winding shown is the part at right angles to the paper, on the outside of the ring armature core. In order that the forces exerted by the conductors under the north pole may be to the right, current in the conductors on the periphery of the armature must flow out from the paper (Fleming's left-

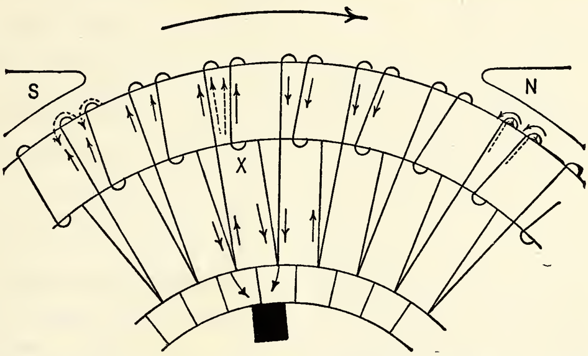


FIG. 7–2. While the coil of a motor armature is short-circuited by a brush, its current must reverse, as shown by the solid arrows. The voltage of self-induction of the coil, acting in the direction of the dashed arrow in coil X, tends to prevent the reversal of the current. The direction of the generated emf in the coils under the poles is shown by the dotted arrows. To overcome the voltage of self-induction, the brush must be moved backwards to bring the short-circuited coil in the fringe of the pole that the coil is just leaving.

hand rule); and, under the south pole, it must flow into the paper. The brush of the motor shown is therefore a negative one.

Coil X is shown in such position that the current in it is beginning to decrease; from what was said on the subject of commutation in the previous chapter, it is evident that the counter voltage of self-induction, induced in the coil as its current reverses, will be in the same direction as the decaying current, as indicated by the dashed arrows within coil X. In a motor the direction of the current in a conductor is opposite to that of the voltage induced by the motion of the conductor in the field. A commutation emf to be opposite to this counter voltage of self-induction, must, by Fleming's right-hand rule, be induced under the south pole, so that, to get the proper condition for sparkless commutation, the brush must be moved backward to such a position that the coil being commutated is moving under the fringe of flux from the south pole. The direction of the voltage

induced in the coils of the armature by their motion in the magnetic field is indicated by the dotted arrows under the south pole.

Commutation flux, in a motor without commutating poles, must therefore be that of the pole which a coil is leaving. If this is applied to the motor with commutating poles, it follows that the polarity of a commutating pole must be the same as that of the main pole just preceding it in the direction of rotation. Figures 6–39 and 6–40 can be made to apply to the commutating-pole motor by reversing the direction of rotation from that shown there for a generator.

12. Effect of Armature Reaction in a Motor. In a generator, the armature current *flows in the direction of the induced emf*, and, in the motor, the armature current *flows against the induced emf* (called the counter emf). We should, therefore, expect the armature mmf of the motor and generator to be in opposite directions, and the main field in the motor to be twisted *against the direction of rotation.*

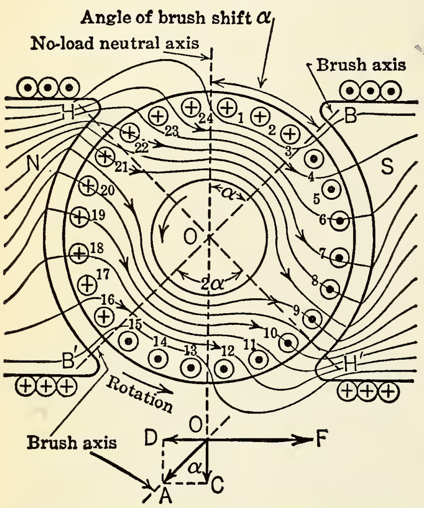


FIG. 7–3. When a motor armature carries current and the brushes are shifted backwards for purposes of commutation, the armature mmf distorts and weakens the field to give a flux distribution about as shown here. This figure corresponds to Fig. 6–27 for a generator.

This may be seen from Fig. 7–3, which represents conditions in a motor, just as Fig. 6–27 represented them for a generator. As in the latter figure, the conductors in the angle *BOH*, combined with those in the angle *B′OH′*, make up the demagnetizing turns. The conductors included in the angle *HOB′*, combined with those in the angle *H′OB*, constitute the cross-magnetizing turns. The main field mmf is shown vectorially by *OF*, and the armature mmf by *OA*; the armature mmf is resolved into its components, the demagnetizing mmf *OD* and the cross-magnetizing mmf *OC*.

Comparison of Figs. 6–27 and 7–3 will show that armature reaction in a motor produces the same effects as in the generator; the main field is both weakened and distorted, the direction of distortion, however, being opposite to the direction of rotation in the motor, whereas it was found to be in the direction of rotation in the generator.

13. The Armature of a Motor as a Rotating Body. To rotate a body requires the application of a driving torque, or turning effort. A resisting torque, due to the friction of the bearings, will always be present in addi-

tion, and will be augmented by the resisting torque due to the load if the rotating body is doing work.

Now, if the driving and resisting torques are exactly equal, the angular speed of the rotating body will remain exactly constant; but if one becomes greater than the other, a change in speed must take place. Such a change in speed will continue as long as the inequality between driving and resisting torques exists. Thus, if a rotating body with constant driving torque has its resisting torque increased so as to be greater than its driving torque, the speed will decrease steadily until the body stops. To start it again, the driving torque must be made greater than the resisting torque; the body will then continue to accelerate.

Consider that a line of shafting, driven by a steam engine, is to be started. Steam is turned on, and, if the steam admitted is sufficient to give the engine enough torque, the shafting starts to rotate; the driving torque applied to the shafting is thus somewhat greater than the resisting torque due to friction of the bearings. If the torque of the steam engine remained constant after the shafting had started to rotate, and if the bearing friction remained constant, the speed of the shafting would gradually increase indefinitely; but as friction in practice generally increases with the speed, the speed finally reaches some definite value. At this point, the resisting torque is just equal to the driving torque. If the speed is to be further increased, or load put on the shafting, the driving torque must be increased by admitting more steam into the engine.

The armature of a motor, being a rotating body, is subject to the same laws. The driving torque of a motor is produced by the forces exerted between the armature current-carrying conductors and the field; its resisting torque will be due to such rotational losses as occur, together with that torque due to the load.

It will be shown that the electric motor is a self-regulating machine. Whenever the resisting torque applied to the motor is changed, a change in the speed will, in general, occur, and this change in speed will automatically bring about reactions within the motor armature which will cause the driving torque again to become equal to the resisting torque.

In considering how a given type of motor will act under load, the student is advised always to begin his reasoning with the ideas on rotating bodies presented above, and then apply whatever conditions are imposed upon the motor.

14. Expression for the Torque of a Motor. In Eq. (8) it was shown that the torque produced by a motor armature *depends only* upon the armature current, the flux per pole, and certain constants of the machine. Representing all the constant terms by K', we may express the torque of a given machine by

$$T = K'\Phi I_a \tag{10}$$

where T = the torque of the motor;
Φ = the flux per pole entering the armature;
I_a = the armature current;
and $K' = 0.1174Zp \times 10^{-8}/m$, all the terms of which are constants for a given machine.

15. Counter EMF of a Motor. It has been shown that, in the case of the motor, the armature turns in the direction in which it is urged by the force acting on the armature conductors. With the field of a motor set up in a certain direction, the direction of the force acting on the armature conductors is dependent on the direction of the armature current, and this must be that in which the voltage of the line, to which the motor is connected, is acting; in a motor the impressed voltage and armature current are in the same direction.

The motor armature conductors, moving in a magnetic field, generate a counter emf, which, being opposite to the direction of current flow, is also opposite to that of the voltage impressed on the armature.

The generated emf of a generator was found in Eq. (2), page 235, to be

$$E_g = \frac{p\Phi nZ}{m \times 60 \times 10^8} \tag{11}$$

in which Z = total number of conductors on the armature;
p = number of field poles;
m = number of parallel paths in the armature winding;
Φ = total flux per pole, entering the armature;
n = revolutions per minute of the armature.

This equation, for a given machine, reduces to

$$E_g = K\Phi n \tag{12}$$

in which $K = \dfrac{pZ}{m \times 60 \times 10^8}$, all the terms of which are constants for a given machine.

This must also be the expression for the counter emf of a motor, since it is the generated emf of the armature. We have then

$$E_c = K\Phi n \tag{13}$$

where E_c = the counter emf of the motor.

16. Types of Direct-current Motors. The classification of motors is generally made according to the kind of field windings they have. The three types are the *shunt, series,* and *compound.* In the shunt-wound machine, the field consists of many turns of fine wire and is connected in parallel with the motor armature. The series motor has a field winding consisting of a few turns of heavy wire connected in series with the arma-

ture. The compound motor has two sets of field coils; one of many turns of fine wire in parallel with the armature, and another of a few turns of heavy wire in series with the armature.

The series winding of a compound motor may be connected so that it *assists* the shunt winding in magnetizing the field, or it may be connected so that the mmf's of the two windings *oppose* one another. The first is called a *cumulative-compound* motor and the second a *differential-compound*, or, as it is generally called, simply a *differential* motor; the latter type is of so little use and of so little practical importance that it is seldom met in practice. The term, compound motor, is therefore used to designate that type in which the series and shunt fields assist one another.

The characteristics of the three types of motors are given here, and will be explained more fully in the later paragraphs.

The shunt motor has a fair starting torque and nearly constant speed for all loads. It is used where the load requires practically constant speed and the starting torque demanded is not excessive.

The series motor operates through a wide range of speed as the load changes, and at very light loads the motor is likely to "run away," i.e., reach dangerous speeds. It gives very great starting torque and is therefore used where a heavy starting torque is demanded and the motor may be positively connected to its load. The principal application of this motor is in railway service.

The compound motor has characteristics between those of a shunt and a series motor and hence has a starting torque greater than that of the shunt motor but less than that of the series motor. It has, however, a definite upper speed limit, and even if all its load is removed it will not run at dangerous speeds. Its principal application is in machine-tool drive, etc., where a fixed speed limit is necessary and considerable starting torque is required. Compound motors are also much used in connection with flywheels and pulsating loads. The number of series turns used on the field coils varies somewhat, according to the service required of the motor; but, in general, we may say that, at full load, the series ampere-turns are 10 per cent to 50 per cent of the shunt ampere-turns. The decrease in speed from no load to full load may vary from 12 per cent to 50 per cent in different motors.

17. Shunt Motor. Current-torque Curve. The relation between the torque developed by a motor and the current flowing through its armature winding is shown by a current-torque curve. This curve has different forms in motors with different styles of field windings.

The simplest form is in the shunt motor. The field current of this machine is independent of the current flowing through the armature, because its field windings are connected directly across the supply line, the voltage of which is assumed constant. Now, if the field current of a motor

is constant, the strength of its magnetic field is practically constant. Hence, from Eq. (10), it is seen that the current-torque curve of a shunt motor must be a straight line, the torque being directly proportional to the armature current. This is shown in Fig. 7–4, in which the result of a laboratory test is given. The curves are practically straight lines, curve A being for a weak field and curve B for a strong field.

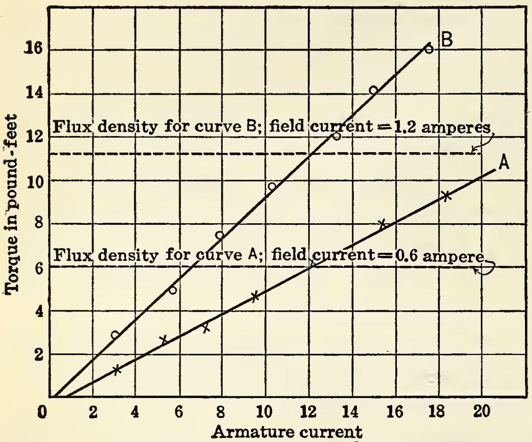


FIG. 7–4. Variation of torque of a shunt motor as the armature current is varied; it is shown for two values of flux density. Although one field current is twice as large as the other, the flux density, and hence torque, is not twice as great, because with 1.2 amperes in the field coils the magnetic circuit is approaching saturation.

The fact that the flux density did not increase proportionately with the field current indicates that the iron of the field circuit is approaching saturation.

18. Shunt Motor. Action under Load. Since the counter emf and the resistance reaction of the armature both act against the impressed emf of the motor, it may be written

$$E = E_c + I_aR_a \tag{14}$$

where E = the voltage of the line to which the motor armature is connected;
E_c = counter emf of the motor;
I_a = the armature current;
R_a = the resistance of the armature.

From the last equation we get

$$I_a = \frac{E - E_c}{R_a} \tag{15}$$

If the load on a shunt motor is increased, its driving torque (i.e., the torque developed by it) must increase; and, since the field flux of a shunt motor is approximately constant, it follows from Eq. (10) that the armature current must increase.

The only variable in the right-hand member of the expression for armature current, Eq. (15), is the counter emf; therefore, if the armature current is to increase, E_c must decrease, and vice versa. From the equa-

tion, $E_c = K\Phi n$, with the field flux constant, a decrease in E_c requires a decrease in speed.

Consider that a shunt motor is carrying a load torque T_1, with an armature current I_1, and running at a constant speed N_1, as represented in Fig. 7–5; the driving torque of the motor is hence also T_1. At time t_1, the load torque is suddenly increased from T_1 to a value T_2. The load, or resisting torque, being thus greater than the driving torque, it follows, from the analysis given above, that the speed of the motor must at once begin to fall, along some such line as AB. Since the field flux will remain

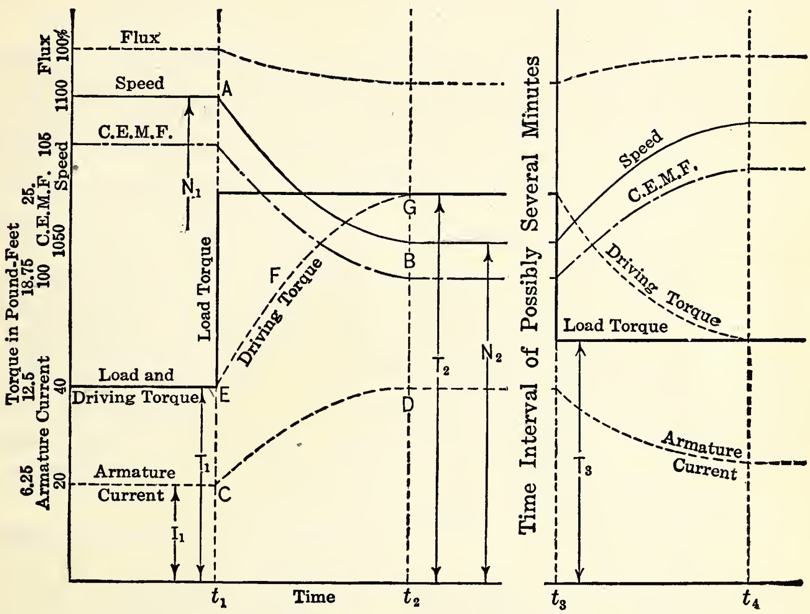


FIG. 7–5. These curves show the cycle of events when the load on a shunt motor is first increased and then decreased. With increased armature current the flux diminishes somewhat, due to increased demagnetization by the armature mmf.

practically constant, the counter emf will also fall, nearly in proportion to the speed, allowing the armature current to increase along some such line as CD; with increasing armature current and field flux practically constant, the driving torque increases along the line EFG. This action continues until, at time t_2, the driving torque is again equal to the load torque, and the speed continues at the constant value N_2.

After an interval of, say, several minutes at time t_3, the load torque drops to a value T_3, and at once the speed starts to increase, the driving torque being greater than the load torque at the instant after the load decreased. The rising speed increases the counter emf proportionally,

which in turn reduces the armature current, so that the driving torque becomes less. At time t_4, the driving torque has decreased sufficiently to be equal to the new load torque, and equilibrium is again reached.

19. Shunt Motor. Speed Regulation. Speed-load Curve. From what has just been said, it is evident that the shunt motor must have a definite no-load speed. At no load a relatively small armature current, sufficient to supply the core, friction and windage losses, is necessary to run the motor; its no-load counter emf, therefore, approaches within about 1 per cent of the value of the impressed voltage; obviously, the counter emf cannot become equal to the impressed voltage if the machine is to operate as a motor.

As the load increases, the speed drops somewhat, decreasing the counter emf and so permitting more armature current to flow. The amount of decrease in speed depends upon the armature resistance, as may be seen from Eq. (14); the greater the resistance the greater the decrease in speed.

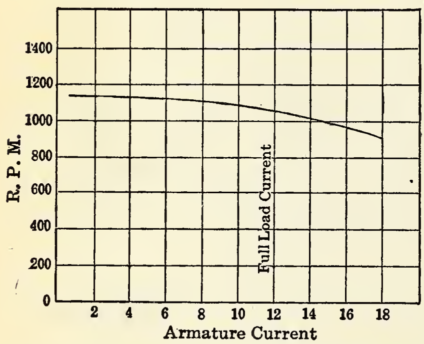


FIG. 7–6. Speed-load curve for a typical shunt motor.

The *speed regulation* of any motor is defined as the per cent rise in speed when full load is removed from the motor. It is thus the difference between no-load speed and full-load speed expressed as a percentage of *full-load* speed. The speed regulation of the average shunt motor would be between 5 per cent and 10 per cent.

The speed-load curve of a typical shunt motor is shown in Fig. 7–6.

20. Calculation of No-load Speed. The no-load speed of a compound or a shunt motor is easily calculated. At no load, there is practically no I_aR_a drop, so that the counter emf must be practically equal to the impressed emf. The no-load speed may therefore be found by calculating what speed the motor would have to run (as a generator) to generate in its armature an emf just equal to that of the supply line.

21. Armature Reaction and Resistance Act to Neutralize One Another. Although the shunt motor is generally considered as having constant flux for all loads, there is some reduction in its field strength with load, owing to armature reaction.

The effect of field weakening, brought about by armature reaction, tends, in general, to cause an increase in the speed. The question of field weakening will be considered in greater detail later on, but its general effect may be seen from the equation for the counter emf, $E_c = K\Phi n$. If the field is weakened, to generate a given counter emf, the motor speed

must increase, inasmuch as armature current is fixed by the torque that the motor must exert.

It appears then that the effect of field weakening by armature reaction could, in shunt motors not equipped with commutating poles, be made sufficient to overcome the effect of armature resistance drop. In that case the speed-load curve would be more nearly flat, giving no speed decrease with increase of load. With the brushes shifted back far enough, the weakening of the field caused by armature reaction could be made sufficient to make the motor *speed up* with increase of load.

In practice, shunt motors not equipped with commutating poles were never designed to operate with so much backward shift of the brushes, as to cause a rising speed-load characteristic, for several reasons. With a rising speed-load characteristic, the operation of a motor is unstable, whereas with a drooping characteristic, the operation is quite stable. To cause sufficient armature reaction to neutralize the I_aR_a drop would require a backward shift of the brushes so large as to cause sparking at the brushes. Since the speed of a shunt motor is affected by the temperature of the field winding (as will be discussed later), the speed-load characteristic of a shunt motor can never be made exactly constant. As the commercial loads driven by shunt motors do not require exactly constant speed, it is better to have a stable drooping characteristic.

In commutating-pole motors, in which the brushes are, in general, placed in the no-load neutral for purposes of commutation, any field weakening effects due to armature reaction will be brought about by the effects of the distortion of the field, as explained in Fig. 6–29. While a slightly drooping speed-load characteristic is preferred for shunt motors, the effect of the armature-circuit IR drop (the commutating-field winding drop being placed in series with the armature is now to be included) may be partly neutralized in another way. As will be discussed later, the brushes may be moved backwards a small amount, and the resultant action is to cause a slight speed rise. The amount of backward brush shift needed is, however, far too small to cause any demagnetizing ampere-turns, the speed increase being produced by another action under the commutating poles.

22. Series Motor. Current-torque Curve. The series motor gives a current-torque curve differing from that of the shunt motor, because the field strength of a series motor depends upon the current flowing through its armature, the field and armature being connected directly in series. The equation for torque involves the product of the field strength and armature current; when the field strength is directly proportional to the current through the field coils, the *torque must vary as the square of the current.*

This is the case with the series motor at light loads. At values of armature current near full load (and for all currents of higher value), the field is approaching saturation, and therefore the field strength is not pro-

portional to the current through the windings. At very high values of current, the strength of the field is practically independent of the current, and thus the current-torque curve tends to become a straight line, similar to that of a shunt motor. A representative torque curve is shown in Fig. 7–7. The magnetization curve of the machine is given for reference, and it is seen that as long as the magnetization curve is a straight line the torque-current curve is parabolic in form, but that, for currents which begin to saturate the field (shown to be about 14 amperes in Fig. 7–7), the curve becomes less steep and approaches a straight line in form.

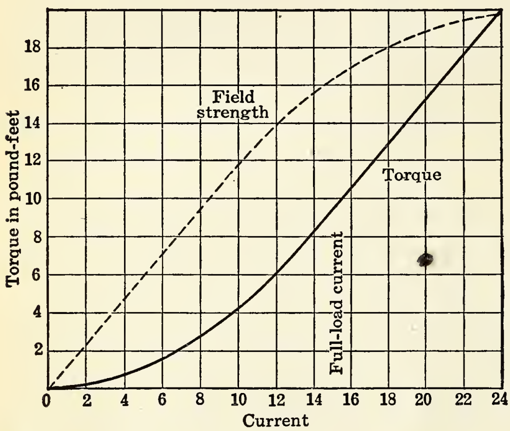


FIG. 7–7. Current-torque curve for a series motor. It is parabolic in form until the current is sufficiently large to produce saturation in the magnetic circuit.

23. Series Motor. Speed-load Curve. The fact that the field strength of a series motor depends upon the current flowing through its armature also gives it a speed-load curve different from that of the shunt motor.

Since the current taken by a series motor flows through both armature and series field, we may write

$$E = E_c + I_a(R_a + R_{se}) \tag{16}$$

where R_{se} = the resistance of the series field, and the other terms have the same significance as in Eq. (14).

The resistance of the series field is always low, less usually than that of the armature; the value of the counter emf at full load will therefore approach within perhaps 15 per cent of that of the impressed voltage.

If the load on a series motor is increased, the speed of the motor must start to fall; this allows the counter emf to decrease, with an attendant increase in both armature current *and field flux.* Since the torque is proportional to the product of armature current *and* field flux, this results in a marked increase of torque. From Eq. (16) it is evident that the counter emf must drop to allow an increased armature current. The speed of a series motor, therefore, must fall a good deal with increased load, to permit the necessary decrease in counter emf, in spite of the increase in flux. In the shunt motor, as the flux remains constant, the speed has to fall comparatively little to allow the counter emf to decrease the proper amount.

The series motor thus decreases its speed very much as its load is increased, as may be seen from Fig. 7–8. As full load is approached, the field generally begins to be saturated, so that the field flux does not increase as much for a given increase in armature current as at light loads. As a result, the speed does not have to fall as much as at light loads, and the speed-load curve tends to straighten out at about full load.

As the load on the series motor is decreased, the speed increases rapidly, and at very light loads the motor runs at speeds far above the safe value. For small values of torque produced, only small values of armature current are required, resulting in low values of flux. If the armature current is small, the value of the counter emf will be nearly that of the impressed voltage; since $E_c = K\Phi n$, a high value of speed is required with a weak field. If all load were taken off a series motor, the speed would rise to such a value that excessively large centrifugal forces would be brought into play, with the result that the winding would be thrown off the armature core and the commutator would probably be thrown to pieces.

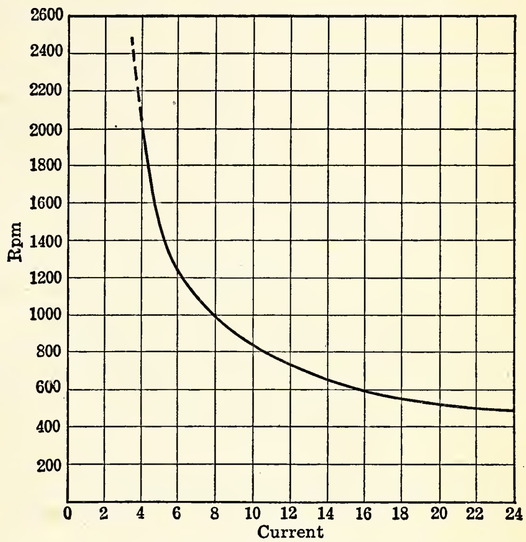


FIG. 7–8. Speed-load curve of a series motor. At light load the motor speed may become dangerously high.

This peculiarity of the series motor limits its use to those classes of service in which the motor may be directly (or by gears) connected to its load, so that under no condition would it be running without enough load to hold its speed down to a safe value. A series motor should never be belted to its load, because, if the belt should run off the motor pulley, the motor would immediately start to "race," and in a few seconds might be damaged. In electric railway installations, the motor is *geared* to the car axle, so that there is never any possibility of its running at excessive speeds.

24. Compound Motor. Current-torque Curve. The compound motor gives a current-torque curve the shape of which is intermediate between those of the shunt and series types. The curves for such a motor are given in Fig. 7–9; the field strength at zero armature current is given by

the shunt coils only, but as the armature current increases the field strength increases somewhat, on account of the mmf of the series coils.

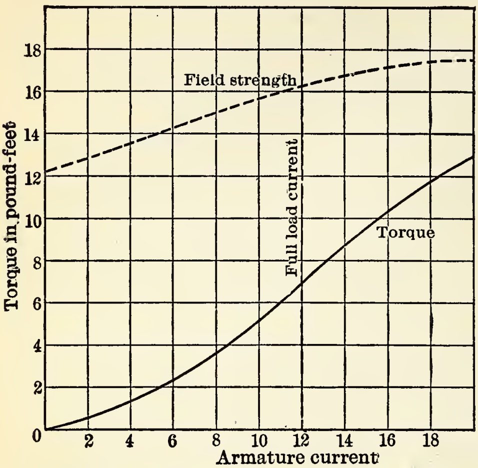


FIG. 7–9. Current-torque curve of a compound motor. The dashed curve shows how the field strength increases with armature current, due to the action of the series field winding, which carries the armature current.

25. Compound Motor. Speed-load Curve. The curve of the compound motor has generally much the same shape as that for the shunt motor, but the speed decrease with increase of load is much more marked, as may be seen from Fig. 7–10. The relative strengths of the series and shunt fields of a compound motor are fixed by the service for which the motor is intended. In most applications, a series field, such that at full load about 40 per cent of the total field mmf is furnished by it, is sufficient. As the series field is made relatively stronger and stronger with respect to the shunt field, the compound motor approaches the characteristics of the series motor. Rolling-mill motors are sometimes so built that the series field furnishes as high as about 80 per cent of the full-load field mmf. Such motors will have practically the characteristics of a series motor, the shunt field being added to prevent them from running away in case they are accidentally disconnected from the rolls.

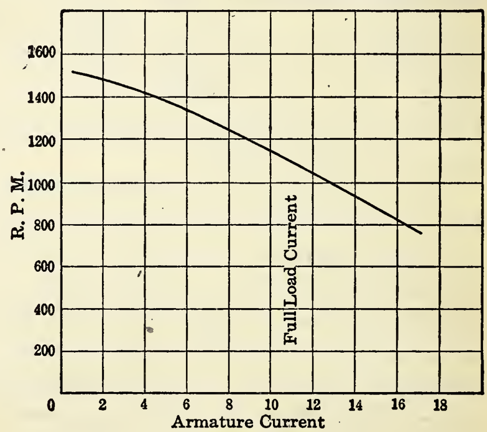


FIG. 7–10. A typical speed-load curve for a compound motor.

26. Comparison of Motor Characteristics. In Figs. 7–11 and 7–12 are shown the respective current-torque and speed-load curves for shunt, series, and compound motors of the same full-load output and speed; the full-load torque of each motor must therefore be the same.

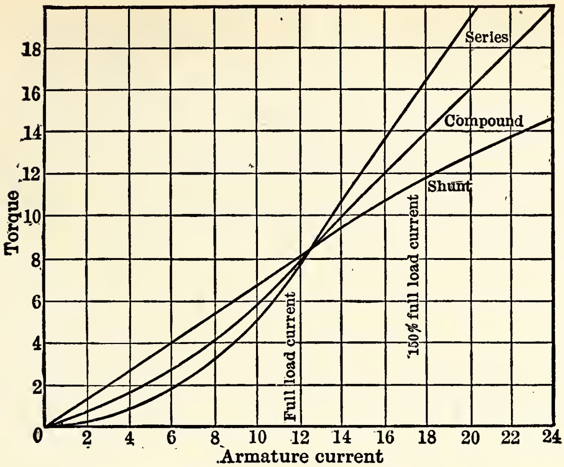


Fig. 7–11. Current-torque curves for shunt, series, and compound motors having the same torque and current at full load.

It has been shown that the torque exerted by a motor is dependent on the value of its armature current and field flux, and independent of speed. The current-torque curves of Fig. 7–11 are therefore characteristic of the respective motors, whether the machines are running or not. The torque exerted by a motor when not running is called its *starting torque*, and it will be seen from the current-torque curves of Fig. 7–11 that with 200 per cent full-load current (which might be safely put through a motor for the short time necessary for starting) the series motor gives much larger starting torque than either the shunt or compound motor, and that the compound motor will exert more starting torque than the shunt motor.

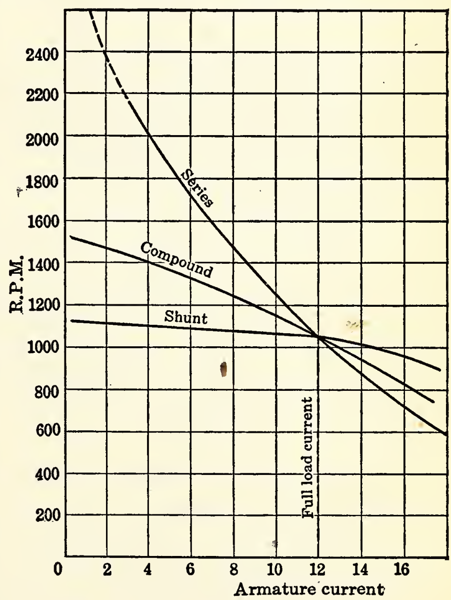


Fig. 7–12. Comparative speed-load curves for shunt, series, and compound motors having the same speed at full load.

Reference to the speed-load curves of Fig. 7–12 will show the wide range of speed of the series motor in comparison with the fairly constant speed of the shunt motor. For the series motor considered, a

speed of about 2150 rpm is considered safe; higher speeds are dangerous. The safe speed for any motor is a matter of design and construction, depending upon the means employed to care for the centrifugal forces developed in the armature and commutator.

The motors chosen for comparison in Figs. 7–11 and 7–12 were all of the same rating, giving equal horsepower output at the same full-load speed, and therefore developing the same full-load torque. Let us also consider a set of three motors, shunt, series, and compound, all of the same horsepower output but having the same speed *at light loads*, as represented in Fig. 7–13.

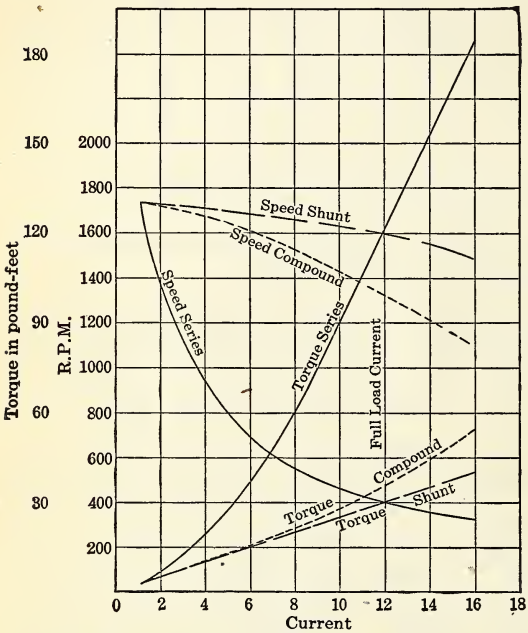


Fig. 7–13. Comparative speed-load and torque-current curves for shunt, series, and compound motors having the same speed at light load. The curves show that the torque, per ampere of armature current, is much greater in the series motor than in the shunt motor.

From Eq. (4), since the three motors have the same output, their full-load torques will be inversely as their full-load speeds. It is obvious, then, from these curves, that the series motor considered is able to start and turn over a very much larger load with a given armature current than the shunt motor with which it is compared, and that it is able to exert the same torque as the shunt motor with much less current, at all but very light loads.

Considering the equation for the torque of a motor ($T = K'\Phi I_a$) in terms of flux and armature current, in connection with the group of motors of Figs. 7–11 and 7–12, since they all develop the same full-load torque with the same value of armature current, it follows that they must all have the same full-load flux; the dimensions of their armatures and field structures will therefore all be about the same.

However, if the series motor of Fig. 7–13 exerts four times the torque at full load, with the same armature current, as the shunt motor of the same figure, it might have four times as much flux threading its field frame and armature. It would probably have twice as much flux, and twice as

many armature conductors. Obviously, either of these conditions requires that the series motor be of much larger dimensions than the shunt motor to which it is compared.

27. Applications of Motors. The principal factor which determines the selection of one type of motor or another for a certain class of work is the variation of its speed as the load on the motor is varied.

The shunt motor is used for such service, where practically constant speed is required regardless of load variation, as driving machine tools, blowers, fans, etc. A special form of the shunt motor, known as the *adjustable-speed* motor, will be considered further on. In this type of shunt motor, the speed can be fixed by the operator at any value between a minimum and maximum value, but when once so set will remain substantially constant for all loads. Adjustable-speed motors are used largely in individual drives for machine tools.

Compound motors are used for driving machines that are subject to sudden applications of heavy loads, as in punches, shears, etc. Their use in connection with flywheels will be taken up later. Compound motors are also useful in cases where large starting torque is required, but where it is desired to have less speed variation under load than a series motor would give.

Series motors are particularly adapted for railway and hoisting service, because of their rapid drop in speed as the torque demanded by the load is increased. Railway practice requires a reasonably high speed on level track and the development of large torque for starting and operation on grades, with as small a current as practicable. That the series motor exactly meets these requirements may be seen by comparing the operation of the shunt and series motors of Figs. 7–11 and 7–12. Both would start their loads equally well, and climb a grade requiring full-load current in about the same time. However, on level track, where the load is light, a car equipped with shunt motors would operate at a very low speed compared with one equipped with series motors.

If motors with characteristics as shown in Fig. 7–13 are used, the speed of the cars on level track would be the same. When called upon to climb a grade, the shunt motor would exert the required torque by excessive increase of its armature current and would continue at practically the same speed as on level track. The series motor, however, would promptly slow down and exert the required torque with very much less current than that which the shunt motor would draw; it would, of course, take longer in climbing the grade. The fact that the series motor can exert a much larger torque with full-load armature current is of great importance, inasmuch as it affects the size of the feeders and generating station.

The characteristic curves of a typical series railway motor are shown in Fig. 7–14.

28. Effect of Change of Line Voltage on Speed. If the voltage impressed on a motor varies, the speed of the motor may be expected to vary correspondingly; from Eq. (14), by substituting for E_c its value as given in Eq. (13),

$$E = K\Phi n + I_a R_a \tag{17}$$

This shows how the speed of a shunt motor may be expected to vary as either E or Φ is changed.

In connection with Eq. (17), it must be emphasized that the value of R_a does not affect the torque, *nor does it determine the armature current.*

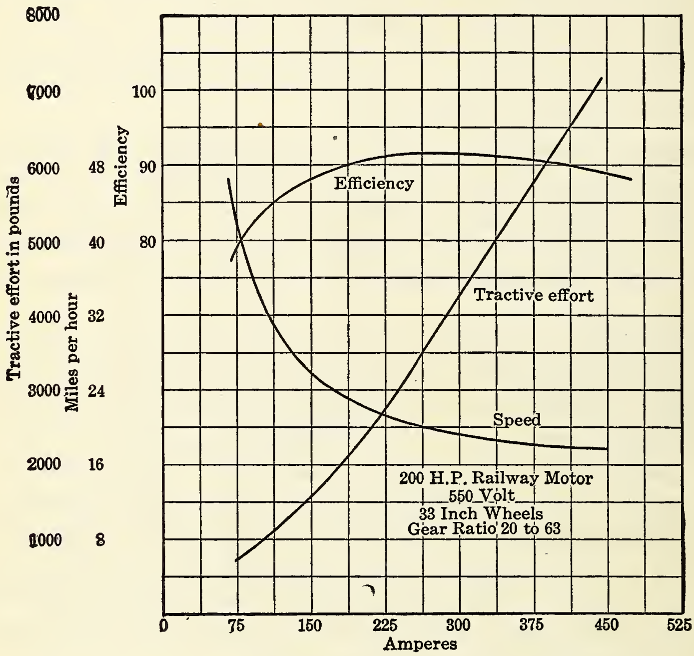


FIG. 7-14. Characteristic curves for a typical railway motor. The motor is geared to the 33-inch car wheel in the ratio of 20 to 63, giving, to the car, speeds and tractive efforts as indicated.

When it is running, a motor must always exert a torque equal to that demanded by its load, and it must draw whatever armature current it requires to build up its torque equal to that of the load.

As the field circuit of a shunt motor is connected directly across the supply line, any change in line voltage must affect the flux of the motor, Φ. Now, from Eq. (17) a reduction in impressed voltage causes a decrease in speed, whereas a reduction in Φ tends to cause the motor speed to increase. If the field of the motor is not operated near saturation, the change in speed for a small reduction in line voltage is not very marked. If E increases, Φ

increases in nearly the same ratio, so that the speed does not increase very much.

By using Eq. (16), we get for the series motor

$$E = K\Phi n + I_a(R_a + R_{se}) \tag{18}$$

In the case of a series motor, there is no shunt field and thus, for a *given current in the armature circuit,* the speed varies in nearly the same ratio as the impressed voltage.

The compound motor, having both shunt and series coils, has a change of speed, with change in line voltage, greater than that of the shunt motor but less than that of the series motor.

29. Effect of Field Heating on Speed. When a shunt motor is first started up after having been idle for several hours, the temperature of its field winding will be the same as that of the room. After it has run for several hours, however, the temperature of the field will rise and its resistance increase, reducing the field current.

A 50 degree rise in temperature will cause an increase in resistance of about 20 per cent and a decrease in field current of about 17 per cent. The resulting decrease in flux depends upon the degree of saturation at which the field operates, but there will usually be sufficient decrease in flux to bring about an increase in speed of about 10 per cent.

It is to be noticed that this action might be compensated by a field resistance which could be cut out as the field heats up. Commercially, however, shunt motors are not usually provided with field rheostats, the effect of field heating being of no importance.

In the compound and series motors, the effect of field heating is less. In the series motor it acts to cause a slight reduction in the speed for a given load.

30. Motors with Commutating Poles. The use of commutating poles in motors enables them to be operated with fixed brush position and to have good commutation over a wide range of loads. Commutating poles also make possible the adjustable-speed motor, providing perfect commutation over a wide range of speeds as well as of loads. The correct setting of the brushes is determined by the manufacturer, and, since it is not intended that the brushes should ever be moved during commercial operation, the brushes are locked in proper position by dowel pins or other suitable devices. These must be deliberately removed before the brushes can be shifted, but they also make it possible to reset the brushes readily in their correct position.

The design of the commutating poles of any motor or generator will be correct when the voltage generated by an armature coil undergoing commutation, as it cuts the commutating-pole flux, is exactly equal to the voltage of self-induction, due to reversal of the current during the same

period. Commutating poles are set exactly midway between the main poles and the brushes should be set so that a coil will be in the middle of its commutating period when it lies under the middle of the commutating pole. This position of the brushes is the no-load neutral; it is shown by the line AA' in Fig. 7–15.

31. Effect of Various Armature Conductors. The automatic adjustment of the torque developed by a motor to be equal to that of the applied load is accomplished by changes in the counter emf of the motor. To develop a definite value of torque requires a definite product of armature current and field strength and hence a definite value of counter emf. The counter emf generated by the armature coils, as they move through the field, varies directly with the strength of the field and with the speed. In the motor the speed is the accommodating factor; it will change if a change in counter emf is necessary or, with constant counter emf, if any of the other factors involved in the counter emf are changed.

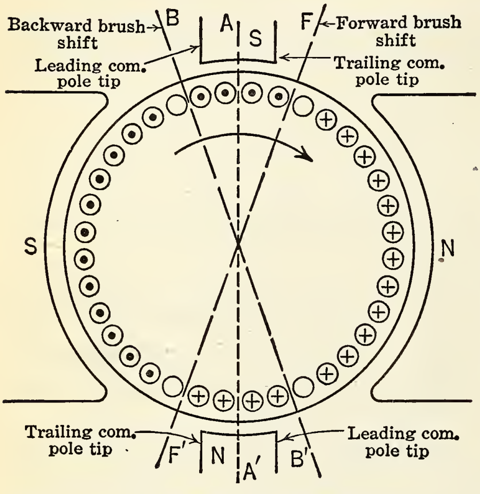


FIG. 7–15. Practically all modern motors are equipped with commutating poles to obtain good commutation. The induced voltages are shown by crosses and dots, the direction of those under the north pole being into the paper. The diagram shows that the voltages induced under the commutating poles contribute nothing to the counter emf of the armature, unless the brushes are shifted from the no-load neutral, AA'.

With the brushes on the no-load neutral (line AA' of Fig. 7–15) the counter emf of the motor is generated in the conductors under the main poles. With the direction of field and rotation as in Fig. 7–15, the induced counter emf under the right-hand north pole will be directed into the paper, as is indicated by the crosses. Under the south main pole the direction of the counter emf is out of the paper, as is indicated by dots.

As a coil moves into the commutating-pole flux before commutation begins, the coil, since it is carrying current, will react with this flux to produce torque and also induce voltage. This torque and voltage will be added to or subtracted from those generated under the preceding main pole, and so have effect upon the total torque and counter emf of the motor.

As the coil continues to move into the commutating-pole flux, commutation begins. The coil is now short-circuited by the brush, and during

the interval the voltage induced, as a result of flux-cutting, is neutralized by the voltage of self-induction. The average torque exerted by the coil during commutation is zero, since the current in the coil reverses.

After commutation is over, and the coil is no longer short-circuited, the coil moves out from under the commutating flux. During this interval under the trailing pole tip, torque and counter emf will again be generated.

With the brushes in the no-load neutral (line AA' of Fig. 7–15) in any path between brushes, torque will be exerted and counter emf generated under a leading commutating-pole tip, under a main pole, and under a trailing commutating-pole tip. The effects under the two commutating pole tips will neutralize, since all commutating poles are of equal strength and are alternately of opposite polarity. Thus with brushes properly set on the no-load neutral, the flux under the commutating poles has no effect upon the speed characteristic of a motor.

If the brushes are shifted forward or backward (shown to an exaggerated degree in Fig. 7–15, by the lines FF' and BB') commutation will take place after or before the armature coil has passed the middle of the commutating pole, respectively. In passing from one brush to the next, the coil will then be under the influence of one commutating-pole tip for a longer time than it is under the influence of the other pole tip, and the pole tip effects will no longer neutralize; there will be a positive effect due to the flux under the commutating poles. Since the commutating-pole strength is proportional to armature current, the effect will vary with load; it is as if an extra series field had been added to the machine.

32. Effect of Forward Brush Shift. With the brushes shifted forward (line FF' of Fig. 7–15) so that commutation takes place after the short-circuited coil has passed the commutating poles, the counter emf and torque generated under the south commutating pole at the top will be added to those under the south main pole. The necessary counter emf for any given armature current may then be induced at a lower speed, and the motor takes on the stable speed characteristic of a cumulative-compound motor.

In addition to the effect just noted, the effect of the IR drop in the armature and commutating field, tending to decrease the speed, is also present. Because the brushes have been shifted forward, there is now a component of armature reaction tending to magnetize the main field; this effect also tends to decrease the speed for a given counter emf.

A given motor with a no-load speed of 1000 rpm, and its brushes on the no-load neutral, might have a full-load speed of 900 rpm. With the brushes shifted forward the full-load speed will be lower than 900, the amount that it is lower depending upon the amount of forward brush shift.

There is no advantage in causing a shunt motor to have the characteristics of a compound motor by shifting the brushes forward. Although

a fair degree of compounding may be obtained, there is the probability that commutation may be impaired; a series field is the preferred arrangement. If the motor is to be operated in both directions of rotation the brushes must be on the no-load neutral, since a forward brush shift for one direction of rotation becomes a backward shift for the other direction of rotation. This is a dangerous condition, as will be seen in the following paragraphs.

33. Effect of Backward Brush Shift. With the brushes shifted backward (line BB' of Fig. 7–15) commutation takes place before the short-circuited coil passes the middle of the commutating poles. The torque and counter emf generated under the upper south commutating pole are now combined with those of the main north pole. Since the poles involved are of opposite polarity the effects under the commutating poles oppose those under the main poles. The counter emf required for any given armature current must then be generated at a higher speed and the motor takes on the unstable speed characteristic of a differential-compound motor.

The effect of the IR drop in the armature circuit is still to cause a decrease in speed, but, with a backward shift of the brushes, armature reaction tends to weaken the field, causing a further increase in speed.

If the armature resistance is high and the main field is strong, while armature reaction and the commutating poles are relatively weak, with a small backward shift of the brushes, the speed will still fall off as load is increased, but not as much as when the brushes are on the no-load neutral. With low armature resistance and a weaker main field, the speed may rise with load increase. Under any conditions, the speed may be made to rise with increased backward brush shift.

A rising speed characteristic is unstable, particularly under the conditions being considered. A sudden application of load will result in a rapid increase of armature current and an equally rapid strengthening of the commutating poles. Because of the differential action of the commutating poles, the net flux as well as the counter emf are reduced, with two effects. The decrease in counter emf results in still greater armature current, still less net flux, and further reduction in counter emf. Meanwhile the motor tries to raise its speed in its efforts to maintain the counter emf, but reduction in counter emf and flux due to current action takes place much faster than the speed can rise. The result is excessive armature current and speed.

34. Brush Setting with Commutating-pole Machines. In motors intended to be operated in both directions of rotation, it is usually essential that, with the same load and voltage, the speed be the same in both directions. If the brushes are not on the exact neutral, this will not be the case. To locate the brushes of such a motor properly, they are first evenly spaced around the commutator. The machine is then successively oper-

ated in both directions with the same voltage, field and armature currents, the load being altered to adjust the armature current if necessary. The brushes are then shifted until the same speed is obtained in either direction of rotation.

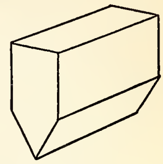


Fig. 7–16. Special brush required to locate the proper position of the brushes of commutating-pole machines.

Another method of adjusting brushes of a commutating-pole machine is often used, with the armature not rotating. The regular machine brushes are removed and are replaced by two special test brushes, one each in brush holders of opposite polarity. These special test brushes (Fig. 7–16) are made by cutting down two brushes until they present a line contact to the commutator, parallel to the bars. The line contact must be in the exact center of the test brushes.

As in Fig. 7–17, a galvanometer is connected across the test brushes, and the field is excited from a suitable source through a switch. The switch is for the purpose of opening and closing the field circuit, or preferably for suddenly reversing the field. In Fig. 7–17 is represented a two-layer drum winding with only a few coils drawn in. If the field switch is closed, causing, say, a growing north pole on the right and a south pole on the left, voltage will be induced in each armature coil to a greater or less degree depending upon how much flux each coil includes. The voltage will act out of the paper in practically all coil-sides below the polar axis AA', and into the paper above that axis.

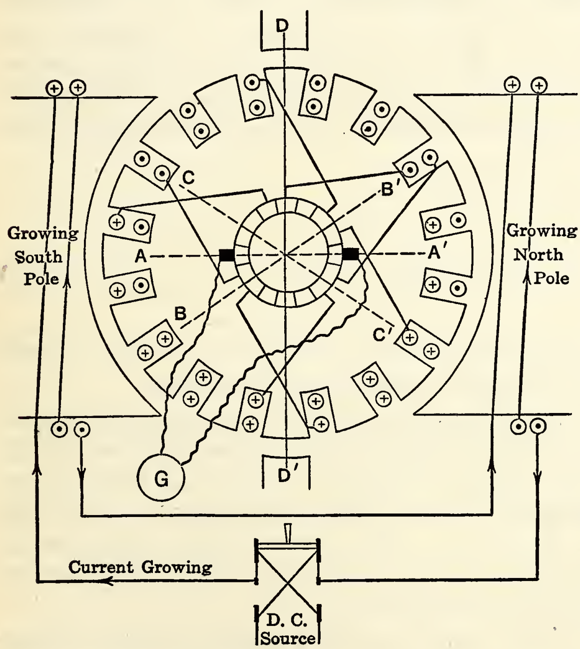


Fig. 7–17. Connections for the test to locate properly the brush setting for a commutating-pole machine.

If the end connections of all the armature coils at the commutator end are exactly equal, the brushes of the machine are on the exact neutral if they are exactly 180° (electrical) apart and placed along

the axis AA'. In this case the voltage induced in each armature path is zero, the sum of all voltages in one direction in a path being equal to the sum of all voltages in the other. But if the brushes are along some axis BB', other than AA', more voltage will be induced in one direction than in the other. Now if the field is made, opened or reversed, there will be a throw of the galvanometer needle. Reversal of the field produces a throw of greater amplitude.

With the brushes on the axis BB' and a growing north pole on the right, the galvanometer throw may be positive. But if the brushes are shifted to the axis CC' with again a growing north pole on the right, the galvanometer deflection will be in the other direction, say negative. The adjustment then consists of moving the brushes until the galvanometer deflection is zero. It is at times difficult to obtain zero galvanometer deflection if the machine is even only slightly unbalanced either electrically or magnetically, or has but one commutating-pole per pair of main poles. In this case two brush positions (BB' and CC') are sought, which give small, equal but opposite galvanometer deflections. The brushes are then set exactly midway.

35. Necessity of a Motor Starting Rheostat. When a motor armature is stationary, there can be no counter emf generated in its windings because there are no conductors cutting lines of force. If, then, the stationary armature of a shunt motor is connected directly to the supply line, the current which will flow in the armature circuit may be calculated from the equation

$$E = E_c + I_aR_a, \quad \text{and as} \quad E_c = 0$$

$$E = I_aR_a \quad \text{or} \quad I_a = \frac{E}{R_a} \tag{19}$$

Now, the current calculated from this equation will be *ten or twenty times the full-load current,* and will be disastrous in its results.

Consider a 110-volt motor, the full-load current of which is 40 amperes. The armature resistance of such a motor would be about 0.2 ohm. If the stationary armature were connected directly to the 110-volt line, the current through the armature would be $110/0.2 = 550$ amperes, whereas the full-load current is only 40 amperes. The current of 550 amperes would burn the brushes, commutator, and winding, and would also blow the fuses and circuit-breakers in the supply line.

After the motor is running, there is not an excessive current flowing through the armature because the current is limited by the counter emf. But while the armature is accelerating, this counter emf is small, and some other means must be employed to limit the starting current. This is the function of the *motor-starting rheostat,* or, as it is frequently called, the *starting box.*

A starting rheostat consists of a variable resistance, placed in series with the armature, which may be gradually cut out as the motor speeds up, and can be cut out altogether when the motor has reached nearly normal speed. The total resistance of the starting rheostat must be of such a value that, when it is connected in series with the armature, directly across the motor supply line, the current which flows through the circuit *will not be greater than about 150 per cent of the full-load current* for the motor. The starting rheostat is sometimes designed so that it limits the current to a value not greater than the full-load current of the motor.

36. Example of a Proper Starting Rheostat. Suppose that a starting rheostat is desired for the 110-volt, 40-ampere motor mentioned in the previous paragraph. If the current in the armature circuit is to be 60 amperes at the start, i.e., 150 per cent of the full-load current, the total resistance of the armature circuit (armature resistance and starting rheostat) must be 110/60 = 1.83 ohms. As the armature resistance is 0.2 ohm, the total resistance of the starting rheostat must be 1.83 − 0.2 = 1.63 ohms. The wire of which the starting rheostat is made must be of sufficient size to carry safely 60 amperes during the short time required for the acceleration of the motor armature.

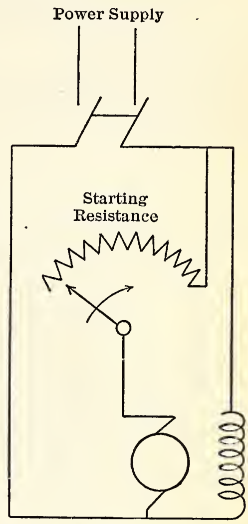


Fig. 7–18. When starting a motor, a variable resistance, called a starting rheostat, or starting box, is inserted in series with the armature circuit.

The connection of the armature to the line through this variable resistance is shown in Fig. 7–18. The field is connected directly across the line, and is at full strength during starting. The wires of which the starting-box resistance is composed are imbedded in sand or enamel or wound on porcelain tubes; they are then enclosed in fireproof material, so that, if the operator keeps the starting resistance in the circuit for a longer time than that for which it was designed, thus overheating and possibly melting the wire, no fire risk occurs.

The resistance of 1.63 ohms would have taps brought out at a number of points, so that it could be cut out by means of a handle operating a sliding contact, as the motor speeds up.

Let us consider what happens as the motor is started up against a constant load torque equal to the full-load torque of the motor. The operator moves the handle, closing the circuit through the armature and all the starting resistance. With 1.63 ohms in series with the armature, the armature current jumps to 60 amperes, as is shown in Fig. 7–19, at time t_1.

With constant field current and flux, the torque exerted by the motor is directly proportional to the armature current; with 60 amperes armature current the motor will exert 150 per cent rated torque or a value 50 per cent greater than the load torque assumed. At once the motor begins to accelerate and generates a counter emf.

From the equation

$$E = E_c + I_a R_a,$$

as soon as E_c starts to increase, with the rising armature speed, the armature current must begin to decrease along some such curve as AB. Increase in motor speed therefore causes a decrease in armature current and in generated torque.

Now, it has been shown that a motor will increase its speed only if its generated torque is greater than the load torque; the motor will therefore

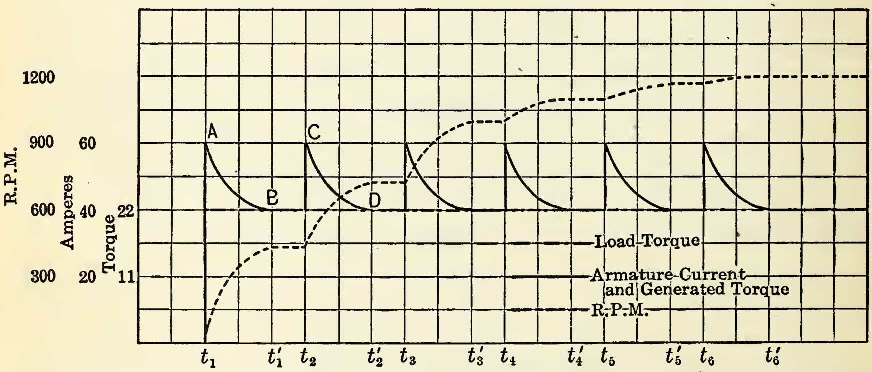


FIG. 7–19. This curve shows how the armature current of a motor, its torque, and speed vary as the starting resistance is cut out, step by step.

increase its speed and decrease its armature current until the latter has fallen to 40 amperes; with this current the motor is developing full-load torque which is equal to the value of the load torque assumed.

If we know the full-load speed of the motor considered, we can calculate the value to which the motor speed will rise at time t_1' with all the 1.63 ohms in series with its armature. With constant field flux, the speed, from the equation for counter emf ($E_c = K\Phi n$), will be directly proportioned to the counter emf.

Let us assume that the full-load speed of the motor is 1200 rpm. At full load, with no added resistance in series with its armature, its counter emf will be $110 - 40 \times 0.2 = 102$ volts.

At time t_1', with 40 amperes flowing and 1.63 ohms in series with the armature, the counter emf will be $110 - 40(1.63 + 0.2) = 110 - 73.2 = 36.8$ volts.

We have then the following proportion:

$$\frac{\text{Speed at time } t_1'}{\text{Speed at full load}} = \frac{\text{cemf at time } t_1'}{\text{cemf at full load}} = \frac{n_1'}{1200} = \frac{36.8}{102}$$

from which

$$n_1' = \frac{1200}{102} \times 36.8 = 433 \text{ rpm}$$

To increase the speed of the motor further, its generated torque must again be increased. This can be done by cutting out a part of the 1.63 ohms in series with the armature, by moving the handle of the starting rheostat farther to the right. This will naturally cause a momentary increase in the armature current, and, if this increase is again to be limited to 60 amperes, the amount of resistance left in series with the armature may be calculated from the equation

$$E = E_c + I_aR_a + I_aR_2$$

where R_2 is the resistance to be left in series with the armature.

Since the value of the counter emf at time t_2 is 36.8 volts, we have

$$R_2 = \frac{110 - 36.8 - 60 \times 0.2}{60} = \frac{61.2}{60} = 1.02 \text{ ohms}$$

Cutting out $1.63 - 1.02 = 0.61$ ohm will therefore cause the armature current to jump to 60 amperes at time t_2. At once the speed will increase, and the armature current will decrease, along some such curve as CD, until it is again 40 amperes. At time t_2', when everything is steady again, the counter emf will be

$$110 - 40(1.02 + 0.2) = 110 - 48.8 = 61.2 \text{ volts}$$

and the speed

$$n_2' = \frac{1200}{102} \times 61.2 = 720 \text{ rpm}$$

The resistance to be left in at time t_3, to allow the current to rise again momentarily to 60 amperes, is then

$$R_3 = \frac{110 - 61.2 - 60 \times 0.2}{60} = \frac{36.8}{60} = 0.61 \text{ ohm}$$

When the current has again fallen to 40 amperes at time t_3', the value of the counter emf will be

$$110 - 40(0.61 + 0.2) = 110 - 32.4 = 77.6 \text{ volts}$$

and the speed

$$n_3' = \frac{1200}{102} \times 77.6 = 914 \text{ rpm}$$

The resistance of 1.63 ohms would therefore have taps at about five points, so that it could be gradually cut out as the motor speeded up. The steps are not even; the above rheostat is divided into steps of 1.63 ohms, then 1.02 ohms, 0.61 ohm, 0.34 ohm, 0.16 ohm, and 0.04 ohm.

Brush drop was neglected in the above calculation for simplicity; it should be included and will affect the values of resistance to a small degree.

37. Necessity of the "No-voltage Release." The object of this device is always to open the armature circuit when the supply line becomes "dead." When this occurs (as, for example, when the circuit-breaker or the fuses on a feeder blow) the motor immediately slows down and stops. If the line is again made alive (circuit-breaker reset or fuses replaced), as the motor armature is directly connected across the line, with no resistance

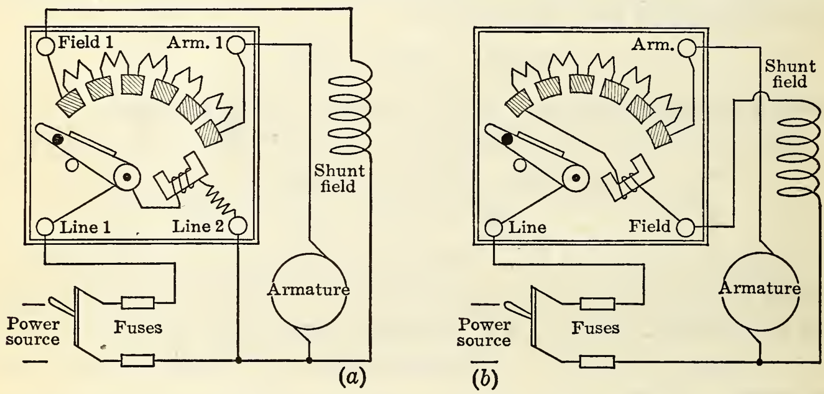


Fig. 7–20. Starting rheostats for shunt motors. That shown in (a) is a four-terminal rheostat with the no-voltage release circuit connected across the line. The rheostat shown in (b) has three terminals with the no-voltage release coil connected in series with the shunt field.

in series, an excessive current will flow through the stationary armature. This excessive current will blow the fuses in the motor circuit, necessitating their renewal.

The no-voltage release consists of a small electromagnet with an iron core shaped so as to attract and hold a small piece of iron on the starting arm of the starting rheostat after it has been moved to the last point (resistance all out), Fig. 7–20. So long as the supply line is alive the solenoid of the no-voltage release is energized and keeps the starting arm in the running position against the action of a spring coiled in the pivot of the starting arm. When voltage fails the solenoid of the release becomes de-energized and the pivot spring returns the starting arm to the "off" position.

The solenoid of the no-voltage release may be connected in series with a resistance and connected across the source as in Fig. 7–20*a*, resulting in a

starting rheostat with four terminals; or the solenoid may be connected in series with the shunt field, resulting in a starting rheostat with three terminals, as in *b*. The latter arrangement has the advantage that the motor is automatically disconnected from the supply if the shunt-field circuit of the motor is accidentally opened, a condition which otherwise would result in excessive current and an attempt by the motor to raise its speed. With the solenoid in series with the field circuit, however, the solenoid must be designed to operate with a given motor. The shunt-field resistance of the motor must be such as to pass the proper current through the no-voltage release coil. Too large a current will overheat the coil; with too small a current the magnet will not hold the starting arm in the running position. This disadvantage is overcome in the four-terminal starting rheostat of

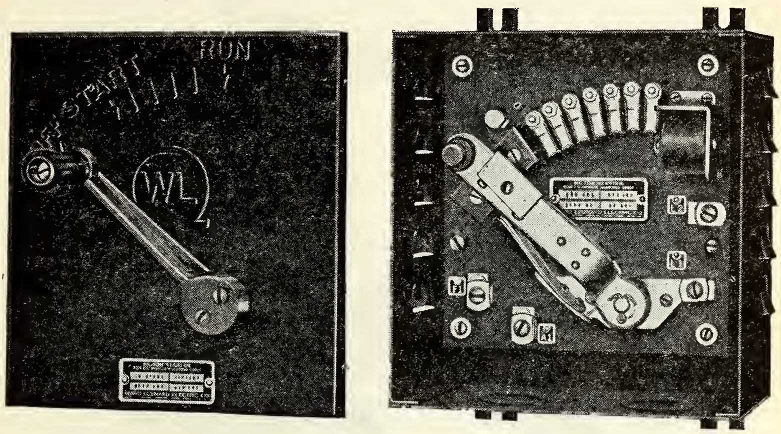


Fig. 7–21. A standard starting rheostat. The resistance may be varied by the handle on the outside of the metal enclosure. *Courtesy of the Ward Leonard Electric Co.*

Fig. 7-20*a*. Motors of a given horsepower output and voltage have very nearly the same value of armature current and, with the no-voltage release coil energized by line voltage, any starting rheostat of a given rating may be used with any motor of the same rating.

A standard starting rheostat is shown in Fig. 7–21. In commercial practice it is customary to place the starting rheostat in a fireproof metal box with a mechanical connection to the starting arm on the front cover. An overload release or circuit-breaker which may be easily and quickly reset is sometimes built into the starting rheostat for motors where overloads are likely to be frequent.

In a starting rheostat for a series motor, the no-voltage release must be energized by line voltage. A four-terminal rheostat is used with the shunt-field connection omitted.

The starting rheostats used with adjustable-speed motors will have the

required resistance, to be inserted into the shunt field, built into the rheostat. A second arm is added, making contact with a second row of contacts by which the field resistance is controlled after all the armature starting resistance has been removed by the first arm. The motor is thus started at its lowest speed and subsequently speeded up as desired.

38. Controllers. Another form of manual motor starter is known as a controller. In this type a drum or shaft supports a number of radially projecting segments of copper which, when the drum is rotated by a handle, come into contact with stationary copper fingers. By properly arranging

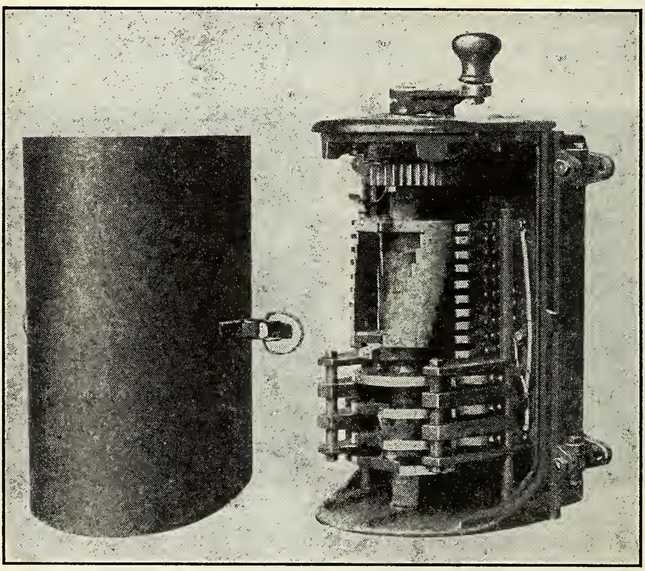


FIG. 7–22. A controller for starting and controlling an electric motor. *Courtesy of the Westinghouse Electric and Manufacturing Co.*

the sequence with which the segments and fingers make contact, the armature-circuit resistance is properly controlled. With controllers the starting resistances to be inserted into the armature circuit are usually separate units outside the controller.

Controllers may be arranged with a no-voltage release, or built so that the handle will remain at any notch to afford speed control by armature resistance (section 44). After all armature resistance has been removed, additional segments and fingers may then insert resistance into the shunt field for further speed control. Additional features may be incorporated into controllers, such as reversing, dynamic braking, etc. Controllers are extensively used on cranes, hoists, machine tools, trolley cars, and also as master controllers in locomotives, cars, etc., to operate mag-

netic or pneumatic contactors, which in turn control the motor circuits. A controller with its cover removed is shown in Fig. 7–22.

39. Automatic Starting of D-C Motors. D-c motors are often started automatically in modern installations. The initiating impulse may be

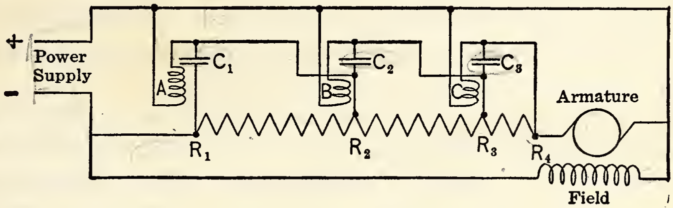


FIG. 7–23. Connections of a counter emf type of automatic starter with three steps. The solenoids A, B, and C operate in sequence, cutting out the starting resistance at the proper time to keep the starting current equal to about 150 per cent full-load value during the starting period.

from a push button, or in an entirely automatic installation, by means of some switch, closed when the motor is to start. As an example, if the motor operates a pump used to fill a storage tank the motor might be started by a float switch when the level of the liquid in the tank falls to a certain point, and again stopped when the liquid has risen to a certain level.

In any case the steps of the starting rheostat are cut out automatically at the proper times after the initiating impulse starts the sequence. This is usually done in one of three ways, known as counter emf starting, time starting, and current limit starting.

40. Counter EMF Starting. With counter emf starting the starting resistance is cut out in steps by contactors actuated in turn by the rising voltage across the armature. A typical starter of this kind is shown in Fig. 7–23. After the main switch is closed, usually by means of an electromagnetically closed contactor, current will flow through resistors R_1–R_2, R_2–R_3, and R_3–R_4 in series with the armature, starting the motor. This would correspond to the point t_1, in Fig. 7–24. The armature current would then fall along the curve AB, the speed would rise along the curve $A'B'$, and the counter emf along the curve $A''B''$. It is to be noted that the curve representing the speed also represents the shape of the counter emf curve and hence, practically, the shape of the

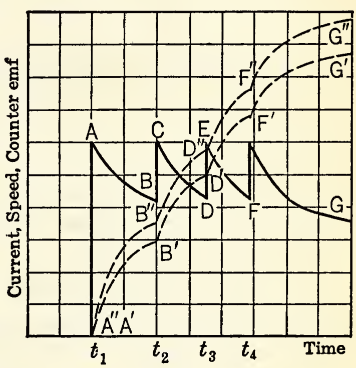


FIG. 7–24. Curves showing the variation of current, speed, and counter emf with time, when a motor is started by an automatic starter.

curve of voltage across the armature, since the counter emf is directly proportional to speed.

When the voltage across the armature rises to a certain value, such as B'' at time t_2, the solenoid A, connected from R_2 to the positive side of the armature, carries enough current to close contacts C_1, short-circuiting the resistor R_1–R_2. This solenoid can be adjusted so that it will just close its contacts when the voltage rises to a certain value. Solenoids B and C will be adjusted so they will not close yet.

When C_1 closes, the current rises immediately to C (Fig. 7–24) then falls along the curve CD, the speed and voltage across the armature rising along the curves $B'D'$ and $B''D''$ respectively. When the voltage across the armature rises to D'', the solenoid B closes contacts C_2, short-circuiting the resistor R_2–R_3. Similarly, in turn, solenoid C closes contacts C_3 short-circuiting the resistor R_3–R_4.

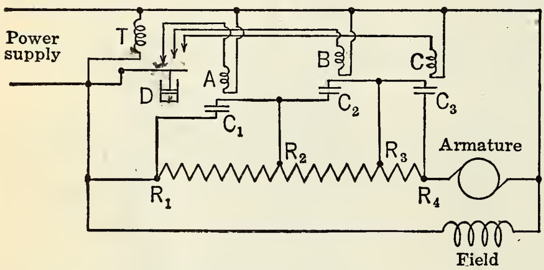


FIG. 7–25. Connections of a time starter. The timing relay, T, has a series of fingers, making contact in turn with an arm, the position of which determines the time interval between the energizing of coils A, B, and C.

This type of starter can be built with any number of steps although it usually is made with two or three, the magnetically operated contactors functioning in sequence, being actuated in turn by the increasing voltage across the armature. It is the least expensive of the types of starters mentioned and is widely used. It can be used in all installations except where the line voltage fluctuates widely.

41. Time Starting. With a time-starting system the steps of the starting rheostat are cut out in sequence at definite time intervals after the switch is closed. The contactors for a typical time-starting system are shown in Fig. 7–25. The solenoids A, B, and C are energized in sequence by the timing contactor T. As soon as the circuit is energized the solenoid on T pulls upward on the arm. This arm is held back by the dashpot D, a piston in a cylinder with only a small hole to admit air, and moves upward very slowly. As A, B, and C are energized in turn they close the contacts C_1, C_2, and C_3, cutting out the starting resistance in steps. Other timing devices may be substituted for an air dashpot: often an oil-filled dashpot is used, sometimes a device similar to the escapement on a clock is used, and sometimes the rate of build-up of electric currents in coils (Fig. 4–7, page 137) to the value where they will operate a contactor is utilized.

All time starters have the disadvantage of removing the resistance of the starter in steps at definite times even if the motor fails to start or is prevented from increasing its speed for any reason.

42. Current Limit Starter. This type of starter has its actuating solenoids in series with the line itself and is so constructed that when the current falls to a certain value the resistance steps of the starter are short-circuited. For example, referring to Fig. 7–24, when the circuit is energized the current rises immediately to A and then falls along the curve AB. When the current falls to B the current-actuated relay causes the first step of the starting resistor to be short-circuited, the current rises to C and then falls along the curve CD, the second current-actuated relay operating when the current falls to D, etc., until all the steps are cut out.

This starter is the most reliable but is more costly than the other two. It has the advantage of not removing resistance from the circuit if the current is high, owing to some fault in the motor.

43. Speed Control of Motors. It is often necessary to vary or adjust the speed of a motor according to the changing requirements of the load. To make the electric motor suitable for railway service, operation of machine tools, etc., it must be possible to change its speed quickly and easily through a wide range; and the scheme for obtaining this variation in speed must be such that the motor operates with a good efficiency at any one of the speeds required.

44. Possibility of Varying Speed. By inspection of the relation

$$E = K\Phi n + I_a R_a$$

it will be seen that for a given load on the motor, the speed, n, may be varied by a change in the voltage impressed upon the armature, E, a change in the value of the field flux, Φ, or an increase in the armature circuit resistance.

Increasing the voltage impressed upon the armature, E, for a given value of armature current, I_a, requires an increase in the speed, n, if the value of the field flux, Φ, remains constant. If the voltage impressed upon the armature is decreased, the speed is reduced.

Increasing the resistance of the armature circuit by the insertion of resistance in series with the armature permits the counter emf to fall; with the same value of field flux, a drop in speed results. The introduction of resistance into the armature circuit results in increased $I_a^2 R_a$ losses (where R_a is the total armature-circuit resistance) so that the efficiency of the motor is reduced. The introduction of resistance into the armature circuit also results in poorer speed regulation. At no load the speed is not much affected, but as load increases the speed falls more rapidly than if no external resistance is added. Speed reduction by the addition of resistance in the armature circuit therefore is seldom used.

45. Speed Control by Field Variation. The most important method for obtaining various speeds consists in weakening the field of the motor; this is called *field control.* When first discussing the speed-load curves of motors it was shown that, under any condition of operation, the counter emf developed in the armature winding must be nearly equal to the impressed voltage. If, with the impressed voltage maintained constant, the field flux is varied, and after the field flux is varied, the load is readjusted so that the armature current is the same as at first, it is evident that the above condition can be fulfilled only if the speed of the motor follows inversely the changes in field strength, a high speed corresponding to a weak field, and vice versa.

Inspection of the relation, $E = K\Phi n + I_aR_a$, together with the torque equation, $T = K'\Phi I_a$, indicates that, with a given load torque on the motor, a decrease in the flux would require more armature current, in order that the value of the generated torque be maintained. An increase in armature current, as has been seen, generally requires some reduction in speed; whether the motor speed increases or decreases depends then upon the relative change in Φ and I_a.

Consider a 110-volt shunt motor, the full-load current of which is 40 amperes, and the armature resistance of which is 0.2 ohm; the full-load counter emf of this motor would be $110 - 40 \times 0.2 = 102$ volts. With no load the motor would require about 2 amperes, so that its no-load counter emf would be $110 - 2 \times 0.2 = 109.6$ volts.

From no load to full load when the armature current changes from 2 to 40 amperes, the counter emf of the motor drops 7.6 volts. In other words, a small change in counter emf produces a very large change in the armature current. Therefore, if a reduction of field flux does cause the motor to draw more armature current, there will be practically no change in the value of the counter emf, and a reduction in flux will then be accompanied with an increase in speed.

46. Methods of Varying the Field Flux of a Motor. The simplest method of changing the flux of a shunt motor is by varying the resistance of its field circuit.

The amount of speed variation which can thus be obtained from a shunt motor of ordinary design, without commutating poles, is not very great. The twisting of the main field by the armature reaction is not enough to be objectionable when the main field is operated somewhere near saturation; but, when this field is weakened, the effect of the armature reaction is much greater and the brushes will spark badly unless properly shifted with every change in load. A point is soon reached, however, where the field becomes so weak that with the greatly increased armature reaction there is not sufficient commutating flux. In the ordinary shunt motor without commutating poles, and hence not designed for the purpose, a

speed increase of about 30 per cent by field weakening is about the limit; beyond this limit excessive sparking will ensue.

Another method of changing the flux of a motor is to change the length of the air gap, thus changing the reluctance of the magnetic circuit. In one such type of "variable air-gap" motors, sometimes used, the armature is slightly conical and is capable of being moved along its shaft relative to the pole pieces, which also have a conical bore. As the armature is moved along the shaft by a handwheel, the length of the air gap is changed; with a constant field current, the flux varies practically inversely as the air-gap length.

The advantage of this method of varying the field flux is that armature reaction is not changed, since the relative strength of the field and armature mmf's is not altered. The disadvantages are that the motor is mechanically complex, and that speed adjustments must be made at the motor, which is not always conveniently located.

47. Commutating Poles Used with Field-weakening Control Scheme. The type of motor best adapted for speed variation by the field-control method is the commutating-pole motor, and practically all motors intended for service where an adjustable speed is desired are equipped with commutating poles. In such motors the commutating poles provide the flux required for sparkless commutation, irrespective of the strength of the main field. Such *adjustable-speed* motors are designed for a speed range as great as 1 : 6; ordinarily, however, the speed variation required is not greater than 1 : 3 or 1 : 4.

48. Design of Adjustable-speed Motors. It is obvious that a motor must be designed so that at the highest speed at which it is to be operated there must be a considerable factor of safety with regard to the centrifugal forces developed in the armature and commutator. Whereas a commutating-pole motor designed to operate at a constant field will, electrically, operate satisfactorily as an adjustable-speed motor, it is not wise to raise its speed more than 40 per cent above the rated value, unless some further guarantee has been received from the manufacturer.

Inspection of the expression for the counter emf of a motor

$$E_c = \frac{p\Phi nZ}{m \times 60 \times 10^8}$$

shows that, for a motor designed to operate at a given impressed voltage, the speed, n, may be made a low value by increasing either the number of poles, p, the flux per pole, Φ, or the number of armature conductors Z. In multiple-pole motors the use of wave windings, in which the number of parallel paths, m, is equal to 2, results in lower speeds. Since increasing the number of poles, the flux per pole, or the armature conductors all tend to increase the size and the amount of iron and copper in the machine,

low-speed motors will be larger and more expensive than machines of higher speed.

Changing the voltage of a machine does not generally affect its size. To change a 110-volt motor into a 220-volt machine of the same horsepower rating may be done by doubling the number of armature conductors. As the armature current will now be only half as great, the conductors may have one-half the area of cross-section, leaving the volume of copper the same.

The size of an adjustable-speed motor with a wide speed range will be determined by its lowest speed, so that these motors will be considerably larger than single-speed motors. Thus the field frame designed for a standard constant-speed $7\frac{1}{2}$-hp motor might be used in the construction of a 5-hp adjustable-speed motor.

49. Field Distribution in Adjustable-speed, Commutating-pole Motors. The need of commutating poles, and the effect of armature reaction in

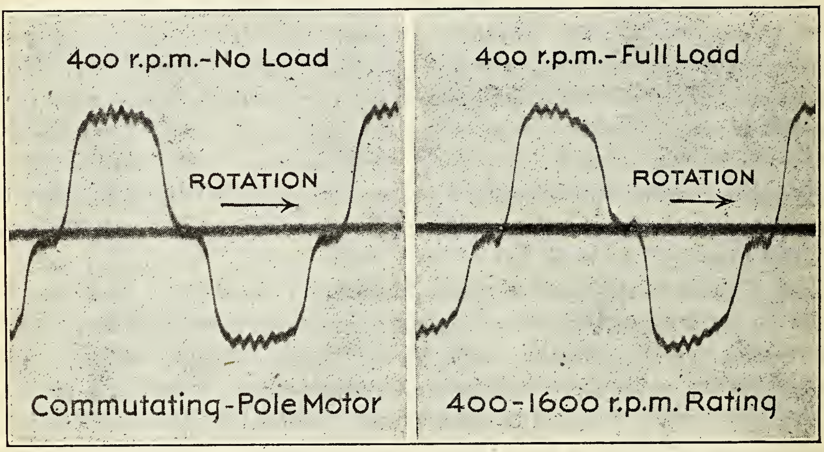


Fig. 7–26. Oscillograms showing the field distribution of an adjustable-speed shunt motor. The rated speed range is 400 rpm to 1600 rpm, and these oscillograms give the conditions, at no load and at full load, for the lowest speed. The main field being very strong, there is but little distortion, even at full load.

twisting the field of an adjustable-speed motor, are shown by the oscillograms of field distribution, given in Figs. 7–26, 7–27, and 7–28. These films were taken by the use of an extra inductor threaded in one of the armature slots; the two ends of this inductor were connected to the oscillograph by two small slip rings put on the armature shaft for this purpose.

In each figure is shown the field distribution for no load and full load, the motor being of modern design and intended for a speed range of 400 rpm to 1600 rpm, by field control. For all cases, the impressed voltage was the same, and the scale for all oscillograms was the same. It may be

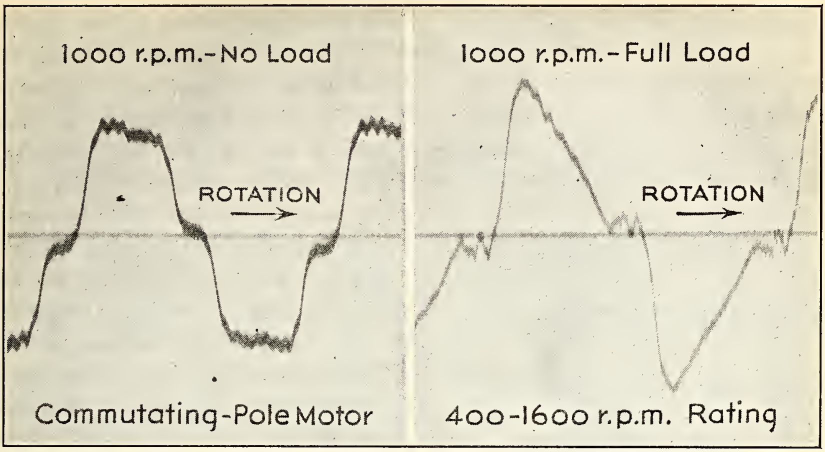


FIG. 7-27. These films are for the same motor as used in obtaining Fig. 7-26. The speed is here 1000 rpm and the main field is about one-third as strong as it was in Fig. 7-26. The field is much more distorted than it was with the strong field used for the speed of 400 rpm.

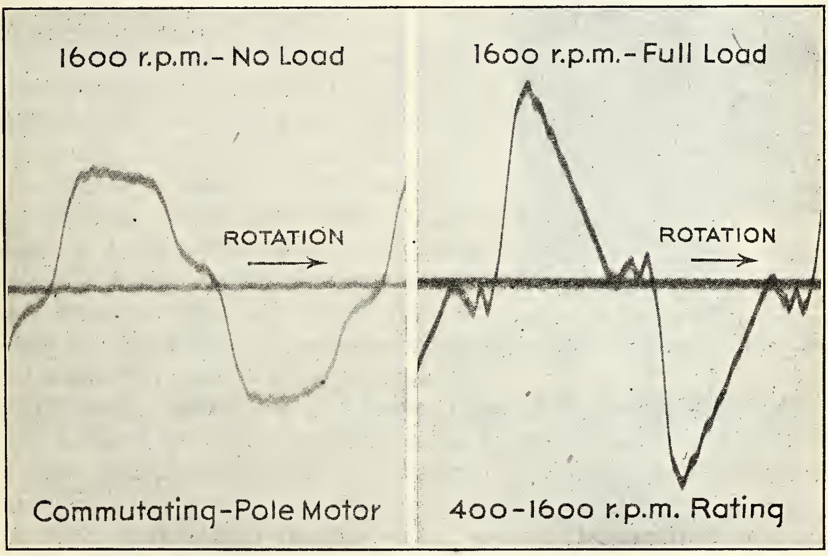


FIG. 7-28. These oscillograms are for the same motor as used for the two previous figures but the speed is here 1600 rpm. The main field is only about one-fifth as strong as it was for Fig. 7-26, and the field distortion for this condition is excessive at full load, being quite noticeable even at no load. Without commutating poles, this motor could not operate at full load. For this, as for the two previous films, it is seen that the commutating poles give at the brushes the proper field for sparkless commutation in spite of the twisting of the main field.

seen that the field is much stiffer at the low speeds than at the higher, this, of course, being due to the greater field current required to bring the speed to the low value. As the full-load armature mmf is of the same magnitude, whatever the speed, it follows that the twisting of the main field increases with speed; and this is seen to be the case by comparing Fig. 7–26 for a speed of 400 rpm with Fig. 7-28 for a speed of 1600 rpm; the higher speed requires about one-fifth as much field current as the lower, and the ratio of armature mmf to field mmf (which is the factor determining the amount of field distortion) is therefore five times as great for the high speed as for the low. The commutating poles furnish the requisite flux for sparkless commutation for all cases; but the motor is much more likely to have commutator trouble at the high speed than at the low, because the maximum number of volts per bar is about twice as much at high as at low speed.

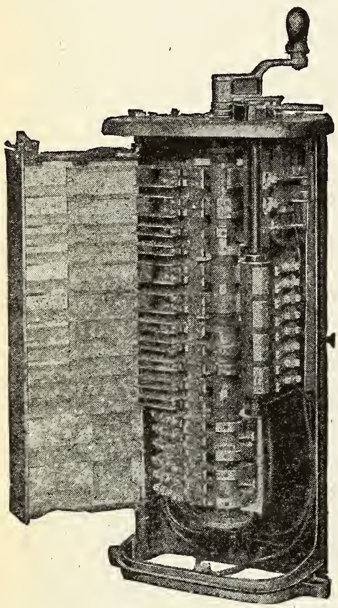


Fig. 7–29. The interior appearance of a typical railway controller.

In the conditions existing for the film of Fig. 7–28, at full load, the effect of armature reaction and commutating poles may actually reverse the direction of the flux in the trailing tips of the main poles, so that as a conductor moves across a main pole it generates voltage in both directions. As the total emf generated in one path of the armature must be equal (practically) to the impressed voltage, it is necessary to have very high voltage generated in some of the conductors to offset the negative or opposing voltage generated under the trailing pole tips. This excessive voltage occurs under the leading pole tip, where it is about twice the average voltage per bar.

50. Adjustable-speed Motor Compared to a Constant-speed Motor. The torque obtainable from an adjustable-speed motor is greatest when the main field has its greatest strength, i.e., at minimum speeds. As the speed is increased by field weakening, the torque decreases in about the same ratio as the speed increases. This is due to the fact that the torque of a motor is proportional to the product of the armature current and the field density; the safe armature current is nearly as much at low speeds as at high speeds, so that the torque goes down as the speed goes up. The product of torque and speed is practically the same whatever the speed, and this means that the motor has the same capacity in horsepower over its whole speed range.

51. Speed Control of Railway Motors. Series-parallel System. The speed control of railway motors is accomplished by the variation of the voltage impressed across the motor terminals by what is called the *series-parallel control.* There are always at least two motors on each electric car, both of the same rating. The necessary switching is performed by means of a controller (Fig. 7–29), which is simply a rotatable switch. It consists of two drums or cylinders mounted in a metal-covered frame, each cylinder carrying a number of copper segments which, as the drum is rotated, make contact with copper fingers, to which the various circuits are connected. The smaller drum reverses the direction of current through either the fields or the armatures of the motors, to control the direction of rotation of the motors. The larger cylinder, which controls the switching of the motors and resistances, may be set into a number of positions, "notches" or "points," each position giving a definite circuit.

In most railway systems the current is supplied from the power source to the locomotives or cars by either a trolley wire or a third rail and returned over the track, the sections of which are always either welded together or bonded by copper cables. The trolley or third rail is usually positive with respect to the track return, the latter being generally designated as the "ground."

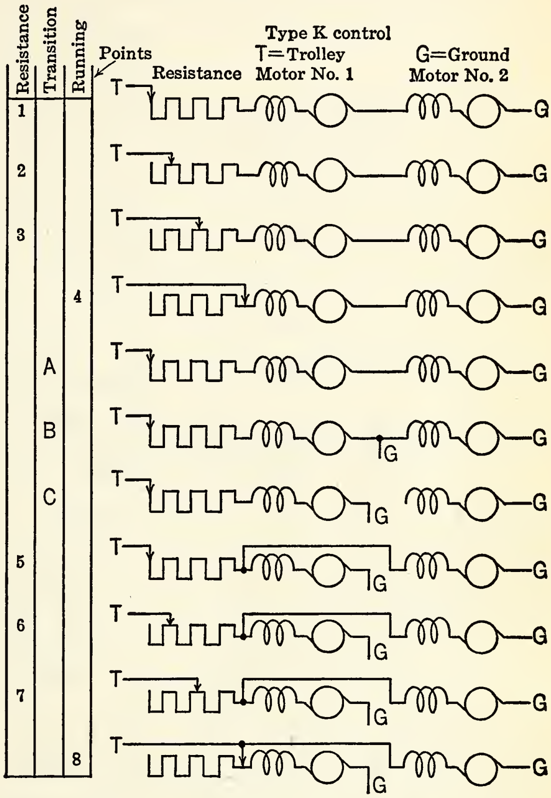


Fig. 7–30. Series-parallel control for series motors. The motors are first operated in series and then in parallel. During the transition period only one motor is exerting torque. This type of control is commercially known as Type K.

In the simplest type of control, known commercially as Type K, and used only for lighter single cars, the order of switching is as is shown in Fig. 7–30. On the first point of the controller, the two motors and all

the starting resistance are placed in series across the supply voltage (600 to 1500 volts). The second, third, and fourth point progressively cut out the starting resistance until it is all out on the fourth point. The first three points are called "resistance" points and are not intended for continuous operation, inasmuch as the operation is not efficient and because the resistances will become hot.

The fourth point is called a "running" point; all resistance having been removed, operation is efficient. As each motor has half the supply voltage impressed, the motors will operate at approximately half speed.

In order to connect the motors finally in parallel without removing power completely, a number of transition points are necessary. As is shown in Fig. 7–30, on the first transition point, *A*, the motor circuits are returned to those of the first point. On point *B* a ground connection is made on the negative side of the first motor, which is the same as short-circuiting the second motor. On point *C*, the second motor is entirely disconnected.

On the fifth point the two motors are in parallel with each other with the starting resistance again in series; this is a resistance point. The sixth, seventh, and eighth points again progressively cut out resistance, with it all out at point eight. On this last point the two motors are operating with full voltage on each motor.

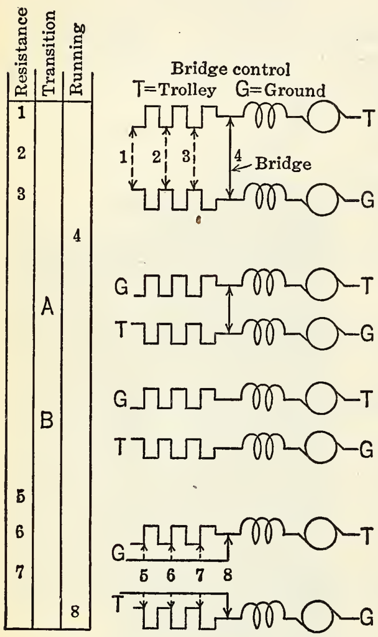


FIG. 7–31. Bridge method of operating series motors in series and in parallel. During the transition period both motors exert torque, giving smoother acceleration than with Type K control.

Consideration of the transition points will indicate that these points must be passed over rapidly. Only one motor is exerting torque and there will be some loss in acceleration of the car during this period. It is because of this lack of smoothness in acceleration through the transition that the "bridge" method of control is preferred.

52. Bridge Method of Series-parallel Control. On the first position of the controller in the "bridge" method of control a series circuit is set up as before with the resistances between the two motors, as shown in Fig. 7–31. The resistance is progressively cut out in points two, three, and four, it being all removed in running position four.

In transition point A, trolley and ground connections are made to the open ends of the resistances. With the "bridge" (the connection which at point four short-circuited the resistances) still left, there are, at point A, two parallel circuits between trolley and ground. The first is through the two motors in series and the second circuit is through all the starting resistances. Both circuits are completed over the "bridge" but the two currents flow in opposite directions. In the second transition point, B, the bridge is removed and each motor is connected across the supply with half the total starting resistance in series.

In the remaining points the resistance is again progressively removed from both motor circuits, with it all removed at running point eight. Additional running points at higher speed may be obtained by placing a diverter around the series fields of the motors.

Because it gives much smoother acceleration and therefore less discomfort to the passengers, the "bridge" control is preferred. It is somewhat more expensive in that the controllers themselves and the wiring between the motors and the controller are more complicated.

53. Multiple-unit Control. As the capacity of motors of a car become larger (300 hp and more) the controllers required to handle the large currents become too bulky and dangerous to place on the vestibules of passenger cars. Further, it becomes impossible to handle the currents required by several cars or locomotives in a single controller.

A multiple-unit system consists essentially of groups of contactors, often called a switch group, located in each motor car or locomotive which, when sequentially operated, control the switching of the motors of the individual car or locomotive, usually according to the "bridge" method. Each car or locomotive has its own connection to trolley and ground, and so far as its own power circuits are concerned it is independent of the other units.

The contactors in each car may be operated either electromagnetically or electropneumatically. An electromagnetically operated contactor has a solenoid which when energized from a control circuit causes the contactor to close either against spring pressure or against gravity. In an electropneumatically operated contactor a small solenoid, when energized, opens an air valve and the contactor is closed by air pressure.

The solenoids of the contactors in each car are controlled from any controller of the train through a "train line," a group of 8 to 12 wires running the length of the train, the connections between cars being made by "couplers."

A simplified diagram of a multiple-unit control with only a few contactors is shown in Fig. 7–32. The train line is made continuous over the length of the train by couplers of flexible wires. The power line is tapped to both controllers, and one of the controllers is energized by closing one of the switches $S1$ or $S2$ in one of the cars.

It will be seen that contactor 1, in each car, connects the armature of motor 1 to the trolley; contactor 2 connects the armature of motor 2 to the ground or rail return. Contactor 3 connects the starting resistances (only two steps are shown) together, and contactors 4 and 5 short-circuit the resistances R_1 and R_2. On the first point of the controller wires 1, 2, and 3 of the train line are energized so that contactors 1, 2, and 3 in each car are closed. This sets up a circuit in each car from the trolley through motor 1, R_1, R_2, and motor 2 to ground. On the second controller point contactor 4 will short-circuit resistance R_1, and on the third point contactor 5 will short-circuit resistance R_2. This will be a running point with the

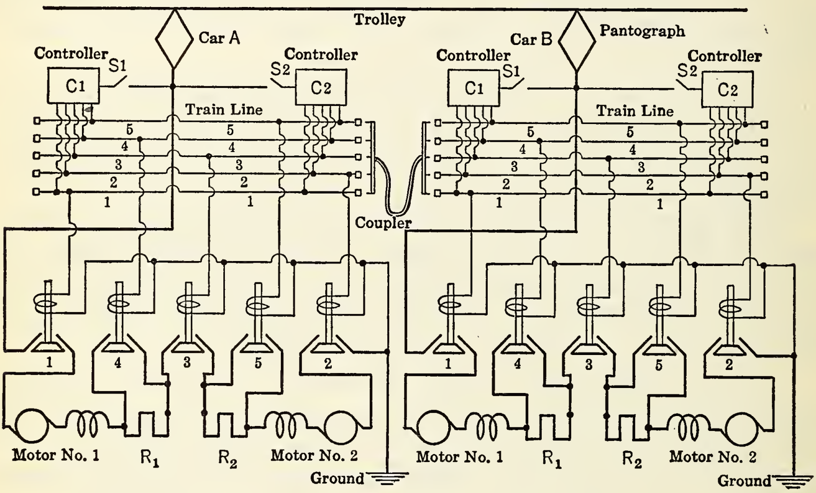


FIG. 7–32. Simplified partial diagram of a multiple-unit control, with only a few contactors shown. Power circuits are shown in heavy lines; control circuits are in light lines.

two motors in series across the source with all starting resistance cut out. Additional contactors will complete the sequence of the bridge control.

54. Manual and Automatic Systems. In the multiple-unit system shown in Fig. 7–32 the sequence of switching is manual or non-automatic, the motorman having it in his power to regulate the current to any value he pleases by moving the controller handle, the motor connections being those determined by the position of the controller handle.

In automatic control certain automatic devices prevent the motorman from causing the motors to take a current greater than a predetermined value. With this method the motors start with a definite current (determined for the first controller point by the total series resistance), but only when the current has decreased to a specified value can a change in connections be made. The motorman may have moved his controller to the

second point or beyond, but the contactors of the second point cannot be closed until the current on the first point has dropped to the specified value.

There is thus an automatic "follow-up" system. The motorman usually puts his controller directly to the last point (two motors in parallel without resistance) but the sequence of switching is fully carried out automatically, the contactors at any point closing only when the current of the previous point has dropped to the specified value. The automatic control thus keeps the rate of acceleration and the current practically uniform throughout the starting period. It is nearly always used in connection with multiple-unit control.

Another automatic device always used with multiple-unit control is the "dead-man's" button. This is a button on the controller handle which the motorman must depress, against spring pressure, to energize the contactors of even the first point. He must continue to keep the button depressed so long as power is applied, for, if the button is allowed to snap open, power is automatically removed and the brakes automatically set.

55. Use of a Flywheel with a Compound Motor. The rate at which power is required by some kinds of machine tools is very irregular. In the punch press, or forming press, for example, but little power is required until the die comes in contact with the metal to be punched or formed. The rolling mill is another instance of a tool calling for an irregular power supply.

If the mass of the moving parts is not very great on such machines, the power consumption of the driving motor will be very irregular. Figure 7–33 illustrates the point; the full-line curve shows the current consumption of a motor driving a shears for cutting large iron bars. Before shearing begins the driving motor runs at practically no load and requires but little current. The shearing process requires a great deal of power and a large current and the driving motor must be made sufficiently large to commutate successfully the *greatest value* of the current as at A. As soon as the shear is completed the motor very rapidly regains its light-load speed.

If the motor driving the shears is a compound-wound motor with a comparatively large number of turns in its series field (say 30-40 per cent compounding), and a heavy flywheel is put on the armature shaft, the current to the motor, while doing the same work as before, will be given by the dashed curve of Fig. 7–33, in which the maximum value is much less than it was before. In fact, this motor equipped with the flywheel might be much smaller than the one without the flywheel. The motor, through which current A' is the maximum value, need be only about two-thirds as large as if current A were the current input.

56. Effect of the Flywheel. The action of the motor equipped with a flywheel is as follows: At time t_1, in Fig. 7–33, the load on the motor suddenly increases and causes the motor to begin to slow down. As it does

so, the rotating flywheel is slowed down also and thus gives up some of its kinetic energy. During the time from t_1 to t_2 the electrical input to the motor is not as large as the power demanded by the load; hence the motor slows down and the retarding flywheel assists the motor to carry the load. During this slowing-down process, the current input to the motor must increase somewhat.

If at time t_1 the speed is 1000 rpm, and at time t_2 the motor has slowed down to 800 rpm, and the speed-load curve of the motor is as given in Fig. 7–34, the current must increase during the same period from OD to OE. In Fig. 7–33 the abscissae at_1 and bt_2 are equal, respectively, to the abscissae OD and OE of Fig. 7–34. At time t_2 (after the cutting operation is over) the power demanded by the load is less than the input to the

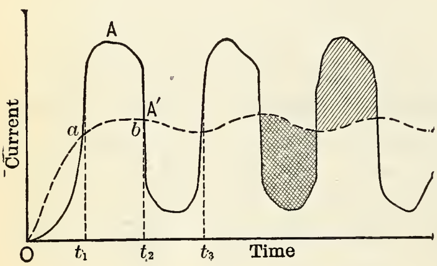


FIG. 7–33.

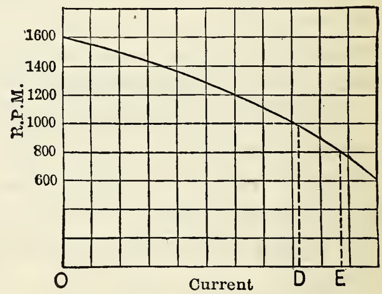


FIG. 7–34.

FIG. 7–33. Typical curve of armature current required by a motor operating a punch press, shears, or similar duty. With no flywheel the current is as shown by the full-line curve and, when a sufficiently massive flywheel is fitted to the motor shaft, the current demand has about the appearance of the dashed curve.

FIG. 7–34. Speed-load curve of a heavily compounded motor.

motor, and so the motor begins to speed up and to increase the energy stored in the flywheel; the current begins to decrease at the same time. At time t_3 the motor has regained its original speed of 1000 rpm and the current has fallen to the value OD. The single-hatched area in Fig. 7–33 represents the amount of energy which the retarding flywheel gives up when the load is heavy, and the double-hatched area represents the energy which the motor returns to the flywheel when the load is light; these two areas are equal.

Consideration will show that neither the shunt nor the series motors are suitable for use with a flywheel, where energy is to be periodically taken from and stored in the flywheel. The amount of energy to be taken from the flywheel will be proportional to the change in its speed. If a shunt motor is used the change in speed from no load to full load of the motor is less than 10 per cent, so that there will still be large current fluctu-

ations. If a series motor is used the speed changes must be very large to cause much change in motor current. The compound motor is the required compromise.

It will also be seen that the amount of compounding used in the motor and the amount of flywheel used will depend upon the time-cycle of operations. If the time between operations is short, less flywheel action is necessary than if the time between operations is long. In the latter case more energy may be taken from the flywheel since the motor will have a longer period to restore it.

57. Dynamic Braking. When a motor is to be started and stopped frequently, the question of getting sufficient retarding force to bring it quickly to rest is to be considered. Ordinary frictional braking is possible, the pressure of the brake shoe being applied by a powerful electromagnet, as shown in Fig. 2–53, page 69. Another method is to make the retarding motor act as a generator by disconnecting its armature from the power supply and connecting the brushes together through a suitable resistance. The amount of current supplied to the resistance depends upon the speed of the motor (acting as a generator and driven by its stored kinetic energy of rotation), the field strength, and the value of the resistance; the braking effort of the motor is proportional to its field flux and the armature current. Sometimes the field circuit is disconnected from the line, leaving in the field poles only the residual magnetism. The armature is then short-circuited, and sufficient current flows to give the desired braking effort.

In electric railroads, regenerative braking is used where the trains are required to descend long grades. When d-c motors are used for the motive power, it is possible to make them pump power back into the trolley system when running downhill, thus not only acting as efficient brakes but also saving the mechanical braking equipment. As a series motor will not reverse the direction of its current, no matter what the speed may be, it is necessary to make the motors, which are to be used for braking, separately excited when braking. One method of obtaining excitation for the motors acting as generators is by a small motor-generator set installed in the locomotive. The amount of excitation current depends upon the speed at which the train is operating and the amount of braking action desired.

While energy may be saved on systems with long grades, the chief advantage of regenerative braking on railroads is found in the saving of brake shoes and wheel rims. Even if no power at all were saved by the scheme, it would still be worth while because of the saving in wear of shoes and wheels.

58. Balancers. A balancer is a motor-generator set used for the purpose of providing a neutral connection and maintaining equality, or approximate equality, between the voltages on the two sides of a three-wire system.

In Fig. 7–35, a three-wire system is shown, the voltage across the outside of which is maintained constant at 220 volts by an ordinary 220-volt generator. The balancer set, consisting of two duplicate machines, mechanically direct-connected, have their armatures electrically in series across the two outside wires; the neutral wire is connected between the two armatures. The shunt fields are also generally connected in series across the outside wires, as shown.

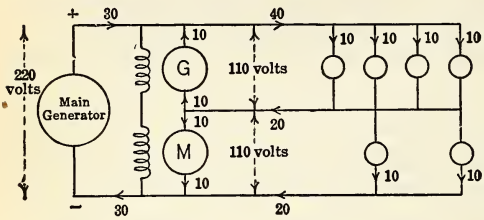


FIG. 7–35. Showing the connection of a balancer set to a three-wire distribution system, the neutral wire carrying 20 amperes, due to the unbalanced load. This diagram assumes that the balancer consumes no power to run itself.

The action of the balancer set in maintaining the potential of the neutral midway between the two outside wires may be seen from Fig. 7–35. When the load is unbalanced, the unit connected across the heavily loaded, lower voltage, side will act as a generator while that across the lightly loaded, higher voltage, side will operate as a motor. It is this difference in voltage, resulting from the unbalance of the load, that determines the function of each machine. Since the two machines are mechanically coupled, the machine with the higher voltage across it will drive the other, and hence becomes the motor. If the machines comprising the balancer set are assumed to have no losses, the neutral current will divide equally between them, and the power absorbed by the lower machine, or motor, will be given out by the upper one acting as a generator. In Fig. 7–35, the current distribution throughout the system is indicated by arrows and figures, the field currents of the machines being disregarded.

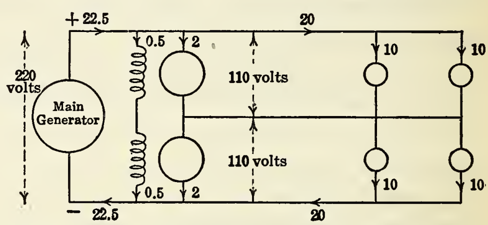


FIG. 7–36. Conditions in a balancer set when the two loads are equal, the neutral carrying no current. It is assumed that the balancer units require currents as shown to supply their no-load losses.

Transferring the unbalance in the loads to the other side will simply reverse the function of the two units; the machine acting before as motor becomes a generator, and vice versa.

When there is no load on the system, or when the loads on the two sides are equal, the two machines of the balancer set, having the same voltage across each, act as motors in series, as shown in Fig. 7–36. In this figure the shunt fields of the two machines are also connected in series

across the outside wires, giving both fields the same constant value of current. It is assumed that the field current is 0.5 ampere, and that with no load on the balancer an armature current of 2 amperes is required for each motor; the main generator therefore furnishes 20 amperes at 220 volts to the load and 2.5 amperes to the balancer set.

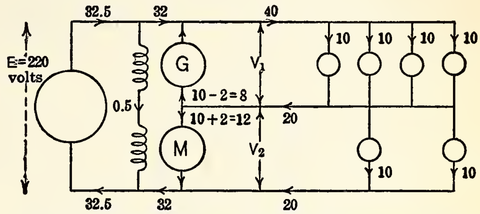


FIG. 7–37. With the amount of unbalance indicated in Fig. 7–35, and with the losses assumed in Fig. 7-36, the current distribution in the system will be as shown here.

If, with the unbalanced load of Fig. 7–35, the losses of the balancer set are assumed the same as in Fig. 7–36, the current distribution will be as in Fig. 7–37; obviously the losses of the entire balancer set must be supplied through that machine which is acting as a motor. Actually the losses will be somewhat greater when the balancer is loaded, causing a difference in the armature currents still greater than that shown.

With an unbalanced load, the terminal voltage of the generator G will be

$$V_1 = E' - I'_a R'_a$$

and that across the motor

$$V_2 = E'' + I''_a R''_a$$

where E', I'_a, and R'_a are, respectively, the generated emf, armature current, and armature resistance of the generator G, and E'', I''_a, and R''_a are corresponding quantities for the motor M. E'' is thus the counter emf of the motor M.

Since both the machines are operating at the same speed and have the same field current, E' must be equal to E''. It follows that the voltage V_1 across the heavily loaded side must be less than the voltage V_2 across the lightly loaded side by an amount equal to the sum of the two IR drops.

Thus, with the field connections so far shown, the automatic response of the balancer set depends for its action upon an unbalancing of the voltage. The amount of voltage unbalance may be somewhat reduced and the operation of a balancer set thereby improved by cross-connecting the shunt fields of the two machines as in Fig. 7–38. Now as the voltage across the loaded side falls, it will weaken the field of the machine operating as motor; holding up the speed of the set. An increase in the voltage across the lightly loaded side will at the same time strengthen the field of the generator, and thereby somewhat increase its generated emf. There must, however, be still some voltage unbalance to initiate the action.

Perfect regulation may be obtained if compound-wound machines are

used, their series fields being connected in series in the neutral in such an order that the neutral current flowing through the two series fields increases the field strength of the generator and weakens that of the motor. The connections for this arrangement are shown in Fig. 7–39, the upper machine being across the heavily loaded side and therefore acting as a generator. The connections are, however, such that, if the load unbalance is shifted and the neutral current reverses, the lower machine, now acting as gener-

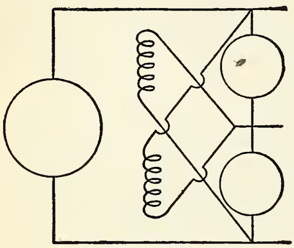


Fig. 7–38.

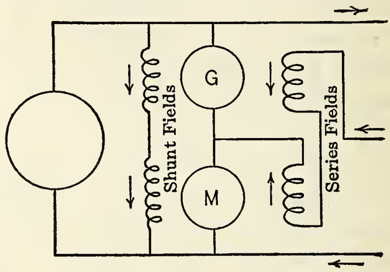


Fig. 7–39.

Fig. 7–38. By cross-connecting the shunt fields of the balancer set, it is possible to reduce the amount of unbalance in the voltage for a given amount of unbalance in current.

Fig. 7–39. By suitably compounding the units of the balancer set and connecting the series field in the neutral wire, it is possible to keep the voltage division between the two sides of the system as nearly equal as may be desired. The voltage on the loaded side may even be made to rise with load.

ator, has its field strengthened and the upper machine, now a motor, has its field weakened.

With this arrangement the actual amount of series-field current that each machine will receive must be carefully adjusted by means of diverters across the series fields. If the series fields are too strong, the voltage on the heavily loaded side will increase.

In practice, balancer sets are not usually called upon to handle more than 15 or 20 per cent unbalance in load; i.e., the neutral current will not be more than 15 to 20 per cent of the full-load current of the main generator. The actual current through either unit of the set will generally be less than the neutral current, so that the balancer units are relatively small machines.

PROBLEMS

7–1. A bipolar motor has an armature 8 inches in diameter and 10 inches long; polar angle is 120°. There are 55 coils of 2 turns each, and the air-gap density is 5500 lines per sq cm. Neglecting pole fringe, how many horsepower are being developed, if the armature takes 65 amperes and turns 1350 rpm? If the friction and core losses are 350 watts, what is the torque at the pulley?

7–2. A motor has a total length of active armature conductor of 1220 inches, lying in a magnetic field of average density equal to 8500 lines per sq cm. The current in the armature conductors is 14 amperes. What is the total force exerted by the armature conductors? If the armature has a radius of 4 inches, how much is the torque developed, in pound-feet? If the motor is turning 780 rpm, how many horsepower are being developed?

7–3. If the above motor is connected to its load by belt, and the belt pulley is 3 inches in radius and the pull of the belt on one side is 220 pounds, how much is it on the other? (Two answers possible.)

7–4. In a multiple-circuit armature, 30 inches in diameter and wound for 8 poles, there are 112 coils of 1 turn each. The length of the pole face (parallel to the shaft) is 13.5 inches. Sixty-five per cent of the inductors lie under the pole face where the flux density is 57,000 lines per square inch and 10 per cent in the pole fringe where the average density is 30,000 lines per square inch. Armature current is 1760 amperes, and the machine makes 900 rpm. What values of torque and horsepower are being developed? What value of counter emf is being generated?

7–5. There are 160 coils of 2 turns each on the multiple-circuit armature of an 8-pole, 600-rpm motor. The armature diameter is 19.5 inches and the length of the pole face (parallel to the shaft) is 12 inches. Sixty per cent of the conductors lie under the pole face in a flux density of 55,000 lines per square inch and 10 per cent lie in the pole fringe in an average density of 30,000 lines per square inch. The armature current is 1600 amperes. What value of counter emf is generated? What is the product, in horsepower, of the counter emf and the armature current? What is the total peripheral force developed by the armature? What are the torque and horsepower developed?

7–6. In a 6-pole motor with a multiple-circuit armature there are 108 coils of 2 turns. The armature diameter is 31.5 inches and the speed is 900 rpm. The length of the pole face (parallel to the shaft) is 13.5 inches. Sixty-five per cent of the conductors lie under the pole face where the flux density is 57,000 lines per square inch and 5 per cent lie in the pole fringe where the average density is 30,000 lines. The armature current is 1830 amperes. What values of torque and horsepower are developed? What value of counter emf is being generated? What is the product of armature current and counter emf, in horsepower? Is all the horsepower developed available at the pulley?

7–7. A 4-pole lap-wound motor has 1.2×10^6 lines per pole, and 55 armature coils of 6 turns each. If the armature current taken from the line is 40 amperes, what will be the torque?

7–8. If the motor of problem 7–7 is turning at 1200 rpm and the armature current

is 35 amperes, how many horsepower are being developed? If the speed is cut down to 600 rpm, by resistance in series with the armature, and the armature current is 60 amperes, how many horsepower are being converted by the motor into mechanical power?

7-9. A 6-pole motor with a multiple-circuit armature has 8.6×10^6 lines per pole, 432 total conductors, and an armature current of 1830 amperes. What are the torque and horsepower output at 900 rpm?

7-10. What values of torque and horsepower are developed at 600 rpm by a motor with a multiple-circuit armature if it has 3.3×10^6 lines per pole, 640 armature conductors, and draws an armature current of 1600 amperes?

7-11. The series-wound armature of a 4-pole, 230-volt shunt motor has an effective length of 4.25 inches and has 29 slots with 10 inductors per slot, the conductor being 30,000 cir mils in cross-section. There are 145 commutator bars, and the total length per coil is 33 inches. The total armature current is 145 amperes and the flux per pole is 1.3×10^6 lines. With the armature at 75 C and brush drop of 2 volts, what is the counter emf, and at what speed is the motor turning?

7-12. A bipolar shunt-wound motor takes 52.2 amperes from a 220-volt line; the field resistance is 125 ohms and the armature resistance is 0.12 ohm. The flux is 3.05×10^6 lines per pole, and the number of conductors is 280. What is the counter emf, and at what speed is the motor turning? Allow 2 volts for total brush drop.

7-13. What is the counter emf of a 4-pole, 230-volt shunt-wound motor with a multiple-circuit armature if the armature current is 75 amperes and the armature resistance is 0.21 ohm? Full-load brush drop is 2.5 volts. If there are 360 armature conductors and the flux per pole is 2.94×10^6 lines, at what speed is the motor turning?

7-14. In the previous problem, if a resistance of 1 ohm were inserted into the armature circuit, the effect would be as if the armature resistance were $0.21 + 1 = 1.21$ ohms. Under these conditions what would be the values of counter emf and speed? Other quantities as in problem 7-13.

7-15. A 4-pole, 220-volt shunt motor with a series or two-circuit armature winding draws an armature current of 275 amperes. The armature resistance is 0.036 ohm, and the full-load brush drop is 2 volts. There are 270 armature conductors and the flux per pole is 4.9×10^6 lines. What is the full-load counter emf, and at what speed is the motor turning?

7-16. What would be the answers to the previous problem if a resistance of 0.15 ohm were inserted into the armature circuit?

7-17. A certain 115-volt shunt motor draws a line current (armature plus field current) of 220 amperes. The shunt-field resistance is 28.75 ohms and the armature resistance is 0.023 ohm. How many watts are used as heat in the field and how many in the armature? What is the brush-contact loss if brush drop is 2 volts? If the core loss (hysteresis and eddy current loss in the armature iron) and friction loss combined amount to 300 watts, how many horsepower are being delivered at the pulley? What is the speed if the torque is 135 pound-feet?

7-18. How much current will the motor of the previous problem draw from the line at no load?

7-19. A certain 240-volt shunt motor has an armature resistance of 0.207 ohm, a field resistance of 76 ohms, and a brush drop of 2 volts. The line current (armature plus field current) is 64 amperes. How many watts are lost as heat in the armature, the field, and at the brush contacts? If the total friction and core loss (hysteresis and eddy current loss in the armature iron) combined are 800 watts, what value of horsepower is delivered at the pulley? What is the torque in pound-feet if the speed is 750 rpm?

7–20. What will be the value of the no-load line current for the motor of the previous problem?

7–21. The field winding of a shunt motor has a resistance of 105 ohms, and is connected to a 240-volt line. The total current taken by the motor from the line is 55 amperes, and the armature resistance is 0.18 ohm. How much power is used as heat in the field, and how much in the armature? If the core loss and friction loss together are 280 watts, how many horsepower are being delivered? What is the speed, if the torque is 110 pound-feet?

7–22. How much current will the motor of the previous problem draw from the line at no load?

7–23. A 110-volt shunt motor has an armature resistance of 0.42 ohm and constant brush-contact drop of 3 volts. Field resistance is 130 ohms. When drawing 39.5 amperes from the line, the motor runs at 1150 rpm. At what speed must the machine be driven as a generator to deliver 45 amperes to a load at 110 volts? What is the difference in the generated voltage when taking 40 amperes from the source as a motor and delivering 40 amperes to a load as a generator, both on a 110-volt line? Consider the field current in all calculations.

7–24. A 120-volt shunt motor has an armature resistance of 0.0035 ohm and a field resistance of 15 ohms. When running at 750 rpm the total current taken from the line is 808 amperes. (*a*) What is the counter emf, if the brush drop is 2.5 volts? (*b*) If the above machine is driven as a generator, what armature current would it have if the load current were 792 amperes at a terminal voltage of 120 volts? (*c*) What is the generated voltage? (*d*) At what speed must it be driven? (If field current is constant, speed and voltage vary directly.)

7–25. When running at 1800 rpm a certain 120-volt shunt motor draws a total current of 82.4 amperes from the line. Its field resistance is 50 ohms and its armature resistance is 0.11 ohm. (*a*) What is the counter emf, if brush drop is 2 volts? (*b*) If the machine were driven as a generator and supplied 77.6 amperes to a load at 120 volts, what would be its armature current? (*c*) What voltage would be generated? (*d*) At what speed must it be driven? (If field current is constant, how will speed and voltage vary?)

7–26. A bipolar, 110-volt shunt motor has a full-load armature current of 95 amperes, an armature resistance of 0.092 ohm, and 280 conductors on the armature arranged in a full-pitch winding. The flux is 3.45×10^6 lines per pole. If the brush-contact drop is 2 volts at no load, and 3 volts at full load, what are the no-load speed and the full-load speed? The friction and core losses are 450 watts.

7–27. In the above problem, the field current is reduced 10 per cent by the field rheostat, and the reluctance of the magnetic circuit thereby is reduced by 2 per cent. Answer same questions as for problem 7–26.

7–28. By how many ampere-turns would the effective field mmf be reduced by armature reaction, if the brushes in the machine of problem 7–26 were moved back by 20°, the armature carrying full-load current?

7–29. A 10-hp, 110-volt, 900-rpm shunt motor has an armature resistance of 0.08 ohm and requires 75 amperes armature current at full load. If the flux per pole were suddenly reduced to 80 per cent of normal, find the value of armature current and torque at the instant the flux is reduced, in percentage of full-load values and also in amperes and pound-feet, respectively.

7–30. A 50-hp, 230-volt, 1150-rpm (at full load) shunt motor has an armature resistance of 0.091 ohm and requires 194 amperes armature current at full load. If the flux per pole were suddenly reduced to 80 per cent of normal, find the value of armature current and torque at the instant the flux is reduced, in amperes and pound-feet,

respectively, and also in percentage of full-load values. Brush drop is constant at 2 volts.

7–31. A 25-hp, 240-volt shunt motor requires an armature current of 94 amperes at a full-load speed of 750 rpm. Armature resistance is 0.2 ohm, field resistance is 76 ohms, brush drop is 2 volts (constant). What armature current would the motor require and what value of torque would it exert at the instant the flux were reduced to 85 per cent of normal? Give values in percentage of full-load values and also amperes and pound-feet respectively.

7–32. A series-wound armature for a 4-pole, 220-volt shunt motor has full-load current of 90 amperes. The field resistance is 40 ohms, and the armature resistance is 0.15 ohm. The flux per pole is 3.8×10^6 lines, and there are 280 conductors on the armature. The no-load input to the armature is 900 watts. What is the no-load speed, full-load speed, and speed regulation? Assume brush drop as 2 volts at no-load and as 3 volts at full-load.

7–33. What would be the speed of the motor of problem 7–32 with full-load current flowing, if there were inserted in the armature circuit an extra resistance of 1.2 ohms?

7–34. A 4-pole, 220-volt shunt motor draws 275 amperes armature current at full load with an armature resistance of 0.052 ohm. There are 270 conductors on its multiple-circuit winding and the flux per pole is 4.9×10^6 lines. Friction, windage, and core loss (practically the entire armature input at no load) amount to 1150 watts. If brush drop is constant at 2.5 volts, what are the no-load and full-load speeds?

7–35. In the motor of the previous problem the field current is reduced by 15 per cent by means of a field rheostat and the reluctance of the magnetic circuit is thereby reduced by 5 per cent. Answer the same questions as for the previous problem.

7–36. A 4-pole, 220-volt shunt motor with 310 conductors in its series-wound armature draws 140 amperes armature current at full load. The armature resistance is 0.075 ohm, brush drop is constant at 2 volts, and the flux per pole is 4×10^6 lines. Friction, windage, and core loss (practically the entire armature input at no load) equal 1200 watts. What are the no-load and full-load speeds?

7–37. If the field current of the motor of problem 7–36 is reduced 12 per cent by means of a field rheostat, the reluctance of its magnetic circuit is reduced 4 per cent. Under such conditions what would be the no-load and full-load speeds of the motor?

7–38. The field-circuit resistance of a motor is 40 ohms at 70 F, and after it has run for 2 hours the resistance of the field circuit is found to be 48.5 ohms. What is the final average temperature of the winding?

7–39. A 110-volt shunt motor having an armature resistance of 0.15 ohm has a normal speed of 950 rpm when carrying full-load armature current of 50 amperes. How much resistance must be put in series with the armature if it is desired to give full-load torque at 450 rpm? Brush drop is 2 volts.

7–40. When drawing a full-load armature current of 96 amperes, a certain 230-volt shunt motor runs at 1750 rpm. The armature resistance is 0.16 ohm and brush drop is constant at 2.5 volts. It is desired to reduce the speed by adding resistance to the armature circuit. How much resistance must be added if full-load torque is desired at 800 rpm? How much with half full-load torque at 1200 rpm?

7–41. A 115-volt shunt motor at full load runs at 1150 rpm and draws an armature current of 96 amperes. Armature resistance is 0.08 ohm. Brush drop is constant at 2 volts. How much resistance must be added to the armature circuit to reduce the speed to 750 rpm with full-load torque? To 500 rpm at 60 per cent rated torque?

7–42. A 550-volt series railway motor is geared to the shaft of the car wheel, the gear ratio being 20 : 63 and the car wheels being 33 inches in diameter. What is the motor speed when the car speed is 20 miles per hour? If a tractive effort of 3500 pounds

is required of the motor and 11 per cent of the motor torque is used up in gear and axle friction, what torque is the motor exerting? What is the horsepower output of the motor?

7–43. Find the speed of a railway motor, in rpm, corresponding to a car speed of 25 miles per hour, the gear ratio being 15 : 64, and the car wheels being 33 inches diameter. Assuming 10 per cent of the motor torque used up in gear and axle friction, what is the motor torque in pound-feet if the tractive effort required of the motor is 2400 pounds? What is the horsepower output of the motor?

7–44. A 600-volt series railway motor is geared by a ratio of 20 : 63 to car wheels which are 36 inches in diameter. If the car speed is 30 miles per hour, what is the motor speed? If a tractive effort of 1300 pounds is required of the motor and 12 per cent of the motor torque is used up in gear and journal friction, what are the torque and horsepower of the motor?

7–45. A 4-pole, wave-wound, 600-volt railway motor has 140 turns per field coil. Armature resistance is 0.21 ohm, and field resistance 0.15 ohm. Full-load current is 90 amperes. Number of conductors on the armature is 912. With 30 amperes flowing, the flux per pole is 1.8×10^6 lines. Assume reluctance of the magnetic circuit constant. What is the speed for 50, 100, and 150 amperes? Assume brush drop constant at 3 volts.

7–46. At what speed will the motor of problem 7–45 run, if 2.5 ohms resistance are inserted in series with it, and the current is 45 amperes?

7–47. In a certain 600-volt railway motor there are 125 turns per field coil and 962 conductors on its two-circuit armature. The field resistance is 0.22 ohm and the armature resistance is 0.18 ohm. With 60 amperes flowing, the flux per pole is 2.22×10^6 lines. As the current through the field-circuit changes, the reluctance of the magnetic circuit changes as follows:

Current in amperes	30	60	90	120
Reluctance of magnetic circuit in per cent	74.0	100.0	129.1	160.9

At what speed will the motor run at each given current? Brush drop is constant at 3 volts, and the motor has 4 poles.

7–48. There are 270 armature conductors on the wave-wound armature and 18 turns per field coil in a certain 4-pole, 220-volt series motor. With 275 amperes flowing, the flux per pole is 4.9×10^6 lines. The armature resistance is 0.05 ohm and the field resistance is 0.01 ohm. As the field current changes, the reluctance of the magnetic circuit changes as follows:

Current in amperes	100	200	275	350
Reluctance of magnetic circuit in per cent	52.0	78.6	100.0	121.7

At what speed will the motor run at each given current?

7–49. The armature resistance of a 20-hp, 250-volt compound motor is 0.158 ohm. The armature current at no load is 4.1 amperes and 70 amperes at full load. The resistances of the series field and commutating field are 0.06 ohm and 0.05 ohm respectively. The resistance of the shunt field (connected long shunt) is 114 ohms. The no-load speed is 1250 rpm. At full load the series field increases the flux by 40 per cent. Calculate the full-load speed.

7–50. Calculate the resistance steps and the steady speed for each step in the starting rheostat for a 15-hp, 115-volt shunt motor under full load, the current inrush at each step to be 150 per cent of full-load armature current. Full-load efficiency of the motor is 84 per cent, the armature resistance is 0.06 ohm, the shunt-field resistance is

62.5 ohms, full-load brush drop is 2 volts, and brush drop at 150 per cent armature current is 2.5 volts. Rated full-load speed is 1200 rpm.

7–51. Calculate the resistance steps in a starting rheostat for a 25-hp, 1000-rpm, 220-volt shunt motor, the efficiency of which is 85 per cent. Armature current is to be limited to 150 per cent. Full-load brush drop is 2 volts and with 150 per cent armature current it is 3 volts. Armature resistance is 0.2 ohm, and the shunt-field resistance is 127 ohms. What steady speed will the motor reach on full load at each resistance step?

7–52. Determine the resistance steps in the starting rheostat for a 1000-hp, 720-rpm, 575-volt shunt motor. Full-load efficiency is 93.4 per cent and brush drop is constant at 2.5 volts. The inrush current is to be limited to 150 per cent of rated current. Armature resistance is 0.009 ohm and shunt-field resistance is 93 ohms. At what steady speed will the motor operate at each resistance step?

7–53. A 4-pole motor with multiple-circuit armature winding draws an armature current of 68 amperes. The self-inductance per coil is 0.8 millihenry and the time of commutation is 0.0012 second. What voltage must the commutating poles induce in the short-circuited coil if the voltage of self-induction is to be overcome, there being 4 commutating poles?

7–54. (*a*) A 600-volt series motor has a rated current of 80 amperes and a full-load rated speed of 660 rpm. The resistances of the armature, series field, and commutating field are respectively 0.3222 ohm, 0.190 ohm, and 0.139 ohm. Brush drop is constant at 3 volts. Calculate (1) the full-load counter emf when 600 volts are impressed and (2) the full-load counter emf and speed when 300 volts are impressed.

(*b*) Two motors as in (*a*) are to be started against rated full-load torque by series-parallel control, the inrush current at each step to be limited to 120 amperes. (1) How much starting resistance is necessary when the motors, in series, are connected to the line? (2) If we assume that the steady conditions of the series running point hold steady throughout the transition points, how much starting resistance is necessary when both the motors are first connected in parallel?

7–55. Two 25-hp, 1200-rpm, 110-volt shunt motors are directly connected mechanically, and their shunt fields are in series on a 220-volt line. At what speed, approximately, will the motors run if the armatures, in series, are connected to a 220-volt line? If connected to a 110-volt line? If the fields are in parallel on a 110-volt line and armatures in series on a 220-volt line?

7–56. For the first case in problem 7–55, the field strength of one motor is reduced by shunting it with a resistance just equal to the field resistance. At what approximate speed will the motors run, and how much power will each motor deliver when carrying full-load current in its armature? Assume permeability constant and disregard brush drop.

7–57. A 110-volt motor is fitted with a steel flywheel weighing 700 pounds, the radius of gyration of which is 15 inches. If the current during the starting period is 20 amperes, and the average voltage on the armature during the starting period is 50 volts, and the armature losses are negligible, how long does it take to bring the flywheel to a speed of 500 rpm? How much longer would it take to get it to 800 rpm?

7–58. The frictional and core losses of the motor in problem 7–57 are 450 watts at 700 rpm and may be assumed directly proportional to the speed. How long will it take for the motor to come to rest after the power is shut off, for the two speeds mentioned in problem 7–57?

7–59. A car weighing 10 tons is running down a 5 per cent grade at a speed of 20 miles per hour, being held at this speed by friction and regenerative braking. If the tractive effort to overcome friction at this speed is 500 pounds, how much current is the motor delivering to the trolley, this being at 600 volts above ground?

7–60. In a 220/440-volt, three-wire system, using a balancer set, the current in the + wire is 150 amperes and the current in the − wire is 100 amperes. What is the current in all parts of the system? If the efficiency of the machines of the balancer set is 82 per cent each, what will be the current in all parts of the system?

7–61. In a 220/440-volt, three-wire system, using a balancer set, the current in the + wire is 125 amperes and the current in the − wire is 100 amperes. The efficiency of the main generator is 88 per cent. In the balancer set the machines have an efficiency of 80 per cent operating as a motor and 82 per cent operating as a generator. Find the currents in all parts of the system. What is the overall efficiency?

CHAPTER VIII

LOSSES, EFFICIENCY, HEATING, AND RATING

1. Generator and Motor Losses. The electric generator or motor is a converter of power. The generator, driven by a prime mover, receives its input in mechanical form and delivers its output in electrical form; the motor receives electrical energy and converts it into mechanical torque and speed.

In both generators and motors a portion of the input is converted into heat in various ways, and thus lost, so that the output of the machine will be equal to its input minus the losses.

The losses occurring in d-c machines may be classified as follows:

- Losses
  - Copper
    - Armature
      - Armature winding
      - Brush contacts
    - Fields
      - Shunt field
      - Series field
      - Commutating field
  - Mechanical
    - Bearing friction
    - Brush friction
    - Air friction or windage
  - Core or iron
    - Hysteresis
    - Eddy current
  - Stray load

Since the mechanical losses due to friction and the core losses are usually determined together, these combined losses are generally referred to as the *stray-power losses. This combined loss is not to be confused with the stray-load loss.*

2. Armature Copper Loss. The armature copper, or $I_a^2R_a$, loss is the loss due to current flowing in the armature. To determine this loss, for either a current which actually existed or for one that is assumed, the armature resistance is measured at the commutator.

This loss, like a number of the losses in an electrical machine, varies with many factors, and may be difficult to define in many cases. As an

example, the armature copper loss, or $I_a^2R_a$ loss, depends obviously upon the resistance of the armature, which in turn depends upon the temperature of the armature. The temperature depends upon a number of factors, as described in later paragraphs, and may not even be uniform throughout the armature windings. To avoid the confusion caused by such a situation, certain conventions have been adopted to describe the behavior of a machine. While these may lead individually to small errors, as a whole the error introduced is very small, and such a system of conventions does make possible a comparison of machines under the same conditions.

The temperature of the armature when the resistance measurement is made must be known, and the resistance corrected to the conventionally chosen standard reference temperature, 75 C. The temperature of the armature when the measurement is made may be taken as that of the surrounding air, if the machine has not been operated for some time.

Since the armature current of a motor or generator increases with the load, the armature copper loss increases more or less as the square of the load. The increase in resistance with heating as load is added is eliminated by conventionally taking the resistance at 75 C.

3. Brush-contact Loss. The brush-contact loss is the product of the armature current multiplied by the resistance of two brush contacts in series, that of a positive and a negative brush. As stated in Chapter V, brush-contact resistance varies inversely with current density and with brush pressure; it also depends upon the speed and condition of the commutator, the material of the brushes, and the temperature of the brushes and the commutator.

Also, as stated previously, the resistance of the brush contacts is depended upon to aid in obtaining good commutation, but as this resistance is increased the energy loss at the contacts also increases. Increasing brush pressure to decrease the contact resistance will increase the brush friction. Soft brushes, in general, have a lower contact resistance and a lower friction loss than hard brushes.

Experience has shown that with brush pressures of 1 to 5 pounds per square inch of contact area, and with current densities of 40 to 60 amperes per square inch of contact area, the best general operating conditions are obtained. With these values the brush-contact resistance varies inversely with the current in such a way that the voltage drop is practically constant over the current range of the average machine. For brushes with pig-tails (a flexible copper connection between the brush and the brush holder) the voltage drop is about 2 volts, and for brushes without pig-tails it is about 3 volts, both values being for a positive and negative brush in series.

Whereas there may be variations in these values up to 50 per cent in different machines. the brush-contact loss is not of such magnitude, com-

pared to other losses, as to require more exact values. Accordingly, the recommendations of the standardization rules of the A.I.E.E. are generally followed, and the brush-contact drop is taken as constant from no load to full load at 2 volts for a positive and a negative brush in series for brushes with pig-tails, and at 3 volts for brushes without pig-tails. The brush-contact loss in watts is then the armature current multiplied by 2 or 3 volts; it then varies directly with the load, as indicated in Fig. 8–1.

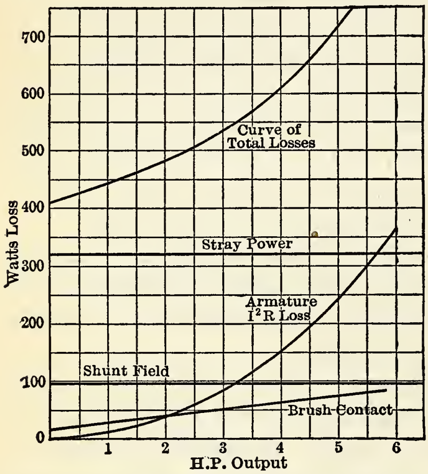


Fig. 8–1. The loss curves for a 5-hp, direct-current motor.

4. Shunt-field Loss. The loss in heat in the shunt field of a machine will be equal to the square of the field current multiplied by the shunt-field resistance. The shunt-field resistance is determined at some known temperature, and the value at the standard temperature of 75 C is calculated.

The present A.S.A. standards * for testing electrical machinery charge all field-rheostat losses to the plant of which the machine is a part and not against the machine. Accordingly, the loss in a shunt-field rheostat is not considered either as input to the machine or as a loss of the machine.

5. Series- and Commutating-field Losses. These losses are equal to the square of the current flowing through the respective fields, multiplied by the resistances. The values of resistances used should be those corresponding to the standard reference temperature of 75 C. Where a diverter is used in parallel with a series field, the resistance of the parallel combination is measured.

As the current in the series and commutating fields varies directly with the load, the losses in these fields will vary as the square of the load.

6. Bearing Friction. The loss due to bearing friction depends upon the area of contact between the shaft and the bearing, the speed of the shaft, and the condition of the lubricant. For sleeve bearings the friction loss will vary about as the 3/2 power of the speed. With ball or roller bearings the friction is less than with sleeve bearings.

7. Brush Friction. This loss depends upon the speed and the condition of the commutator and upon the pressure of the brushes. The stand-

* American Standards Association, American Standards for Rotating Machinery, C50–1936, paragraph 2.107.

ards of the A.S.A. suggest the conventional formula for the brush friction of carbon and graphite brushes, in watts, as $8.0 \times 10^{-3}\ AV$, where A is the total brush-contact area in square inches and V is the peripheral velocity of the commutator in feet per minute. Brush-friction loss may be determined experimentally as will be shown later.

Brush friction as well as bearing friction and windage losses are affected by load changes only so far as such changes may affect the speed.

8. Windage. The power required to overcome the resistance of the air surrounding the rotating armature of a machine is the air friction or windage loss. Since air is used to carry away the heat generated within a machine, the windage loss in a machine will depend upon its ventilation. In open machines fan blades are frequently attached to the armature to force a stream of air through the machine. In a closed machine, fan blades will cause an air stream to circulate through the armature and over the cooler inside surfaces of the frame. The greater the stream of air, the greater the windage loss, but the greater the amount of heat carried away per second.

The windage loss of a motor or generator depends upon the speed, the variation in loss being at some exponent between 2 and 3 of the speed.

9. Hysteresis Loss. When a unit volume of the armature core or teeth lies under the center of a north pole, it will be magnetized in one direction with maximum flux density. As the unit volume passes through the next interpolar space, the flux density will decrease, reverse, and build up in the opposite direction, as it moves under the next pole. Thus each time that the armature moves past a pair of poles, the core and teeth will pass through a complete hysteresis cycle. As was shown on page 75, the hysteresis loss depends upon the composition of the iron, varies as the 1.6 power of the maximum flux density reached during the cycle, and varies directly as the number of flux cycles passed through or the frequency (the last being the number of pairs of poles passed per second). Because of the smaller volume and higher flux densities used in the teeth than in the iron below them, the hysteresis loss in the teeth is the more important.

The iron used for the laminations, of which the core is constructed, generally contains silicon, and a compromise is made between the cost of the iron and the loss. In general the lower the hysteresis constant of the iron is made, the more expensive the iron becomes.

Owing to the teeth, there are also fluctuations in the flux density in the pole faces, as was shown on page 172, so that there will be in addition a small hysteresis loss in the pole faces.

10. Eddy-current Loss. The presence of eddy currents in an armature core was shown on page 167, and in the pole faces on page 172. It was explained further that this loss could be reduced by building up the core and poles of laminations. The loss varies with the composition of the iron,

as the square of the thickness of the laminations, as the square of the flux density, and as the square of the frequency or speed.

11. Core Loss. The hysteresis and eddy-current losses of a machine, since they occur for the most part in the armature core, and because it is difficult to separate them, are generally combined and referred to as the core or iron losses. As was mentioned above, hysteresis and eddy-current losses are functions of the speed and flux of a machine.

Both hysteresis and eddy currents produce a mechanical force which opposes rotation; the energy to supply them is part of the mechanical input of a generator, or the mechanical power developed by a motor.

12. Core Loss as a Function of Flux and Speed. The flux in an unloaded machine is determined by the field current, the relation being given by the magnetization curve of the particular machine. As load is added, armature reaction results in additional mmf's which further affect the flux. In a machine without commutating poles, if the brushes are shifted to improve commutation there will be both distorting and demagnetizing armature ampere-turns. In commutating-pole machines there will be distorting armature ampere-turns and the presence of the commutating-pole flux.

Since there are no simple methods of measuring flux or determining its change due to armature reaction, there is no way of determining directly the flux which a given field current will produce under load. It is, however, possible to express the total flux as a function of the ratio of voltage to speed.

The core loss is given by

$$\text{Core loss} = f(\Phi, n) \tag{1}$$

where Φ is the flux per pole and n is the speed in rpm.

The induced voltage of a machine

$$E_g = K\Phi n \tag{2}$$

where E_g is the generated voltage of a generator, or the counter emf of a motor, and K is a constant involving the number of armature conductors, parallel armature paths, number of poles, etc.

From Eq. (2)

$$\Phi = \frac{E_g}{Kn}$$

The ratio of E_g/n thus becomes a measure of the flux per pole since K is a constant for a given machine. Substituting the value E_g/n for Φ in Eq. (1), and changing the function (to account for the constant K),

$$\text{Core loss} = f_1\left(\frac{E_g}{n}, n\right) \tag{3}$$

If then the ratio E_g/n and the speed n are the same for two operating conditions, the flux and therefore the core loss will be the same. In other

words, if a machine is operated at no load with the same speed and induced voltage it has (or is assumed to have) at full load, the core loss measured at no load will be the same as at full load. The same reasoning applies to fractional loads.

The reason for defining the core loss of a machine in terms of no-load value is that at no load the entire input to the machine represents losses and all of them except the core loss may be calculated. If the sum of all the known losses is subtracted from the measured no-load input, the remainder represents the core loss.

According to this definition, the core loss of a shunt motor decreases with load since both its speed and induced voltage will fall as load is increased. Similarly the core loss of a generator with constant or increasing terminal voltage will increase, as its generated voltage will increase with load.

13. Stray-load Losses. As load is added to a generator or motor, armature reaction and the commutating poles (if present) will distort the flux, causing the flux density to increase under the trailing pole tips of a generator and to increase under the leading pole tips of a motor. So even with equal values of total flux at no load and at full load, the maximum flux density in some regions will be greater at full load than at no load. Since the hysteresis loss is dependent on the 1.6 power of the *maximum* value of flux density, this loss will be greater when the machine is carrying load. Also since the eddy-current loss varies as the square of the flux density, the loss for a distorted field will be greater than for an undistorted one, even if the total flux is the same.

Actually then the hysteresis and eddy-current losses will increase with load so that the core losses at full load will be greater than at no load. The core loss has been defined as the hysteresis and eddy-current loss at no load when the machine has the same total flux and speed as it has when loaded. *Any increase in these losses due to flux distortion under load is then, by definition, a part of the stray-load loss.*

The distortion of the field under load also causes other losses, such as eddy-current losses in the armature conductors, and there are in addition losses due to the short-circuit currents during commutation.

All these losses produced by the addition of load, except copper and brush-contact losses, are defined as the "stray-load losses." Since their determination is very difficult, and so far no commercial method for measuring or computing them has been found, the present A.S.A. Standards recommend that the stray-load losses of d-c machines be assumed to be 1 per cent of the output.

14. Efficiency. The efficiency of any device is the ratio of the energy output to the energy input, both quantities being expressed in the same units. If the energy input and output are being supplied and withdrawn at constant rates over a time interval, since energy is the product of power

and time, we may express the efficiency as the ratio of power output to power input, the time factor in the two energy quantities canceling. If the rate of energy supplied or withdrawn is not constant during the time interval considered, the energy ratio must be used to determine the overall efficiency.

We are usually interested in the performance of electrical machinery at some constant given load, for example at full load, and the power ratio is then sufficient. Thus the efficiency is given by

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} \tag{4}$$

which may also be written as

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{\text{input} - \text{losses}}{\text{input}} \tag{5}$$

The efficiency of a motor or generator is one of its most important characteristics. To illustrate the importance of a high efficiency assume that a factory requires a 100-hp motor which will be fully loaded for 10 hours for 300 days a year, and that the cost of power is 3 cents per kwhr. If a motor with an efficiency of 90 per cent is purchased, the input at full load is 100/.9 = 111.1 hp, which is equal to 82.88 kw. The cost of operation for one year would be 82.88 × 3000 × $0.03 = $7459.20.

If a motor with an efficiency of 80 per cent is purchased, its full-load input would be 100/.8 = 125 hp or 93.25 kw, and the cost of operation for one year would be 93.25×3000×$0.03 = $8392.50. The difference in cost in operating these two motors for a year would be $933.30.

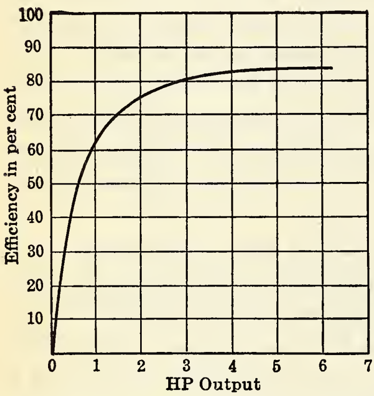


Fig. 8–2. Efficiency curve for the 5-hp motor of Fig. 8–1.

Consideration of the last calculation will indicate that high efficiency is important when the cost of the input power is high, and when the machine is fully loaded for long periods rather than for short periods. In general the higher the efficiency of a machine, the higher will be its cost, since higher efficiency is the result of a more liberal use of material, the use of more expensive material, and better workmanship. To justify the increased cost of a motor of higher efficiency the saving in energy over the life of the motor must be a little greater than the increased amortization charges due to the greater cost.

The average full-load efficiencies of d-c motors will be about 75 per cent for 1-hp machines, increase to about 86 per cent for 10-hp machines, 92 per cent for 100-hp machines, and reach 94 per cent for the larger ones.

The present standards of the A.S.A. recognize two methods of determining the efficiency of an electrical machine, a *direct* and a *conventional* method.

15. Direct Efficiency. In determining the direct efficiency, the machine is loaded to the value for which the efficiency is desired, and the input and output measured directly. The electrical output of a generator or the electrical input to a motor may be measured easily and accurately by electrical instruments. The mechanical output of small motors may be obtained by friction brakes of various types, all of which require some form of pulley which may be water-cooled, since the entire output is dissipated as heat at the pulley.

To determine the mechanical output of motors up to about 100 hp, or the mechanical input of generators up to about 75 kw, the electrodynamometer or cradle dynamometer may be used conveniently.

It is obvious that methods of determining efficiency by actually loading, unless some special "pump-back" test is used, will be wasteful of energy and therefore costly. For this reason and also because of the difficulty in providing loads for them, the efficiency of large machines is determined by conventional methods and the efficiency so determined is called a *conventional efficiency.* In many cases the conventional efficiency is determined for even small machines in preference to the direct efficiency.

16. The Electrodynamometer. The electrodynamometer or cradle dynamometer is a d-c machine, the frame of which is suspended in bearings and is free to move through a small limited arc. The armature of the dynamometer is direct-connected or belted to the device which is to be tested. An arm attached to the frame is supported by a scale or spring balance, so that the pull may be determined.

If the machine is driven as a generator, by a prime mover such as an engine or electric motor, the stator or field frame tends to follow the armature and so exerts a torque equal to that applied to drive the armature by the prime mover. If the dynamometer is operated as a motor to drive a fan, pump, or an electric generator, the reaction on the stator tends to turn it in the opposite direction from that in which the armature is rotating and exerts the same torque as the armature exerts upon its load. Thus the dynamometer is in reality an electric brake and has the advantage that the pull it exerts upon the scale or spring balance is very steady. The lever arm of the dynamometer is the distance from the center of the shaft to the scale support.

In using a dynamometer it is not necessary to determine its electrical input or output, although it is advisable to measure its armature current

and voltage so that these quantities do not exceed the rated values of the machine. Two sources of error in the use of a dynamometer are the friction of the stator bearings and the windage loss due to the rotation of the armature. Although these errors are usually neglected, corrections may be made if necessary.

17. Conventional Efficiency. In determining the efficiency of a machine by the conventional or indirect method, Eq. (5) is used, the electrical input to a motor or the electrical output of a generator being assumed and the corresponding losses calculated or determined from tests. Since these losses will vary with load, and measurement of them be made at a different load from that at which the efficiency is to be calculated, some method or convention must be adopted to account for this variation and lead to a uniform method of testing. These tests are all made electrically, so that the accuracy of the loss measurements is high, and with no load on the machine, hence little power is required. Furthermore, since the losses are a small percentage of the output (or input), a given percentage error in their determination results in a much smaller error in the efficiency.

If the efficiency of a generator were 80 per cent, an error of $2\frac{1}{2}$ per cent in the loss determination would lead to only a $\frac{1}{2}$ per cent error in the value of the efficiency; if the generator were 90 per cent efficient, a 5 per cent error in the loss determination would lead to $\frac{1}{2}$ per cent error in the value of the efficiency.

Thus we see that, even if some error is made in assuming certain conventions, only a small error will result in the determined efficiency.

18. Determination of Copper Losses. As previously stated the calculation of the I^2R losses in the field and armature requires that the resistances be known at 75 C. The name plate of a shunt or compound motor will give the full-load line current drawn at rated load. The armature and commutating-field current and the series-field current of a compound motor will equal the line current minus the shunt-field current. If a field rheostat is not used, the shunt-field current will equal the rated line voltage divided by the shunt-field resistance (at 75 C); if a rheostat is to be, or was, used in the shunt field, the rated shunt-field current will be obtained from a no-load test made in connection with the determination of stray power, as will be shown later.

The current through the armature and the commutating field of a generator will be equal to the load current plus the shunt-field current. Load currents may be assumed since the full-load line current of a generator is given on its name plate.

The full-load shunt-field current of a shunt generator is usually obtained from actual load tests although there are methods for calculating it. The no-load shunt-field current of a compound generator will be equal to the rated no-load voltage divided by the resistance of the shunt-field circuit.

This value of current, and therefore the resistance of the field circuit, R_{fc}, will be obtained when the generator is operated at no load. At full load, assuming a short-shunt connection of the shunt field, the shunt-field current will be equal to

$$I_f = \frac{E_t + I_{se}R_{se}}{R_{fc}} \tag{6}$$

where E_t is the rated terminal voltage of the generator and I_{se} and R_{se} are respectively the current through, and the resistance of, the series field. With a short-shunt connection of the shunt field the current through the series field is the load current.

The resistance of the shunt field alone must also be obtained, so that the shunt-field copper loss may be calculated.

If a test has been made upon the compound generator to determine its compound characteristic, its terminal voltage at each load will be known. But if no such test has been made, the compound characteristic may be approximated. The name plate of the generator will give its rated voltage at no load and at full load. If the machine is flat-compounded it may be assumed that its voltage at one-half load is 2 per cent higher than at full load and 2 per cent lower at 125 per cent full-load output. If the compound generator is over-compounded it may be assumed that at half load the voltage will have increased 60 per cent of the rise between no load and full load and that the voltage at 125 per cent load is the same as at full-load value.

19. Determination of Core and Friction Losses. Four methods for determining the combined core and friction loss are recognized, as follows:

(1) *The electrical-input method.* In this method the machine is operated at no load as a motor and the input measured by electrical instruments. The input to the machine being all dissipated as heat, the combined core and friction loss is determined by subtracting the copper losses from the input. This method is much used for testing d-c motors and generators.

(2) *The mechanical-input method.* In this method the machine under test is driven by a motor, usually a d-c shunt motor, in capacity 5 to 10 per cent of the machine under test. The losses and efficiency of the small driving motor are known at all loads, so that its output is determined from its input. If the test machine is driven with its field unexcited and its brushes lifted, the output of the driving motor represents the bearing friction and windage loss of the test machine. If the brushes of the test machine are dropped and the test is repeated, the brush friction of the test machine is determined by the difference. Similarly the core loss of the test machine is determined by the difference by exciting its field. This method of testing is much used for large machines both d-c and a-c.

(3) *Retardation method.* In this method the speed of the test machine, operating as a motor, is brought up to a value 10 per cent above normal,

the supply power cut off, and simultaneous readings of speed and time taken, as the speed of the machine decreases. If the stored rotational energy is known, usually by knowing the weight and radius of gyration of the rotating mass, the losses may be determined from the rate of loss of energy of rotation, as evidenced by the slowing down of the rotation. If the field is unexcited the combined friction losses at any speed may be determined; if the test is repeated with the field excited, the core loss is determined by the difference. The method is particularly adaptable to machines of large inertia. Although sometimes used with large d-c generators, it finds its greatest use for large a-c generators.

(4) *Calorimeter method.* This method consists in suitably enclosing the machine and measuring the temperature rise and the volume of the cooling medium. It is applicable to machines built to have special ventilating systems, in which the air is recirculated through the machine and a cooler, or fresh, air is taken from outside the building, rather than the ordinary arrangement for taking the air from the room in which the machines are installed. For use with the usual machine a special enclosure must be built, so that air can be passed through the machine. Arrangement must be made to measure the ingoing and exhaust air temperature and to minimize the effect of heat loss through the enclosure.

20. Calculation of Internal Voltage. The standards of the A.S.A.* define the core loss of a d-c machine as "the difference in power required to drive the machine at normal speed, when excited to produce a voltage at the terminals corresponding to the calculated internal voltage, and the power required to drive the unexcited machine at the same speed." These standards state further that "the internal voltage shall be determined by correcting the rated terminal voltage for the resistance drop only."

According to this definition, the induced or counter emf of a motor is to be calculated from the expression

$$E_g = E_t - I_a R_a - I_a R_c \tag{7}$$

where I_a is the armature current, R_a is the armature resistance measured at the commutator, and R_c is the resistance of the commutating field.

It will be realized that brush drop is disregarded in the determination of the induced voltage so that the procedure becomes empirical or conventional to some extent.

The induced voltage of a generator, according to the definition, is to be determined from the expression

$$E_g = E_t + I_a R_a + I_a R_c + I_{se} R_{se} \tag{8}$$

where $I_{se}R_{se}$ is the drop in the series field. The brush drop is again neglected.

* American Standards Association, American Standards for Rotating Electrical Machinery, C50–1936, paragraph 2.112.

21. Determination of Operating Speed. The operating speed of a generator is given on the name plate and is taken as such for all load values. A 4 per cent drop from no load to full load is sometimes allowed for the automatic operation of the governor of the prime mover.

In the case of a shunt motor, although the full-load speed will be given on the name plate, it is often necessary to check the value by calculation. The motor is operated at no load with rated voltage, E_t, impressed upon the armature circuit, and with a field current equal to that which flows when the field is at 75 C. The armature current, and speed, n_o, are then carefully determined.

The no-load internal or counter voltage, E_{go}, is determined according to Eq. (7), and then the induced voltage, E_g, corresponding to full-load armature current, using the same equation. It is then assumed that, if the motor has the same voltage impressed and has the same field current, the speed will vary directly as the internal or counter emf. Accordingly, the full-load speed, n, may be obtained from the relation

$$\frac{n}{n_o} = \frac{E_g}{E_{go}} \tag{9}$$

Some error is made, however, in the last assumption, since the effect of armature reaction is neglected. The flux at full load will in general be less than the value at no load so that the result gives a full-load speed less than the actual value.

22. Determination of Combined Core and Friction Loss by the Electrical Method. When a machine is operated at no load as a shunt motor the entire input represents losses. The total input to the motor will be armature-circuit input plus shunt-field input; but, since the latter is all lost in heat as I^2R loss, and can be easily calculated, only the armature input need be considered, so that

$$\text{Armature input} = \begin{cases} \text{Armature loss} = I_a^2 R_a \\ \text{Brush loss} = I_a \times (2 \text{ volts}) \\ \text{Commutating-field loss} = I_a^2 R_c \\ \text{Core loss} \\ \text{Friction (brush, bearing, and air) loss} \end{cases}$$

Now the brush loss of a motor operating at no load is usually neglected, it being a very small loss because the current is small and the brush drop is actually only about one volt per pair of brushes instead of the load value of two volts. The loss in the commutating field, $I_a^2 R_c$, is also small.

The definition of core loss is "the difference in power required to drive the machine at normal speed, when excited to produce a voltage *at the terminals* corresponding to the calculated internal voltage, etc." The

definition thus expressly neglects the drop across the commutating field, I_aR_c, and as well the loss in that field, considering the loss as core loss.

The $I_a^2R_a$ loss in the armature is also small, but not as small as that in the commutating field. It is sometimes considered and sometimes neglected.

If now brush loss and commutating-field loss are neglected,

$$\text{Armature input} = \text{armature } I_a^2R_a \text{ loss} + \text{combined core and friction loss}$$

and if the armature $I_a^2R_a$ loss is neglected, the entire armature input at no load becomes the combined core and friction loss of the machine.

23. Example of the Electrical Input Method for a Motor. A certain 75-hp, 240-volt, 1200-rpm motor has an armature resistance of 0.0241 ohm, a commutating-field resistance of 0.00635 ohm, a shunt-field resistance of 160 ohms, all at 75 C. Rated line current is 250 amperes.

When operated at no load with 240 volts impressed, the armature current was 12.04 amperes and the speed 1240 rpm. The no-load internal or counter emf was, therefore, $240 - 12.04\ (0.0241 + 0.00635) = 240 - 0.37 = 239.63$ volts.

With a shunt-field current of $240/160 = 1.5$ amperes, and 250 amperes line current, the full-load armature current would be 248.5 amperes, and the full-load induced or counter emf, from Eq. (7), is

$$240 - 248.5\ (0.0241 + 0.00635) = 240 - 7.57 = 232.43 \text{ volts}$$

From Eq. (9) the full-load speed is

$$n = \frac{1240 \times 232.43}{239.63} = 1203 \text{ rpm}$$

so that the name-plate value for the full-load speed of 1200 is substantially correct.

To determine the combined core and friction loss of the motor, it was next operated with 232.43 volts impressed and with such field current as would cause it to run at 1203 rpm. Under these conditions the armature current was 11.05 amperes.

With the brush-contact loss (of about 11.05×1 volt $= 11$ watts), the commutating-field loss (of $11.05^2 \times 0.00635 = 0.8$ watt), and the armature $I_a^2R_a$ loss (of $11.05^2 \times 0.0241 = 3$ watts) all disregarded,

$$\text{Combined core and friction loss} = 232.43 \times 11.05 = 2568 \text{ watts}$$

and it is evident that, had the three losses (15 watts) been subtracted from the input, the result would have been changed by about one-half of one per cent.

If it is now assumed that at full load the motor draws 250 amperes from the line, of which 1.5 amperes is the shunt-field current, then the

losses, with the exception of the stray-load loss, are

Armature $I_a^2R_a = 248.5^2 \times 0.0241$	$= 1488$
Brush contact $= 248.5 \times 2$	$= 497$
Commutating field $= 248.5^2 \times 0.00635$	$= 392$
Shunt field $1.5^2 \times 160$	$= 360$
Core and friction	$= 2568$
	5305

With an input of $240 \times 250 = 60{,}000$ watts, a trial output would be $60{,}000 - 5305 = 54{,}695$ watts. With 1 per cent of this trial output (547 watts) as the stray-load loss, the total losses become $5305 + 547 = 5852$ watts, and the true output $60{,}000 - 5852 = 54{,}148$ watts or $54{,}148/746 = 72.6$ hp; and the efficiency is $54{,}148/60{,}000 = 90.25$ per cent.

A conclusion to be reached from this particular problem is that the input current of the motor is slightly under-rated. If this is raised a few per cent the motor will give its rated output of 75 hp at about the same efficiency.

24. Simplified Electrical Input Method. This method is frequently simplified by considering the input to the motor, when operating at *rated voltage at no load,* as the combined core and friction loss, rather than when operating at an impressed voltage equal to the full-load internal voltage. The result is to make the full-load core and friction loss slightly higher.

In the previous example the armature current when operating at 240 volts was 12.04 amperes with an input of 2890 watts. When operating at 232.43 volts, the armature current was 11.05 amperes with an input of 2568 watts. If the input of 2890 watts is taken as the full-load core and friction loss, the stray-load loss becomes 544 watts, so that the total losses are increased by 319 watts and the efficiency becomes $53{,}829/60{,}000 = 89.71$ per cent, a decrease of one-half of one per cent.

25. Example of the Electrical Input Method. Generator. A certain 500-kw, 900-rpm, compound generator has a rated voltage of 250 volts at no load and 275 volts at full load. The resistances, all at 75 C, are as follows: armature, 0.0018 ohm; commutating field, 0.0004 ohm; shunt field, 47.5 ohms; series field, 0.0002 ohm.

The no-load shunt-field current, since there will be a shunt-field rheostat, may, of course, be found by driving the machine as a generator and exciting it to rated no-load voltage. It may also be found by operating the machine as a motor by impressing 250 volts on the armature (inside the commutating and series fields) and then adjusting the shunt-field current until the machine runs at 900 rpm. Neglecting a small brush drop and a small armature I_aR_a drop, the induced voltage is then 250 volts and the shunt-field current the same as if it were driven as a generator. When so tested

the shunt-field current was 4.167 amperes, and the resistance of the shunt-field circuit (field plus rheostat) was $250/4.167 = 60.0$ ohms.

The output current of the generator is $500{,}000/275 = 1818.2$ amperes, so that the armature current is $1818.2 + 4.2 = 1822.4$ amperes. The full-load internal or generated voltage is then

$$275 + 1822.4\,(0.0018 + 0.0004) + 1818.2 \times 0.0002 = 279.4$$

The voltage applied to the shunt field at full load (since it is connected across the armature and commutating field, i.e., inside the series field) is $275 + 1818.2 \times 0.0002 = 275.4$. The full-load shunt-field current will be $275.4/60 = 4.59$ amperes.

The generator was operated as a shunt motor with 279.4 volts impressed upon its terminals and the shunt-field current adjusted to cause the machine to run at 900 rpm. The armature current was 55.5 amperes, so that the full-load combined core and friction loss was $279.4 \times 55.5 = 15{,}507$ watts.

The shunt-field current is thus 4.59 amperes at full load instead of the no-load value of 4.2, used in the above calculation for determining the full-load armature current. An added refinement could be made by recalculating, using the corrected value of armature current, $1818.2 + 4.59 = 1822.8$ amperes, but the change in values would be insignificant.

Assuming rated output of 500,000 and adding the corresponding losses

Output		500000
Armature loss = $1822.8^2 \times 0.0018$	=	5981
Brush loss = 1822.8×2	=	3646
Commutating-field loss = $1822.8^2 \times 0.0004$	=	1329
Series-field loss = $1818.2^2 \times 0.0002$	=	665
Shunt-field loss = $4.59^2 \times 47.5$	=	1001
Core and friction loss		15507
Stray-load loss		5000
Input		533129

$$\text{Efficiency} = 500{,}000/533{,}129 = 93.78 \text{ per cent}$$

26. Determination of Core and Friction Loss by the Mechanical Method. In this method the test machine is direct-connected or belted to a small motor, the rating of which is 5 to 10 per cent of the rating of the test machine. The small driving motor must be calibrated, i.e., its losses or efficiency must be known at all operating conditions. The operating speed and the internal voltage of the test machine are determined as before and the machine driven at the required values.

If the test machine is first driven with its field unexcited and with its brushes lifted, the output of the driving motor will represent the combined bearing and air friction. If the brushes are dropped, the additional output of the driving motor will be the brush friction. If then the test ma-

chine is excited to generate the full-load internal or generated voltage, the additional output of the driving motor will be the core loss of the generator. The last two steps are the basis of the definition for the core loss of a machine as given on page 376.

27. Example of the Mechanical Input Method. Consider that the generator of the last problem was tested in this way. The data for the generator are: 500-kw; no-load voltage, 250 volts; full-load voltage, 275 volts; armature resistance, 0.0018 ohm; commutating-field resistance, 0.0004 ohm; shunt-field resistance, 47.5 ohms; series-field resistance, 0.0002 ohm.

To drive the generator a 25-hp, 240-volt, 900-rpm shunt motor was used; its armature resistance was 0.10 ohm. The generator was driven at 900 rpm according to the following table, and the armature voltage and current of the driving motor read

	Generator Operation	Driving Motor		
		Armature volts	Armature current	Internal voltage
A	Generator unexcited, brushes lifted........	226.3	28.6	223.4
B	Generator unexcited, brushes down........	231.3	45.0	226.8
C	Generator excited, brushes down..........	237.0	72.1	229.8

To determine the combined core and friction loss of the driving motor, it should have been operated at no load (generator disconnected) at 900 rpm with each of the three values of internal voltage impressed and the input measured. Because the change in core loss is small for the change in internal voltage, and to simplify the problem, it was operated at 900 rpm with the mean value of 226.8 volts impressed upon the armature. The armature current was 4.083 amperes making the core loss of the motor $226.8 \times 4.083 = 926$ watts.

The core and friction losses of the generator at full load may now be determined, as in the following table in which the settings correspond to those of the previous table.

	Driving Motor					Generator Losses		
	Armature input watts	Armature $I_a^2R_a$	Core loss	Brush watts	Output	Bearing friction and windage	Brush friction	Core
A	6472	82	926	57	5407	5407	....	....
B	10408	203	926	90	9189	5407	3782	....
C	17088	520	926	144	15498	5407	3782	6309

The combined core and friction losses are 15,498, practically the same as in the last example, so that the calculation of the efficiency is exactly as in that example.

28. Calibration of a Machine. When a given motor is to be used frequently to determine the losses of other machines, it will be calibrated, i.e., its losses over a wide range of operating conditions will be determined.

If the intent of the standardization rules is followed, the machine is operated as a motor at no load and the input determined over a wide range of speed (obtained by shunt-field adjustment), the internal or counter emf being maintained constant throughout a given run. The internal voltage is then changed a small amount and the input determined for another range of speed with this new value of internal voltage. The procedure is repeated to cover the range of voltages and speeds which may be needed. The voltage impressed across the armature only is read, to eliminate the commutating field from the calculations.

The combined core and friction loss of the machine is calculated for each condition and a family of curves plotted between watts loss and speed, each curve for a constant value of internal voltage. From these curves, others may be plotted between loss and internal voltage, each curve at a constant speed.

A curve of armature $I_a^2R_a$ loss plus brush watts is also plotted against armature current up to 150 per cent of rated value. A curve of armature I_aR_a plotted against armature current is also useful to determine the internal voltage.

Another method of calibrating a motor which is quite commonly used, but not quite in accordance with the intent of the A.S.A. standards, is to operate the machine as a motor with a given constant value of field current and then to vary the speed over a wide range by varying the impressed voltage. The input to the motor is, as before, the combined core and friction loss. This procedure is repeated with a number of different field currents, each held constant over a wide speed range.

A family of curves is then plotted between combined core and friction loss and speed, each curve representing values for a definite field current. In using the motor to drive a generator to determine the losses of the generator, it is advantageous always to operate the motor at values of field current for which a loss curve has been determined.

29. Pump-back Tests. A method of testing motors and generators, in which much of the required energy is saved, is by means of pump-back, loading-back, or opposition tests, in which a generator is driven by a motor. The energy required to drive the motor is drawn from a source of supply and the energy output of the generator is put back either into the same source from which the motor was fed or into some other source.

In Fig. 8–3 is shown a shunt motor which is driven from a suitable d-c

source. Connected to the motor is a generator of the same voltage rating as the motor. The motor is brought up to speed by means of the starting rheostat, *SR*, and the generator excitation adjusted until the voltage across the switch S is zero, when it may be closed. The generator may now be considered as operating in parallel with the supply; if its excitation is increased it will send current into the source. As the generator is loaded the motor must draw more current to drive the generator. Speed is maintained by adjustment of the motor field.

The net energy required from the source will be the total losses of both machines. By connecting the two fields across the source, the field losses,

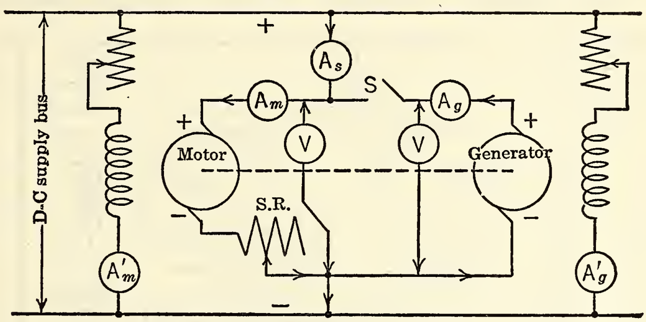


FIG. 8–3. A pump-back test in which a motor drives a generator, the generator in turn sending its energy back into the same source from which the motor is driven. Both machines may be operated at full load, and only the total losses of both machines are wasted.

since they are all lost as heat, will not be included in the reading of ammeter A_s.

Because of the saving in energy, pump-back tests are widely used in making temperature tests on machines, in which the test machine is operated at full load or overload for such a time that the temperatures of the stationary parts become constant. Usually only one of the machines is under test, the other being a suitable shop machine used for such testing. It is not necessary in making temperature tests that the generator feed back its power into the source from which the motor is driven. Thus a 230-volt motor operating from a 230-volt source might drive a 115-volt generator which is operated in parallel with a 115-volt source. But if loss measurements are to be made, both machines must operate from the same source.

With a source voltage V, the input to the armature circuits is VI_s, where I_s is the supply current as read on ammeter A_s. This input will be equal to the combined stray powers of the two machines plus the armature-circuit losses of the two machines.

The division of the total stray-power loss, including stray-load loss,

between the two machines, is not simple, since the motor is more heavily loaded than the generator since it must supply the generator losses. With duplicate machines, if the total stray-power loss is divided between them in proportion to their internal voltages, the error will not be large.

Another scheme of connections, used when the voltages of the two machines are only approximately equal or when two identical machines are to be tested, is shown in Fig. 8–4.

Consider that a 110-volt, 1200-rpm generator is to be tested and that only a 125-volt, 1150-rpm shunt motor is available, and that the power source is 125 volts. To make up the 15-volt difference between the generator and source voltages, a booster, capable of carrying the output cur-

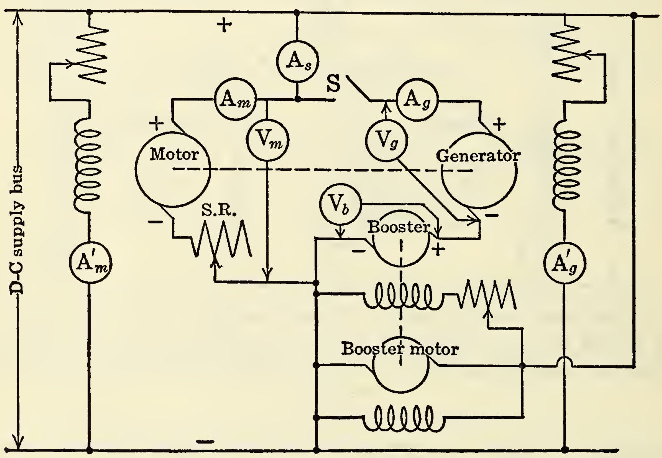


Fig. 8–4. A pump-back test in which the terminal voltage of the generator is raised by a booster to be equal to the supply or motor voltage.

rent of the generator, is wired in series with the generator. The booster is driven by a shunt motor and is separately excited.

After the set has been started, the generator is built up to rated voltage and the excitation of the booster built up until the voltage across switch S is zero, when the switch is closed. The excitation of both generator and booster is raised until the generator supplies rated current and has its rated terminal voltage. Speed of the generator is maintained by adjusting the motor field current. The armature-circuit and stray-power losses of the set are now supplied, partly from the source directly and partly through the booster.

With two identical machines operating from a source of the same value, the combined core, friction, and stray-load losses of each machine may be determined with no great error by operating the generator at the same field current as the motor requires to run the set at rated speed. The booster,

as before, is excited to cause rated generator current to flow, but the motor will be overloaded.

It is assumed that, since both machines have the same speed and field current, they will have the same stray-power losses. The combined stray-power losses will be the direct source input (V_mI_s, where I_s is the source current in Fig. 8–4), plus the booster input (V_bI_g), minus the total armature-circuit losses of both machines. The stray-load loss of each machine is one-half the total. The error made is that the stray-load loss of the motor is greater than that of the generator because it carries more load.

30. Allowable Operating Temperatures. The losses in electrical machinery all appear as heat which ultimately must be dissipated to the surrounding air. There will be a rate of generation of heat depending upon the load and a rate of dissipation depending upon the temperature difference between the machine and the surrounding air. So long as the rate of generation is greater than the rate of dissipation, the temperature of the machine will increase and with it the rate of dissipation. A balance is finally reached at some ultimate value of temperature which will remain constant for a given load and at a given temperature for the surrounding air.

The *capacity* of an electrical machine is limited by the ability of its insulation to withstand, continuously, and without deterioration, the maximum temperature caused by its losses. Experience has shown that, for each type of insulation, there is a certain limiting temperature above which the insulation will deteriorate more or less rapidly. As far as the useful life of the insulation is concerned, however, there does not seem to be any advantage in operating at temperatures below this safe limit.

31. Classes of Insulation. The standards of the A.S.A.* and of the N.E.M.A.† group insulating materials into two classes. *Class A* insulation consists of cotton, silk, paper, and similar materials, when so treated or impregnated as to increase the thermal limit (of the unimpregnated material), or when permanently immersed in oil; also enameled wire and enameled silk- or cotton-covered wire. It is generally considered that the limiting temperature of Class A insulation is 105 C, and this limit was formerly stated in the standards. It is still implied, as will be seen, when the limiting temperatures for machine parts are considered. *Class B* insulation consists of mica, asbestos, glass, and other materials capable of resisting high temperatures, in which any Class A material or binder is used for structural purposes only, and may be destroyed without impairing the insulating or mechanical qualities of the insulation. The limiting temperature for Class B insulation is generally taken as 125 C.

* American Standards Association, American Standards for Rotating Electrical Machinery, C50–1936.

† National Electrical Manufacturers Association, Motor and Generator Standards 34–22, 1934.

32. Ambient Temperature. Since the insulation of a machine may not be subjected to a temperature above a certain limit, it becomes evident that the amount of load which a machine may carry will depend upon the temperature of the ambient or encompassing air, the so-called *ambient temperature.* A given motor operating in air which has a temperature of 10 C will not reach as high a final temperature as one operating in a hot boiler room.

In order to standardize the power ratings of electrical machinery, electrical manufacturers have accepted 40 C as the maximum ambient temperature which will satisfy practically all requirements, and all guarantees are based on that value. If the guarantee states that the temperature of the armature will not rise more than 55 C, it is meant that if the machine is operated with the ambient air at 40 C the temperature of the armature copper will not be more than 95 C. But if the ambient temperature is 20 C the temperature of the armature copper will be 75 C, it being assumed that the rise in temperature will be the same for all ambient temperatures up to 40 C.

On page 387 is given a table of limiting observable temperature rises for machines for both classes of insulation; it applies to all ambient temperatures not over 40 C.

33. Methods of Measuring Temperature Rise. There are three methods by which temperature rise in a machine may be determined. They are (1) thermometers, (2) resistance measurements, (3) imbedded detectors.

34. Thermometers. The bulb of the thermometer is placed in contact with the surface, the temperature of which is desired, and covered with either a felt pad which is cemented to the surface, or a small mass of putty or cotton waste.

When a thermometer bulb can be brought in contact with the bare copper of a winding, such as an edgewise wound series or commutating field, the reading of the thermometer is considered as the temperature. In column 2, for Class A insulation the allowable temperature rise for bare copper windings is 65 C, which, with an ambient temperature of 40 C, would reach a final temperature of 105 C.

Where a thermometer bulb is placed upon the insulation over an armature or a field winding, the temperature of some parts of the copper will be higher than the outside of the insulation. In column 2, row 1, the permissible rise in temperature is given as 55 C over an ambient temperature of 40 C, giving a final temperature of 95 C. If the maximum temperature allowed for the Class A insulation is 105 C, a 10-degree drop in temperature through the insulation has been allowed. In other words, with temperature measurements by thermometers placed over the insulation of a winding, the temperature of the copper is considered to be 10 C higher than the thermometer reading.

TABLE A

LIMITING OBSERVABLE TEMPERATURE RISE IN D-C MACHINES IN °C †

The temperature rises in degrees centigrade in this table apply to the several classes of machines as given at heads of the respective columns.	Generators (including those for Electrolytic Service) and General Purpose Motors*	Totally-enclosed, and Totally-enclosed Fan-cooled, Motors		Generators Having a Two-hour 25 Per Cent Overload				Motors and Generators Other Than Columns 1 to 9 Inclusive	
				At Continuous Load		At End of 2-hr Overload			
Item	Col. 1 Class A Insulation	Col. 2 Class A Insulation	Col. 3 Class B Insulation	Col. 6 Class A Insulation	Col. 7 Class B Insulation	Col. 8 Class A Insulation	Col. 9 Class B Insulation	Col. 10 Class A Insulation	Col. 11 Class B Insulation
1. Armature windings, wire field windings and all windings other than 2	40	55	75	40	60	55	75	50	70
2. Single-layer field windings with exposed uninsulated surfaces and bare copper windings	50	65	85	50	70	65	85	60	80
3. Cores and mechanical parts in contact with or adjacent to insulation	40	55	75	40	60	55	75	50	70
4. Commutators and collector rings**	55***	65	85	55	70	65	85	65	85

5. Miscellaneous parts (such as brush-holders, brushes, pole tips, etc.) may attain such temperatures as will not injure the machine in any respect.

* These low temperature rises are provided to allow a greater factor of safety where the load conditions are unknown. No other temperature rating or temperature guarantee is given in conjunction with the general purpose rating.

** The class of insulation refers to insulation affected by the heat from the commutator which insulation is employed in the construction of the commutator or is adjacent thereto.

*** The limiting observable temperature rise for commutators of generators for electrolytic service shall be 50 C.

† Taken from A.S.A. Standards, C50, 1936, p. 24. Columns 4 and 5, which apply to special generators for mining service, have been omitted.

The standards of both the A.S.A and the N.E.M.A. recommend that all temperature readings with d-c machines be taken by thermometers.

35. Resistance Measurements. If the resistance of a winding is measured at a known temperature and measured again at an unknown temperature, the average unknown temperature may be calculated by the methods given in Chapter III. Some parts of a winding will be hotter than others, so that the calculated final temperature is only an average. Accordingly it is customary to consider that the temperature of the "hot spots" will be a little higher than the value by resistance measurement.

The standards recommend the method of determining temperature rise by resistance measurements for a-c synchronous motors and a-c generators.

36. Imbedded Detectors. An imbedded temperature detector is either a resistance, the temperature of which is determined by measuring its resistance, or a thermocouple. Imbedded temperature detectors are used to measure temperatures only in stationary members and so are suitable only in transformers and in the stationary armatures of synchronous alternators and motors. They are placed between coil-sides in the slots of two-layer windings, or at the bottom of the slots of single-layer windings. Because of their position these temperature detectors will give readings nearer the true temperature of the copper than do thermometers. The leads from the detectors are brought out to special instruments which are usually calibrated to read temperature directly. The temperature of a stationary armature may thus be determined after it has been installed and the load adjusted so as to prevent overheating of the insulation.

The resistances used as temperature detectors are usually of copper and have a resistance of about 10 ohms at 25 C. The currents used to determine their resistance are very small and flow only while a measurement is being taken.

37. Rating. The rating of an electrical machine is an arbitrary designation of the power which the machine can safely deliver when operated under definite conditions as set by the manufacturer.

The *name plate* of a d-c motor will specify the voltage at which it is to be operated and state the horsepower output which the machine will deliver without reaching a dangerous temperature, the speed at which it will deliver its rated output, and the current it will draw from the source at rated load. It may specify also the maximum temperatures to be expected and the length of time the machine may be operated for it to reach these temperatures.

The name plate of a d-c generator will specify its continuous power output in kilowatts, the speed at which it is to be operated, its no-load and full-load voltages, and its current output.

The rating of a machine will depend upon the duty for which it is

intended and the conditions under which it must operate, such as the presence of dust, gases, moisture, etc., in the cooling air.

38. Duty Classification.* The duty of a machine is a requirement of the service it must render and defines the degree of regularity of the load.

Continuous duty is a requirement of service that demands operation at substantially constant load for unlimited periods.

Intermittent duty is a requirement of service that demands operation for alternate periods of (1) load and no load; or (2) load and rest; or (3) load, no load, and rest, the periods so apportioned and regulated that the temperature rise of the machine at no time exceeds that specified for the machine.

Periodic duty is a requirement of service that demands operation for alternate periods of load and rest in which the load conditions are well defined and recurrent as to magnitude, duration, and character.

Varying duty is a requirement of service that demands operation at loads, and for periods of time, both of which may be subject to wide variation.

It is evident from the above classification that the rating of a motor will be affected by its duty. Consider a motor which can deliver 25 hp continuously without reaching a dangerous temperature. If this motor were called upon to deliver 40 hp it would heat very rapidly, but, if at the time its temperature reached its limiting value it were shut down and then left to cool for a period, it would operate successfully with periodic load and rest intervals.

39. Heating and Cooling. The rise in temperature of a machine depends upon the rate of generation of heat, upon its heat-storage capacity, and upon the facility with which heat is dissipated. When the rates of heat generation and dissipation become equal, the temperature will become constant. The form of curve showing the temperature rise is a logarithmic curve of exactly the same general shape as that given for the rise of current in an inductive circuit.

The rate at which heat is dissipated from any body (e.g., an armature) depends upon the difference in temperature between it and the ambient air, and upon the amount of air that is carried over the radiating surface. This second condition is really involved in the first, because if but little air is supplied it soon gets hot and so reduces the temperature difference, whereas if much air is supplied, as in forced ventilation, the air is carried away from the dissipating surface before it has time to get hot.

40. Effect of Ventilation upon Capacity. In Fig. 8–5 are shown curves for heat production and for heat dissipation. The curve OAB gives the rate of heat production plotted against the armature current as abscissa. The lines OC, OD, and OE show the rate of dissipation for good, medium,

* A.S.A. and N.E.M.A. Standards.

and poor ventilation; the abscissae for these curves are the difference between the armature and the room temperature. It is supposed that the air for ventilation is taken directly from the room and is therefore at the same temperature as the room.

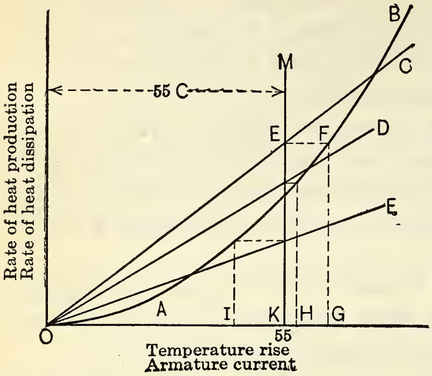


Fig. 8–5. Curves showing the factors which determine the temperature rise of a machine.

If the highest temperature rise in the hottest part of the particular machine is fixed at 55 C above a room temperature of 40 C, the line KM is erected at this value. With good ventilation the rate of dissipation at this temperature is equal to the rate of heat production by the current OG, and so we say the safe current, in so far as heating is concerned, is OG. If the ventilation is medium, the safe current is OH; and if the ventilation is poor, the safe current is OI. This diagram shows the enormous advantage of forced ventilation, gained by equipping a machine with a ventilating fan and proper air ducts. All machines are designed with the idea of getting the best ventilation possible at a given cost for the machine.

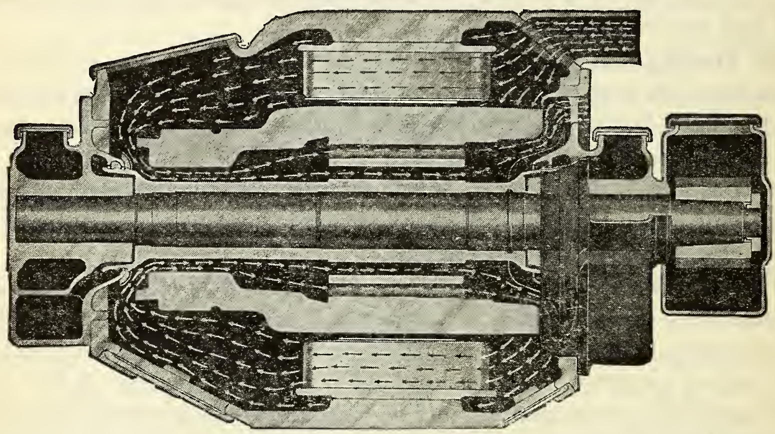


Fig. 8–6. Showing the paths taken by the cooling air in a semi-enclosed motor. *Courtesy of the General Electric Co.*

41. Ventilation and Enclosure of Machines. The ventilation of a machine may be accomplished by means of fans integral with the machine, in which case it is said to be *self-ventilated*. Or the ventilating air may be circulated through proper admission and discharge openings in the ma-

chine, by a fan or blower external to the machine, in which case the machine is said to be *separately ventilated.* Practically all d-c machines are self-ventilated.

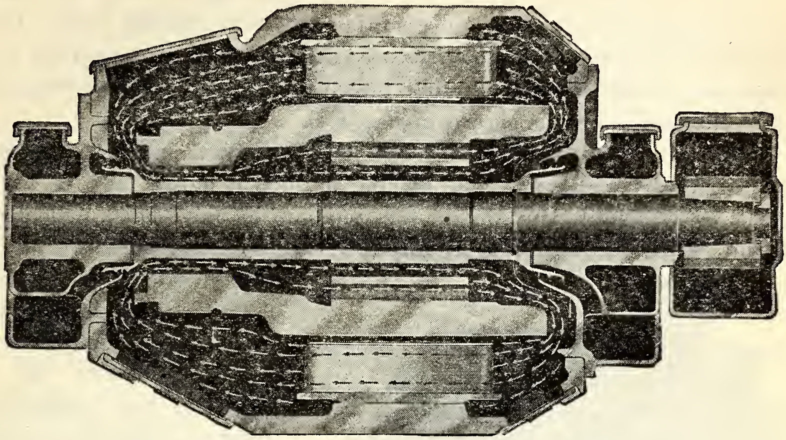


FIG. 8–7. In a totally enclosed motor the air currents circulate about as shown here; the only way the heat can escape from the motor is by the comparatively poor conduction of heat through the steel frame. The air heats up as it passes through the armature and partly cools off as it flows by the frame while in the outer part of its path. The safe rating of such an enclosed motor is comparatively low. *Courtesy of the General Electric Co.*

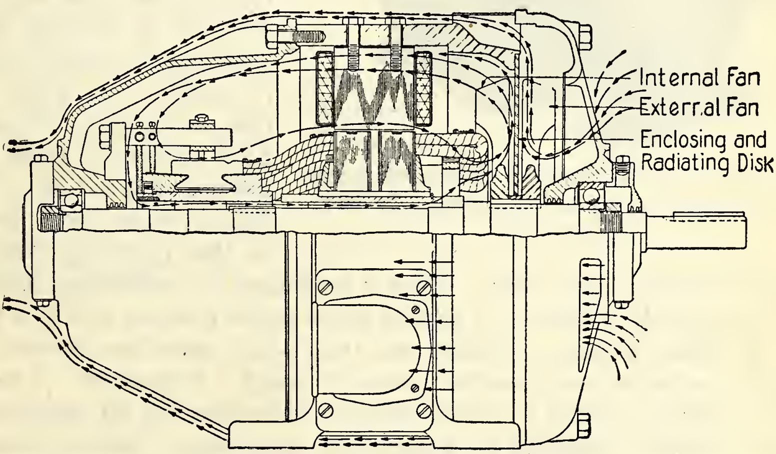


FIG. 8–8. Section of an enclosed, fan-cooled motor, showing the paths of the air currents. *Courtesy of the General Electric Co.*

If a machine is installed in surroundings where the air is clean, it may be left open, with no restrictions to the passage of the air through the windings other than those imposed by mechanical construction. Such a machine is known as an *open machine.*

Where a machine operates under conditions in which there is much dust in the air, it becomes necessary to protect the motor from dust as much as possible. In such cases, only one or two openings, covered with screens are provided for the entering air, resulting in a *semi-enclosed* motor. Other examples of semi-enclosed machines are those in which the openings are so placed as to make them drip-proof, splash-proof, etc. A cross-section of a semi-enclosed motor is shown in Fig. 8–6.

If the ambient air is laden with fine dust, moisture, or fumes, the motor must be totally enclosed. The cooling of such a motor may be accomplished by circulating air within the machine by fan blades attached to the end of the armature, as in Fig. 8–7. In Fig. 8–8 is shown another design with air currents inside and outside of a well-ventilated, totally-enclosed motor. Between the end bell and the armature is placed a disk of metal which seals the motor. An internal fan causes the inside air to circulate within the motor; an external fan between the disk and the end bell draws in air through holes around the bearing, forces it radially out along the disk, and expels it through holes between the frame and the extended end bell. The exhaust holes direct the air so that it sweeps over the outside surface of the frame. Heat is exchanged by conduction through the frame of the machine. A picture of the motor is shown in Fig. 8–9.

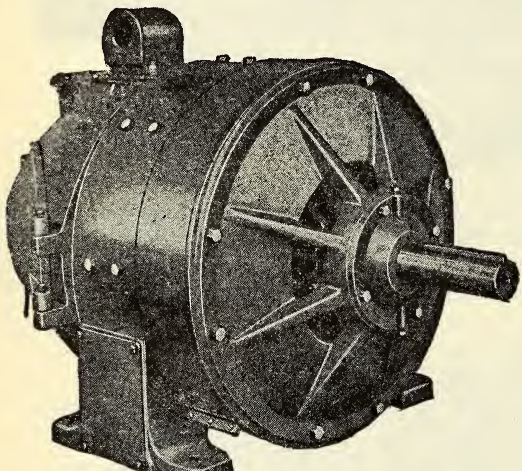


FIG. 8–9. Totally enclosed, fan-cooled motor, shown in section in Fig. 8–8. *Courtesy of the General Electric Co.*

42. Motor Ratings. Besides the rating which takes into account the type of enclosure used, a motor is rated to adapt it to its duty. A motor with *continuous* rating is able to carry its rated load for an unrestricted period without exceeding its temperature limitations. Motors intended for intermittent, periodic, and varying duty are given an *intermittent* or *short-time* rating, the short-time rating defining the load which can be carried for the time specified in the rating, without causing any of the temperature limits to be exceeded. With such service, heat is generated intermittently, but is dissipated at a more or less steady rate. Experience has shown that such service may be approximated, so far as heating is concerned, by allowing the machine to reach its limiting temperature after

steady operation at rated load in a definite time. Thus a motor which, operated at full load, reaches its limiting temperatures in 30 minutes, is given a 30-minute rating. Standard periods for short-time ratings are 5, 10, 15, 30, 60, and 120 minutes.

Motors intended for intermittent duty (especially enclosed motors) are often built with more iron than is necessary for magnetic or structural requirements, in order to increase their heat-storage capacity. There is obviously no gain in increasing the heat-storage capacity of a machine that is to be given a continuous rating.

A *general-purpose* motor is defined by the A.S.A. Standards as any motor of 200 hp or less and 450 rpm or more, having a continuous rating, and designed, listed, or offered in standard ratings for use without restriction to a particular application. These are of the open type and generally use Class A insulation.

The temperature limits for motors are shown in the table on page 387. It will be seen that the limiting temperature rises in columns 1, 2, 3, 10, and 11 are 40, 50, 55, 70, and 75 C and the types of motors as shown under these columns are frequently referred to as 40-degree rise, 50-degree rise, etc., motors.

43. Generator Ratings. Generators, except those used for automobiles, train-lighting, etc., are generally of the open type. D-c generators and exciters for a-c synchronous generators and motors, for which the service conditions are unknown, a class equivalent to general-purpose motors, are usually built with Class A insulation. Their temperature limits are shown in column 1 of table A on page 387.

For usual service in central stations, d-c generators are given a *continuous, with 2-hour, 25 per cent overload rating.* This rating defines the load which can be carried continuously, followed by a 25 per cent overload for 2 hours, without causing the temperature limits to be exceeded. The temperature limits for such rating are shown in columns 6 to 9 in the table on page 387.

PROBLEMS

8–1. What is the full-load efficiency of a motor with a name-plate rating of 115 volts, 38 amperes, 1200 rpm, 5 hp?

8–2. If the only variable loss in a motor is that due to armature I^2R loss, prove that the maximum efficiency occurs when this loss is equal to the sum of all the other losses.

8–3. A 40-hp, 125-volt shunt motor is to be purchased to drive a machine at full load for 10 hours per day, 300 days per year. Two motors, X and Y, are offered. Motor X is guaranteed to have a full-load efficiency of 90 per cent, and motor Y, 92 per cent. Motor Y will cost $50 more than motor X. If power costs 2 cents per kwhr, and the money to buy the motor is borrowed at 6 per cent per year, which motor is the cheaper?

8–4. A factory desires to purchase a 10-hp, 110-volt shunt motor which is to drive a load for 300 days per year; the load requires each day an armature current of 100 per cent for 2 hours, 125 per cent for 2 hours, 60 per cent for 4 hours, 30 per cent for 4 hours. In addition, the motor will run 12 hours at practically no load. Power costs 3 cents per kwhr, and the money to pay for the motor may be borrowed at 6 per cent per year. Two 10-hp motors, A and B, each of 86 per cent full-load efficiency, are offered for purchase. Motor A has an armature resistance of 0.09 ohm and field resistance of 55 ohms. Motor B has an armature resistance of 0.07 ohm, and field resistance of 65 ohms, but costs $25 less than motor A. Which motor should be purchased? Disregard brush drop, obsolescence, etc.

8–5. The motor of a motor-generator set takes 110 amperes at 240 volts and delivers 91.2 per cent of its input to the generator. The latter has an efficiency of 92 per cent and is furnishing power to a 110-volt line. What is the current output of the generator?

8–6. A 40-hp, 230-volt shunt motor has a full-load efficiency of 90 per cent. The shunt-field resistance is 190 ohms, and its combined armature and commutating-field resistance is 0.0514 ohm. Brush drop is 2 volts. Calculate the value of stray power. About what value of line current should this motor draw at no load?

8–7. The full-load efficiency of a 50-hp, 230-volt motor is 88 per cent. Shunt-field resistance, hot, is 70.0 ohms, the combined armature and commutating-field resistance, hot, is 0.059 ohm, brush drop is 2 volts. What is the stray power in watts? About what value of line current would this motor draw at no load?

8–8. The name plate of a 240-volt motor states that the full-load line current is 67 amperes. On test at no load at rated voltage, it required 3.35 amperes armature current and 3.16 amperes field current. Armature resistance, hot, was 0.207 ohm (no commutating field). Brush drop 2 volts. If stray power is assumed constant from no load to full load, what are the horsepower and efficiency at full load?

8–9. The data for a 10-hp, 220-volt motor are: rated armature current 40 amperes; shunt-field resistance 250 ohms; armature resistance 0.30 ohm; commutating-field resistance 0.05 ohm; stray power 460 watts. Plot curves (points for every 10 amperes input, to 50 per cent overload) of armature loss, commutating-field loss, shunt-field loss, brush-contact loss, stray power, total loss, and efficiency.

8–10. A 10-hp, 110-volt, 900-rpm shunt motor has an efficiency of 86 per cent, an armature resistance of 0.07 ohm, a commutating-field resistance of 0.01 ohm, and a

shunt-field resistance of 55 ohms. When the load torque is of such value as to cause 50 per cent rated armature current to flow, what will be the horsepower output and the efficiency? Brush drop with rated armature current is 2 volts, and with half-rated armature current is 1.75 volts.

8–11. A 110-volt motor has a shunt-field resistance of 68.3 ohms, and an armature resistance of 0.052 ohm. Rated full-load current is 110 amperes. When running at no load, the motor draws 5.2 amperes from the line. Assuming that the combined core and friction loss remains constant from no load to full load, calculate the efficiency and horsepower output with an input of 50 amperes. With 100 amperes.

8–12. A 50-hp, 230-volt shunt motor at no load takes 9.8 amperes total line current. The shunt-field resistance is 53.5 ohms, hot, the armature resistance is 0.038 ohm, hot, and the commutating-field resistance is 0.010 ohm, hot. The name plate states that the full-load line current is 182 amperes. Brush drop is 2 volts. Calculate the horsepower output and efficiency for line currents of 182 and 91 amperes. Is the name-plate rating for horsepower correct?

8–13. Two 25-kw, 240-volt generators were tested by a pump-back test with both machines connected across a 240-volt supply (Fig. 8–3). When the generator was adjusted to supply rated current, the supply current was 23.4 amperes. The generator field current was 4.31 and the voltage across the generator field was 200 volts. The motor field current was 2.38 amperes and the voltage across the motor field was 133.2 volts. Each armature has a resistance of 0.103 ohm. Brush drop is 2 volts. What is the efficiency and output of each machine?

8–14. A 240-volt, 250-hp shunt motor has a field resistance of 36 ohms, an armature resistance of 0.0032 ohm, and a commutating-field resistance of 0.0008 ohm. Brush drop is 2 volts. The machine under test does not have a shunt-field rheostat. When driven without load by another motor at full-load speed and at such excitation that the terminal voltage was equal to the full-load internal voltage, the armature current of the driving motor was 62.5 amperes with 120 volts impressed upon its armature. With the motor under test unexcited, but driven at its full-load speed, the armature current of the driving motor was 39 amperes with 115 volts impressed upon its armature. Running free at the same speed as when driving the machine under test, the armature current of the driving motor was 5.7 amperes with 110 volts impressed. The armature resistance of the driving motor is 0.06 ohm. What is the efficiency and total friction loss of the motor under test at full load?

8–15. The losses of a 2000-kw, 300-rpm compound generator are to be obtained by means of a 100-hp, 240-volt shunt motor. The no-load voltage of the generator is 500 volts, its full-load voltage is 575 volts, its armature resistance is 0.00285 ohm, its series-field resistance is 0.00076 ohm, its commutating-field resistance is 0.00055 ohm, and its brush drop is 2 volts. The armature resistance of the driving motor is 0.0133 ohm and its brush drop is 2 volts. When the generator is driven with no load upon it at 300 rpm and generating its full-load internal voltage, the driving motor draws an armature current of 268.5 amperes with 234.5 volts impressed upon the armature. When the generator is driven unexcited, the driving motor takes 107 amperes armature current at 222.5 volts. When the driving motor runs free at the same speed, it requires 16.1 amperes at 216 volts. What are the full-load efficiency, the total friction loss, and the induced voltage of the generator?

8–16. Calculate the efficiency of the generator of the last problem at one-half rated load.

8–17. The armature copper of a certain machine rises 10 C with a current of 75 amperes. Assuming a safe temperature rise of 55 C, how much current can the machine carry? (Neglect the effect of core loss in this problem.)

8–18. The type and speed of a 400-kw, 550-volt machine are such that a temperature rise of 45 C in the armature would dissipate 0.25 watt per sq cm. The available radiating surface is 62,000 sq cm; the armature resistance is 0.0135 ohm, and the iron loss is 9000 watts. What will be the rise in armature temperature at full load?

8–19. A 6-pole, 150-kw, 250-volt generator has an armature surface and ventilation such that it radiates 0.05 watt per square inch per degree C rise. The armature resistance is 0.0095 ohm, the hysteresis loss in the teeth is 610 watts, and in the core proper 1240 watts; the eddy-current loss in the teeth is 50 watts and in the core proper 90 watts. The radiating surface is 2600 square inches. What is the temperature rise at full load? What is the safe current through the armature, if a rise of 55 C is allowed?

8–20. A rectangular field coil, 9 inches by 15 inches, carries a current of 10 amperes and has a resistance of 3.9 ohms. The heat dissipated is 0.003 watt per sq cm per degree C rise. How long must the coil be, if a temperature rise of 50 C is allowed?

8–21. By putting in a fan to assist in cooling a motor, the armature temperature rise is dropped from 40 C to 30 C. If a 40 C rise is safe, by how much may the safe rated armature current be increased, after the fan is in use?

8–22. An armature reaches its maximum safe temperature of 90 C when carrying full load of 500 amperes, the ambient temperature being 20 C. Its core loss is 2200 watts and copper loss 1850 watts. What should be its rated full-load current if it is operated where the ambient temperature is 35 C?

8–23. A generator is correctly designed to deliver 50 kw without having its armature temperature rise to greater than 90 C (which is its maximum safe temperature), with the temperature of the surrounding air equal to 25 C. The machine is to be used in the southwestern part of the United States where the ambient temperature holds at 45 C for periods of 10 hours and more. By what percentage must the full-load rated current be reduced if the armature temperature is not to exceed its safe value?

8–24. In problem 8–23, how much must the field current be reduced if the field-coil temperature is to be no greater than it was at the lower ambient temperature? What will this do to the voltage rating of the machine? How much will be the kilowatt rating of the machine at the higher temperature?

8–25. The inside temperature of an electric refrigerator is held at 40 F by the electrically driven compressor. To hold this temperature with the ambient temperature at 70 F the compressor has to run 15 minutes of every hour, and the average temperature of the electric motor is 70 C. The refrigerator is used at a place where the ambient temperature is 120 F; how much of each hour must the compressor work, and what will be the average motor temperature? (Assume that average motor temperature rises directly with the proportion of the time it is operating and that compressor efficiency remains constant.)

CHAPTER IX

DIRECT-CURRENT INSTRUMENTS

1. Classes of Instruments. Electrical quantities are measured by means of instruments. An instrument is a general term indicating any type of electrical measuring device, but is specifically applied to measuring devices giving the value at any definite time, the so-called "present" value.

An indicating instrument is one in which the magnitude of the quantity is indicated by a movable pointer or needle moving relative to a divided scale. A recording instrument records the value of the quantity at a series of values of time, usually by drawing a record on a moving paper or chart.

The quantities generally measured by d-c instruments are current and voltage. The instruments for measuring these quantities take their names from the units of the quantity they measure and are called ammeters and voltmeters.

Devices that integrate the electrical quantity to which they respond with respect to time by means of some totalizing mechanism are called meters. The quantities generally measured by d-c meters are electrical energy and quantity of electricity; the meters are known as watthour meters and ampere-hour meters respectively.

In addition to the above-named instruments, the electrical engineer uses several other types for special purposes, such, for example, as the oscillograph; these will be taken up later.

2. Requirements for Indicating Instruments. There must first be an actuating force which is proportional in some manner to the quantity to be measured. In order that the needle or pointer may come to rest at some definite point on the scale, some counter or restoring force, the intensity of which will increase in proportion to the displacement of the pointer, must be provided. Since the needle or pointer must move over a scale, bearings of some sort are necessary. That readings may easily be taken, some sort of device is required to rapidly damp out oscillations of the needle; the instrument must be "dead beat." A uniform scale is generally desirable, as it makes interpolation between divisions easier, but is not always possible; this feature depends upon the variation of the actuating force with the quantity to be measured. In some types of instruments, the actuating force varies as the square of the current or voltage, as the case may be.

3. Actuating Force. Direct-current indicating instruments obtain their actuating forces in one of three ways, the most important being that employed in the type known as the *D'Arsonval*, in which a movable coil, carrying a current proportional to the quantity to be measured, is placed in the field of a permanent magnet. In the *movable-core* type of d-c instrument, one or more pieces of soft iron are acted upon by the electromagnetic field of a coil, carrying current proportional to the quantity to be measured. In the third type, known as the *electrodynamometer*, the force between two or more conductors carrying current is utilized. These three types of instruments are illustrated by the drawings of Figs. 9–1, 9–9, and 9–10. Instruments are generally classfied according to their actuating forces, and these three types will be described in greater detail later.

4. Restoring Force. As restoring or counterbalancing forces are used: (1) the resisting force of a spring; (2) the attraction of gravity; (3) the torsion of some filament. The spring is most frequently used; instruments depending upon gravity must be carefully leveled, while a torsion filament is usually very fragile.

5. Damping Devices. The means employed for this purpose may be either the attraction of currents, induced in properly placed conductors by motion of the moving element, or the air friction of a suitable fan or vane.

6. D'Arsonval Instrument. In Figs. 9–1 and 9–2 are shown the general features of this type of instrument. A rectangular aluminum frame, on

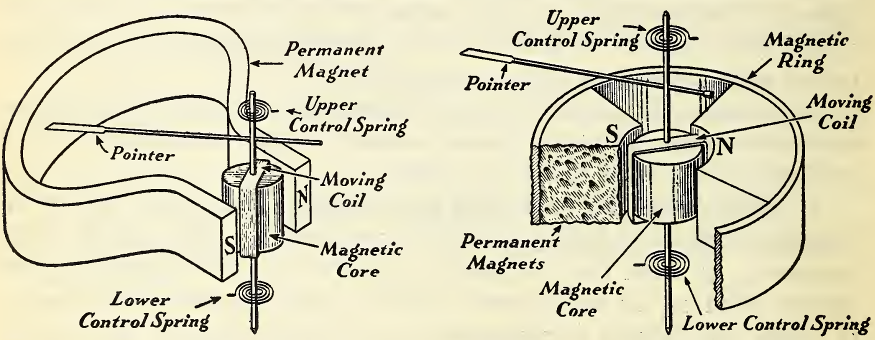


Fig. 9–1. Two forms of the D'Arsonval type of permanent-magnet instrument are shown. The circular type of magnet leads to a more compact construction. *Courtesy of the General Electric Co.*

which is wound a coil of very fine insulated copper wire, as shown in Fig. 9–3, is pivoted between the poles of a permanent horseshoe magnet, these poles being shaped, by the addition of soft-iron pole pieces, so that the clearance between the coil and the pole-piece faces is constant. Within the coil and supported by a plate of non-magnetic material attached to the pole pieces, is a soft iron cylinder, which helps to complete the magnetic

circuit. The air gap between the cylinder and the pole pieces, within which the coil moves, is thus of constant length, and the coil can move for a considerable arc in a radial field of uniform flux density. Two spiral springs, as shown in Fig. 9–1, one at the upper, the other at the lower end of the shaft, furnish both the restoring force and a means of current connection between the coil and the external circuit. The springs are arranged so that, as the coil is deflected, one is coiled up and the other uncoiled, compensating for errors due to elongation brought about by temperature changes. The moving element is supported at both top and bottom by hardened steel pivots turning in cup-shaped jewels, which are generally white sapphires. The pointer is made of very thin aluminum tubing and is balanced by small counter weights.

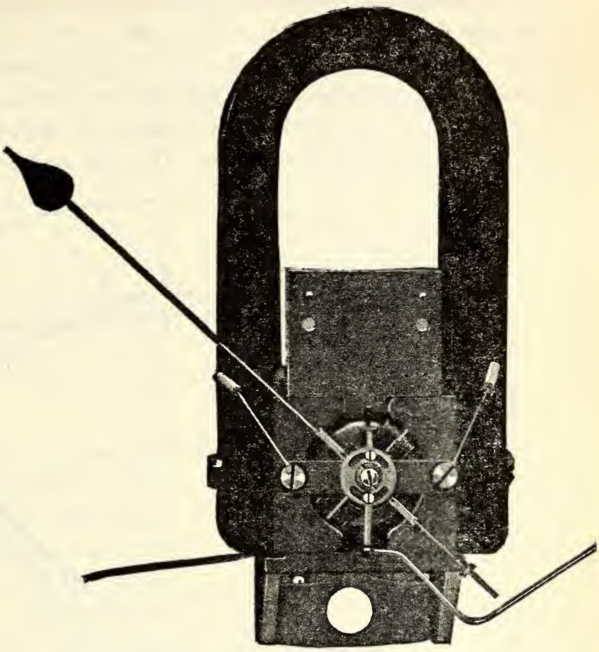


Fig. 9–2. This cut shows the construction of the moving parts of a D'Arsonval type of instrument. *Courtesy of the Weston Electrical Instrument Corp.*

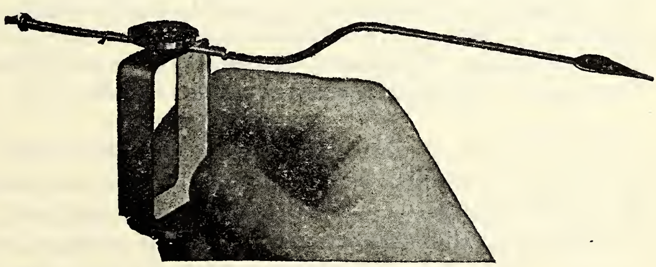


Fig. 9–3. The moving coil of an ammeter or voltmeter of the D'Arsonval type. *Courtesy of the Weston Electrical Instrument Corp.*

Since the turns of the coil are in a uniform magnetic field, when current flows through it a force results which is directly proportional to the cur-

rent. By the use of spiral springs, the restoring force is made directly proportional to the deflection, and a uniform scale results.

The rectangular aluminum frame on which the coil is wound serves also as the damping device. When the current in the coil changes, the equality between the actuating and restoring forces is destroyed, and the coil must move until this equality is again restored. If the moving element is very light, the coil moves very quickly; in any case, without a suitable damping device, it would overshoot the new point of equilibrium and con-

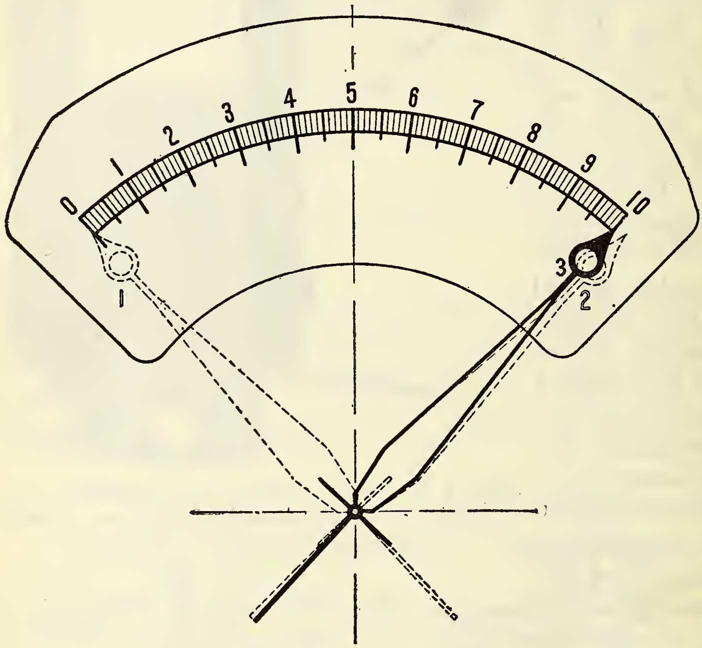


Fig. 9–4. The moving system of an instrument should be damped sufficiently so that it overshoots its proper position only a small amount, coming to rest with not more than one or two small oscillations. A reasonable amount of overshooting is shown in the above figure. *Courtesy of the Weston Electrical Instrument Corp.*

tinue to oscillate until air and bearing friction brought it to rest. Within the aluminum frame (on which the coil is wound) eddy currents are induced while the coil is moving. The direction of these currents is always such that, reacting with the magnetic field, a force is produced which opposes the motion. This form of damping device is very effective, the needle coming to rest with only one or two oscillations, as indicated in Fig. 9–4. Overdamping is never desirable, as it is likely to cause sluggishness; slight underdamping gives opportunity to observe that the movement of the needle is perfectly free.

The moving element of good commercial instruments of the D'Arsonval

type is made to deflect to full scale value with a current of 0.00005 to 0.010 ampere through the coil. A permanent magnet being used, the coil will move in a desired direction only when the current flows through the coil in the correct direction. If the current is reversed, the direction of deflection will be in the opposite direction, or backwards.

Instruments are usually made to deflect in one direction only, a stop being placed to prevent motion in the other direction. Some instruments, however, are made to deflect in either direction from a zero position at the center of the scale; such instruments are called *zero center* instruments. It is customary to indicate the polarity of the terminals of d-c instruments by a plus sign on the proper post.

If the current through the coil is varying, the needle will tend to follow the variations. If these variations are too rapid for the needle to follow, the needle will read the *average* value of the varying current. If an alternating current is impressed on the instrument it will read the average value of the alternating current, or zero.

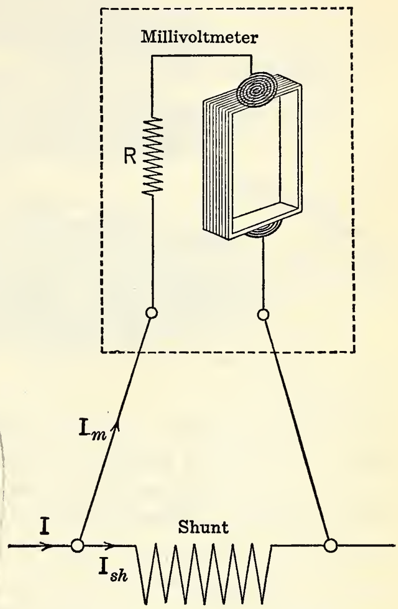


FIG. 9–5. When a D'Arsonval type of instrument is used as an ammeter, connections are made so that only a small fraction of the current to be measured goes through the moving coil. The moving coil is connected in parallel with a low-resistance shunt which generally carries more than 99 per cent of the line current.

7. Ammeters and Shunts. In order that the instrument described above may be used as an ammeter to measure currents greater than, say, 0.010 ampere, it is placed in parallel with a resistance, known as a *shunt*, as in Fig. 9-5. In order that the instrument may be deflected to full scale when a line current, I, flows through the combination of instrument and shunt, a required value of current, I_m, must be passed through the instrument. This leaves a current, I_{sh} (equal to $I - I_m$), to flow through the shunt. The drop in potential across the shunt is

$$E_{sh} = I_{sh}R_{sh}$$

and this is also the voltage impressed upon the instrument. Hence, the resistance of the instrument, and the leads that connect it to the shunt, must be

$$R_m = \frac{E_{sh}}{I_m} \tag{1}$$

In the best types of D'Arsonval instruments, the drop across the shunt, when it is carrying its rated current, is made 0.050 to 0.200 volt or 50 to 200 millivolts. In practically all ammeters the drop is 50 millivolts; in milliammeters, 100 or even 200 millivolt drops may be used. Assuming that the current required to cause the needle to go to full-scale deflection is 0.010 ampere, when 50 millivolts are impressed across the instrument, the resistance of the instrument together with the leads must then be 0.050/0.01 or 5 ohms. The moving coil, although wound of fine wire, has a resistance usually lower than 2 ohms; in order to bring the resistance of coil and leads to exactly 5 ohms, or whatever value is necessary, sufficient resistance is put in series with the coil, as is shown at R in Fig. 9–5.

If the instrument just described is intended to be used as a 10-ampere ammeter, the current through the required shunt would be $10 - 0.01 = 9.99$ amperes, and the resistance of the shunt, to give a drop of 50 millivolts, would be $0.050/9.99 = 0.005$ ohm, very nearly.

The same instrument, when used as a 200-ampere ammeter, would require a shunt with a resistance of 0.050/199.99 or 0.00025 ohm, approximately.

The instrument in operation really reads the voltage drop across the shunt. If the line current is one-half the value for which the shunt is rated, and its rated current gives a drop of, say, 50 millivolts, the drop across the shunt is 25 millivolts, and only 0.005 ampere passes through the instrument, causing the needle to stop at half scale. This would indicate a current of 5 amperes if the shunt used is a 10-ampere one, or 100 amperes for the shunt having a rating of 200 amperes.

The instrument alone, together with its leads, is called a millivoltmeter, and it is thus possible to use a millivoltmeter as an ammeter of any range, provided it is used with the proper shunt. In commercial instruments, in smaller ranges, it is customary to place the shunt inside the instrument case and to calibrate the scale of the instrument directly in amperes. Such instruments are called *self-contained* and are made for as high as 500 amperes by some manufacturers. To make the supply of instruments as flexible as possible, it is frequently the practice in laboratories to have all shunts separate from the instrument. Wherever external shunts are used, however, the leads must have a certain resistance to make the instrument read accurately.

Shunts are usually made of a metal the resistance of which does not change with temperature to any appreciable degree, such as manganin (see Fig. 3–17, page 101); for large values of current these shunts become very large and bulky. There must be enough cross-sectional area to carry the current at a low current density to limit the heat produced and a large surface to dissipate the heat readily. A 1200-ampere shunt weighs 6

pounds, a 12,000-ampere shunt weighs 114 pounds. Shunts ranging from about 10 amperes to several thousand amperes are shown in Fig. 9–6.

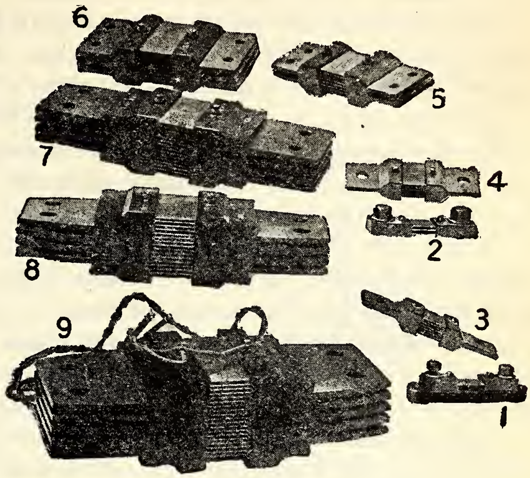


FIG. 9–6. Shunts intended to carry large currents must be suitably designed. To prevent undue heating, with resultant error in the instrument reading, much metal must be used for the large currents and a liberal amount of heat-radiating surface allowed. *Courtesy of the Weston Electrical Instrument Corp.*

One reason for using as high as 50 millivolts drop across the shunt for the best millivoltmeters is that the change in resistance (due to temperature variation) of the copper wire used on the moving coil can be compensated. The adjusting resistance is made of a metal with negative temperature coefficient; enough metal must be used for proper compensation, and the increased resistance requires the use of a fairly high drop across the shunt. In less expensive instruments, this compensation is not added to the instrument.

8. Voltmeters and Multipliers. To use the D'Arsonval type of instrument as a voltmeter, it is necessary only to place in series with the moving

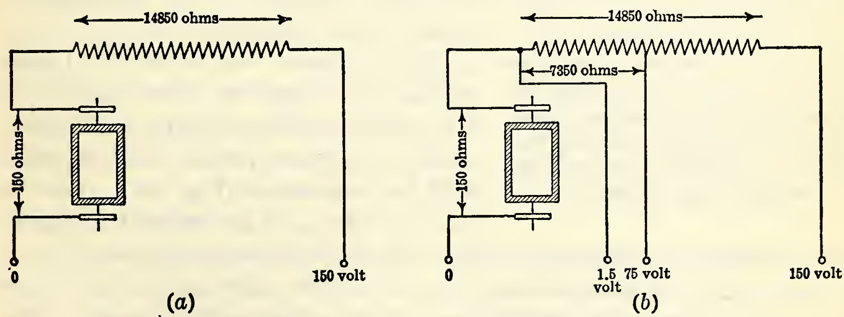


FIG. 9–7. When a D'Arsonval type of instrument is used for a voltmeter, a suitable resistance must be used in series with the moving coil. In case several voltage ranges are to be obtained with the same instrument, suitable taps from this added resistance may be brought out to binding posts.

element a high resistance, as in Fig. 9–7. The moving coil in voltmeters is sometimes wound with more turns, and of finer wire, than that used in

millivoltmeters, and the needle will therefore go to full-scale deflection with smaller currents. However, the value of 0.01 ampere is quite common. The resistance of the coil might be 150 ohms.

To draw a current of 0.01 ampere from a 150-volt source of potential, a total resistance of 15,000 ohms is required. If the resistance of the moving coil is taken as 150 ohms, 14,850 ohms of additional resistance must be added in series, as shown in Fig. 9–7*a*. This series resistance is ordinarily of a metal having a zero resistance-temperature coefficient. The scale of the instrument is naturally calibrated in volts, the current taken (and therefore the deflection) being proportional to the voltage impressed.

It is a very simple matter to provide a number of different ranges on the same voltmeter. If a 75-volt range is desired, it is necessary only to bring out a tap from the added resistance, so that 7350 ohms are added to the coil. With a coil resistance of 150 ohms, using the coil alone, a 1.5-volt range results. (Fig. 9–7*b*.) In the best commercial voltmeters the resistance of the moving coil may be as low as 12 ohms.

Such an instrument as the one just described is designated as having 100 ohms per volt resistance. It is often desirable to have high-resistance voltmeters so that the current taken from the circuit by the voltmeter does not appreciably change the currents in the circuit. This is especially true if the currents in the circuit are small and the circuit resistances high. A value of 100 ohms per volt is quite common, although values as high as 20,000 ohms per volt are fairly common, and considerably higher values have been used.

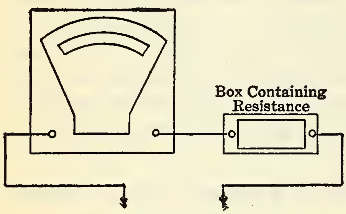


Fig. 9–8. In case a single-range instrument is to be used for voltages higher than its rated range, a suitable external resistance, called a multiplier, may be connected in series with it. This resistance must be able to carry safely whatever current is required to give full-scale reading of the instrument.

If a voltmeter is to be used for higher ranges of voltage than the instrument was built for, proper external resistances, called *multipliers*, can be used in series with the instrument (Fig. 9–8). For the case considered, if an external resistance of 15,000 ohms is connected in series with the instrument, the combination of instrument and multiplier can be used for 300 volts, the reading of the instrument being multiplied by 2 to get the voltage of the circuit. The resistance of the multiplier must always be matched to that of the instrument so that a known proportion of the total voltage appears across the instrument.

9. Movable-core Type Instruments. To this type of instruments belong all those in which the actuating force is obtained by the action of a coil carrying current, on a piece of soft iron. One type is shown in Fig. 9–9;

it is readily seen that the current magnetizes the soft iron core and rotates it into the axis of the coil. The restoring force is again the spiral spring. The force rotating the core is almost proportional to the square of the current; doubling the current doubles the flux through the iron (provided it does not become saturated) and quadruples the force. This results in a scale of uneven divisions, which is objectionable, the divisions for low values of current being generally very small. When the instrument is used as an ammeter, the entire current to be measured is passed through the coil; shunts are never used in connection with it. The actuating force is independent of the direction of the current, so that these instruments do not possess polarity.

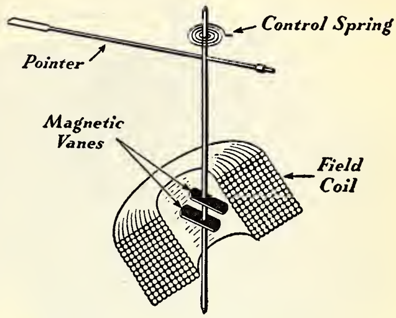


Fig. 9–9. One type of movable iron-core instrument. When the coil is energized it pulls the iron vanes into alignment along the coil axis against the tension of the control spring; the greater the current through the coil, the nearer the iron vanes are pulled into the coil axis. Since the vanes are of soft iron and initially unmagnetized, the instrument will read regardless of the direction of the current through the coil. *Courtesy of the General Electric Co.*

Modern instruments employing this principle are entirely satisfactory except for their uneven scales. However, for d-c purposes, instruments on the D'Arsonval principle are so nearly perfect and, for the same accuracy and sensitivity, so cheap, that at present this is practically the only type employed.

Movable-core type instruments are extensively employed in a-c circuits, simply because there are no other types that can be made at moderate cost. The forms so employed are discussed in Volume II.

10. Electrodynamometer Instruments. As noted before, the principle of operation is the mutual attraction and repulsion between circuits carrying currents, or the interaction of the magnetic fields produced by the currents. As no iron is employed, the action is called electrodynamic, instead of electromagnetic.

The principle of operation, as it might be applied in a voltmeter, is illustrated in Fig. 9–10, which gives a sketch of an instrument of this type. Two stationary coils and one movable coil are connected in series the movable coil being pivoted to rotate between the two stationary coils. Current flowing through the three coils causes the movable coil to rotate against the tension of springs.

Evidently, since the force between the coils varies as the current squared, such an instrument will also have an uneven scale. To set up forces without the use of iron requires comparatively high values of cur-

rent, resulting in an instrument which, compared to those utilizing the D'Arsonval principle, requires considerably greater power.

It will readily be appreciated that it is almost impossible to pass any but very small currents through the moving coil; the connections to the coil can be only of the lightest, and the coil itself must also be very light. The use of this principle for portable ammeters is thus impracticable except where small currents are to be measured. For measuring large currents with instruments of this type, suitable shunts are sometimes used, so that only a small fraction of the total current flows through the movable coil.

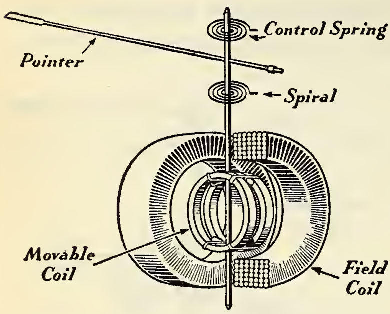


FIG. 9–10. In the electrodynamometer type of instrument, the field is supplied by a pair of coils instead of a permanent magnet. (In the figure one half of the front coil has been cut away.) This instrument does not have polarity. When used as a voltmeter, the movable and field coils are connected in series. *Courtesy of the General Electric Co.*

For portable instruments, the electrodynamometer principle is seldom used at present in d-c ammeters and voltmeters, being generally inferior to the D'Arsonval. However, standard instruments built on the electrodynamometer principle are very convenient for laboratory work, being used to compare alternating currents with direct currents, that is, as transfer instruments.

11. The Wattmeter. The electrodynamometer type of instrument finds its greatest application as a wattmeter, an instrument to measure directly the power in a circuit. The current flowing into the circuit is passed through the stationary coil, and the potential across the circuit is impressed upon the moving coil with a suitable resistance in series. The deflection is proportional to the product of current and voltage and the instrument can be calibrated to read directly in watts. Wattmeters are not widely used in d-c circuits as the power is readily obtained from the product of the current and voltage.

12. Special Uses for Indicating Instruments. Electric instruments may be used for many special purposes in which the measurement of the electrical quantity is incidental to the measurement of some other quantity.

Temperature may be measured by indicating the changing resistance of a resistor as its temperature is changed. This could be done by impressing a fixed voltage on the resistor and an ammeter in series. The changing resistance would be indicated by a changing ammeter reading; the ammeter could be calibrated to read directly the temperature of the resistor in degrees.

A thermocouple (see Chapter XI) in conjunction with a millivoltmeter,

or short-circuited by a microammeter, can also be used to measure temperature.

A small d-c generator, as shown in Fig. 9–11, using a permanent-magnet field structure, is often used for speed measurements. A voltmeter connected across its armature will indicate the speed at which it is turning and can be calibrated to read speed directly. The resistance of the armature is made low so that the terminal and generated voltages are essentially equal and this results in a uniform speed scale on the voltmeter.

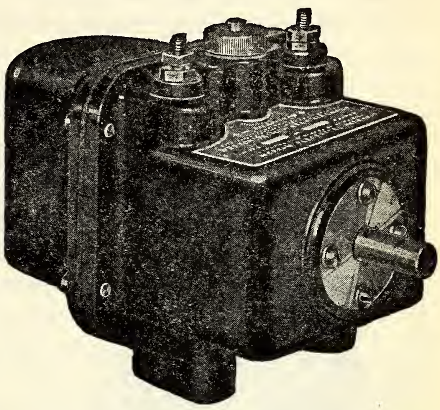


Fig. 9–11. This small d-c generator, called a magneto because its field is supplied by permanent magnets, is used for measuring speed. When connected to a voltmeter, the reading is a direct indication of the speed. *Courtesy of the Weston Electrical Instrument Corp.*

13. The Watthour Meter. In connection with the supply and sale of electric energy, a meter to record the total amount of energy used is necessary. It is standard commercial practice to measure electrical energy in kilowatthours, the kilowatthour being defined as the total amount of energy supplied in one hour to a circuit in which the average rate at which the energy is expended is 1000 watts.

A watthour meter consists essentially of (*a*) a small electric motor as actuating force, the torque of which is proportional to the power taken, (*b*) a brake system, so constructed that the retarding force is proportional to the speed of the motor armature spindle to which it is attached, and (*c*) a system of gears with graduated dials for registering the number of revolutions of the spindle.

When the speed of the rotating spindle has reached a constant value, the actuating torque must be exactly equal to the retarding torque. If the driving torque of the motor is made proportional to the power taken, and the retarding torque of the brake system is proportional to the speed of rotation of the spindle, the speed of the spindle is proportional to the actuating torque and also to the power.

If speed of the spindle is proportional to power, each revolution represents a certain number of watthours passed through the meter, whether the power is variable or steady, so that the total number of revolutions of the shaft during a given interval is a measure of the total energy during the interval.

The two common types of d-c watthour meters are the Thomson or "commutator," and the mercury flotation. The Thomson is almost always used and will be described below.

14. Brake System. As the brake system of all modern watthour meters is virtually the same, it will be taken up first. An aluminum disk is mounted on the motor armature spindle so as to rotate between the poles of one or more permanent magnets. The motion generates eddy currents within the disk which, reacting with the magnets, produce a drag on the disk. The strength of the magnets and the resistance of the eddy-current paths being constant, the magnitude of the eddy currents, and hence the force between them and the field of the magnets, will be proportional to the speed. Therefore, the drag on the disk is directly proportional to the speed.

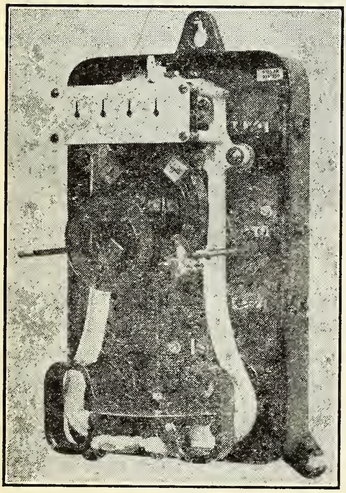


FIG. 9–12. View of the most common type of d-c watthour meter. *Courtesy of the General Electric Co.*

15. Thomson, or Commutator, Watthour Meter. In this type, a motor having fields and an armature with a tiny commutator is employed. The field coils, of low resistance, are placed in series with the line supplying the current and since no iron is used in the construction of the motor, their flux is directly proportional to the current.

The armature is placed directly across the line and its current (which is reduced to a very low value by the use of a high series resistance) is then directly proportional to the voltage of the line. The torque of a motor being proportional to the product of armature current and field strength, it follows that the actuating torque of the meter is directly proportional to the product of line voltage and line current, or to the power supplied to the load.

In Fig. 9–12 is shown a commercial form of a Thomson or commutator type of meter, with the cover removed; in Fig. 9–13, the circuits of such a meter are shown diagrammatically. A vertical spindle, S, supported at the top by a guide bearing, and at the bottom by a sapphire or diamond bearing, carries the armature, A, made in spherical form to fit closely to the fields, and the aluminum disk D.

The main field coils, FF, are in series and carry the entire current passing through the meter. The armature, or potential, circuit, shown in dotted lines, starts at one side of the line, and passes successively through the compensating field C, the armature, and a fixed high resistance, R,

ending on the other side of the line. In some meters all the necessary resistance in series with the armature is contained in the compensating field and no extra resistance, R, is used.

The function of the compensating field is to compensate for the friction of the main bearings, the brushes on the commutator, and the gear train. The friction is reduced to a minimum by using an extremely small commutator (about one-tenth inch in diameter), jewel bearings, and a very

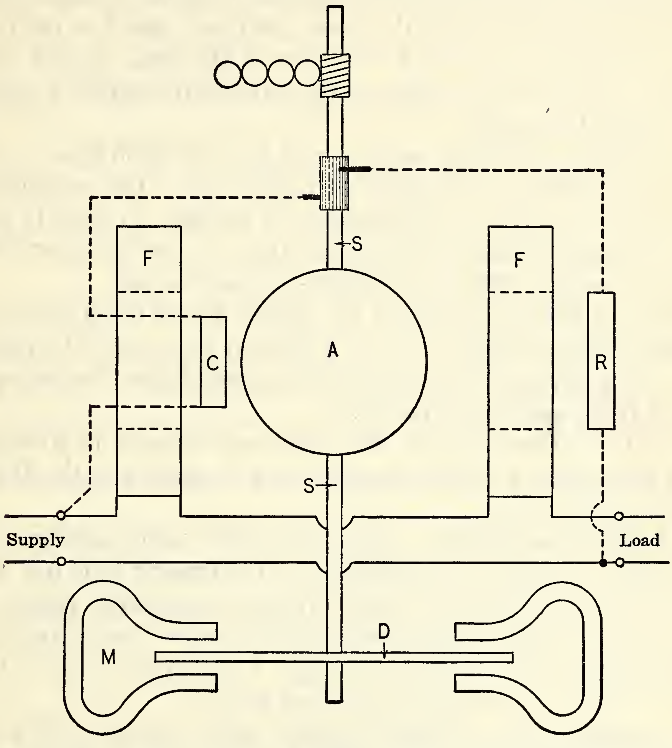


FIG. 9–13. Schematic diagram of the Thomson watthour meter, showing the field coils, FF, connected in series with the load, and the armature circuit, consisting of the armature, A, compensating winding, C, and extra resistance, R, connected across the line.

light gear train. The material used in the commutator is generally pure silver, to maintain a smooth surface and prevent oxidation, and the brushes are generally silver-tipped for the same reasons.

The strength of the compensating field is not adjustable, but its effect on the armature may be changed by varying its position with respect to the armature; enough of its flux is made to pass through the armature to create sufficient torque to just balance friction, so that the meter is just on the point of starting at no load. If the compensating coil is too close to the armature, more torque than necessary results, and the meter tends

to "creep," or rotate slowly, without any load current through its fields, and register "fast," or too high, at light loads.

As the load on the meter increases, the main fields become stronger and stronger, and the effect of the compensating field comparatively less and less. In order to control the speed of the disk for loads above 10 per cent, the position of the permanent magnets is changed. If they are moved out from the center of the disk, the speed of the disk will decrease for a given load, for the same rate of cutting of the flux from the permanent magnets can now be maintained with the lower rotational speed of the disk. On moving the magnets in toward the center of the disk, the disk will speed up. At very light load, the effect of the permanent magnets is insignificant because of the low speed.

The resistance of the armature circuit is about 2500 ohms for 110-volt meters, about 5000 ohms for 220-volt meters, etc. The resistance of the armature proper is about 1200 ohms for all voltages, so that the resistance of the compensating field and fixed resistance, R, must be about 1300 ohms for 110-volt meters, 3800 ohms for 220-volt meters, etc.

Customary full-load speeds of the spindle are 25 to 50 rpm; with such low speeds and weak fields (no iron is used in the motor) the value of the counter emf generated in the armature is insignificant, the entire voltage impressed being used up as IR drop.

In small watthour meters the dials read directly in kilowatthours, while in larger sizes a "dial" constant must be applied to the dial reading to get kilowatthours.

16. Ampere-hour Meter. The ampere-hour meter measures quantity of electricity, and finds its application in the present wide use of storage batteries in central stations, electric vehicles, submarine vessels, lighting equipments on steam railroad cars, etc.; in order that storage batteries may be operated to the best advantage, it is essential that a record be obtained of their ampere-hour input and output.

The watthour meter, being an energy meter, measures the product of voltage, current, and time, or voltage and quantity; in the ampere-hour meter, quantity, or the product of current and time, is measured. In order to make the torque of the motor element proportional to current only, a constant field strength is necessary, and is obtained by the use of permanent magnets. Any d-c watthour meter may be transformed into an ampere-hour meter by replacing its electromagnets by permanent magnets.

17. Insulation Testing. The function of the insulation in electrical machinery is to keep the current in the right path. To this end it must possess dielectric strength against breakdown under the static strain produced by voltage, and also resistance to the passage of leakage currents. If, by reason of moisture, temperature rise, dirt, etc., the resistance

of the insulation is much decreased, the leakage currents will become appreciable. This increase will produce a further rise of temperature and possibly charring, which further decreases the resistance, and, the effect being cumulative, a complete breakdown of the insulation may result.

It is therefore advisable that measurements of the resistance of the insulation of machines, cables and other apparatus be taken periodically to determine its condition. Such measurements should precede a high-voltage test to determine the dielectric strength, inasmuch as trouble which can easily be located and remedied may cause breakdown if present while high voltage is applied.

Instruments much used for these periodic tests, known as Megger* insulation testing instruments, give direct readings of resistance. An instrument of this type, as diagrammatically shown in Fig. 9–14, consists of two permanent bar magnets, MM, between the poles of which, at one end, is the armature, G, of a small hand-driven generator, while at the other end is the moving system of the instrument. The latter consists of three coils, A, B, and B', rigidly fixed together and capable of freely rotating about the axis, O. No restoring or controlling springs are used, so that the pointer may stand anywhere over the scale when the generator is not being driven. It will be seen that coil B' slips over the pole tip of the upper magnet pole, and coils B and A both slip over the C-shaped piece of soft iron.

Coil B' is wound so that, with passage of current, its field is opposite to that of the magnet pole; a force is then developed tending to force the coil off the pole tip. Coil B is similarly wound and, with passage of current, it tends to set itself in a position where minimum flux from the permanent magnets passes through it, or directly opposite the gap in the C-shaped piece of iron. Coils B' and B, therefore, when energized, move the pointer counter-clockwise. Coil A is wound so that with passage of current it tends to move in a clockwise direction.

There are two circuits in the Megger tester, the potential circuit consisting of the coils B and B', and the resistance, R, in series, and the current circuit through the insulation to be tested across the test terminals, the resistance R' and coil A.

If the test terminals are open and the generator driven, current flows through coils B and B' and they set themselves as described above, opposite the gap in the C-shaped piece of iron along the dotted line. This corresponds to infinite resistance. If a sample of insulation of suitably high resistance is connected across the test terminals, current from the generator will flow over both circuits; that through coil A and the test insulation causes the moving element to move clockwise, while the current through coils B and B' produces a torque in the opposite direction. As

* Trademark, James G. Biddle Co.

the moving element moves clockwise, the coils B and B', moving into a stronger field, offer an increasingly stronger restraining torque until the torque due to coil A is balanced and the needle comes to rest over a point on the calibrated scale which correctly indicates the value of the resistance of the insulation being tested. Short circuit of the test terminals, corresponding to zero resistance, causes a current through coil A great enough to overpower the torque of coils B and B', and the moving element comes to rest as in Fig. 9–14.

Since both circuits are supplied by the same generator, any voltage change in the latter affects both circuits in the same proportion; the position of the pointer is therefore independent of the voltage or speed of the generator. However, for testing circuits or apparatus containing elec-

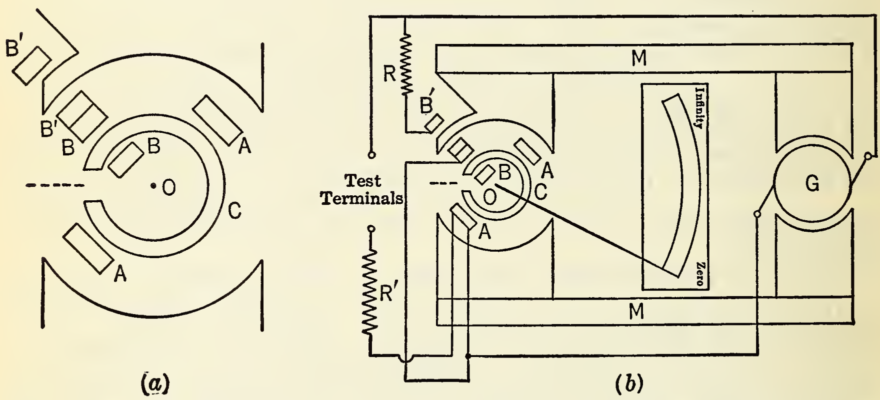


Fig. 9–14. The construction and connection scheme of a Megger insulation testing instrument. An enlarged view of the moving element is shown in (a).

trostatic capacity such as cables, machines with mica insulation, etc., it is advisable that the generator speed be constant; variable speed, causing variable voltage, results in charging and discharging currents, so that the readings vary. For this reason, in Megger testers to be used for determining the insulation resistance of large machines, long cables, etc., the armature and crank shafts are connected through a special friction clutch; for crank speeds above a definite value the clutch slips, insuring constant armature speed and constant voltage. This feature of a slipping clutch being unnecessary in tests where the electrostatic capacity is very small (less than 0.5 microfarad), a less expensive tester may be built. In these instruments the voltage of the generator varies with speed.

A phantom view showing the working parts of one type of Megger testing set is pictured in Fig. 9–15 and a smaller, more compact set is shown in Fig. 9–16.

18. The Oscillograph. This is an instrument which permits the observation and recording of the forms of all kinds of currents and voltages,

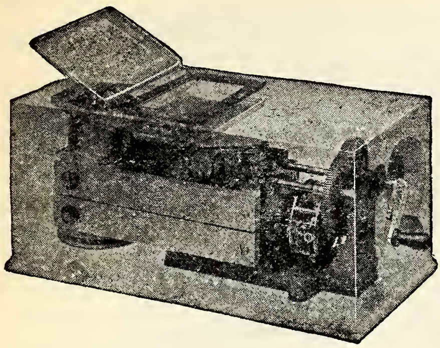


FIG. 9–15.

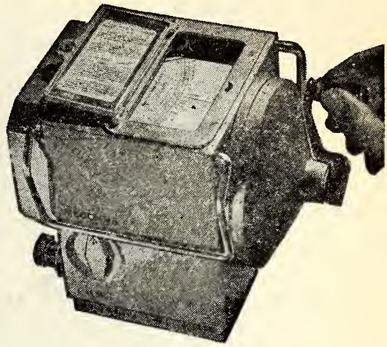


FIG. 9–16.

FIG. 9–15. Phantom view showing the working parts of a Megger Testing Set. *Courtesy of the James G. Biddle Co.*

FIG. 9–16. A "Meg" Insulation Tester with the scale calibrated in ohms. This is a lighter and more compact instrument than that shown in Fig. 9–15, and has separate magnets for the generator and the moving element. *Courtesy of the James G. Biddle Co.*

especially those that vary with time; it has probably done more in advancing the exact study of electrical phenomena than any other piece of apparatus. Photographic records of wave forms, called *oscillograms*, have been freely used in this book to substantiate theoretical predictions.

The oscillograph element, as ordinarily made, consists essentially of a two-strip suspension, mounted in a magnetic field, about as indicated in Fig. 9–17. The two wires, A–B, really form two sides of a single-turn loop, its upper ends being fastened to terminals M–N, and its lower end being taken around a small pulley C, which holds the loop tight by being pulled downward by a suitable spring. Across the loop, at its middle point, is cemented a small mirror D. The plane of the loop is parallel to the magnetic field of the pole pieces, N–S.

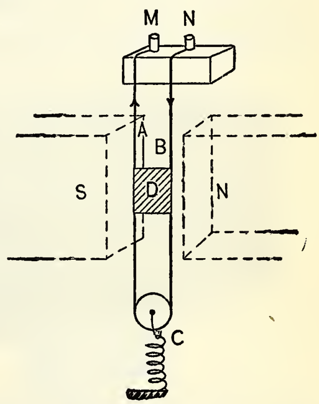


FIG. 9–17. Elementary sketch of the essential part of the Duddell oscillograph. The mirror, D, oscillates if alternating current is sent through the wire suspension A–B.

Current, the form of which is to be determined, goes down one wire and up the other; as the wires are very light and flexible, the force set up moves one of them forward and the other backwards, thus producing a

rotation of mirror D. Provided the changes in current are not too rapid, the position of the mirror, and hence that of a beam of light reflected

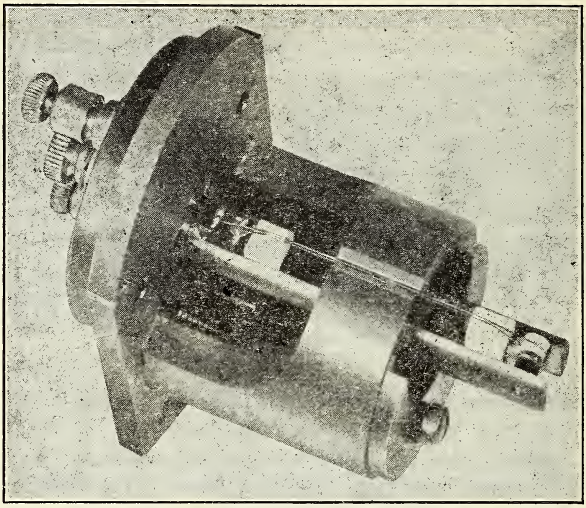


FIG. 9–18. A vibrator or element of an oscillograph, showing the loop of wire carrying a mirror. The wires are here placed in the field of a powerful permanent magnet. *Courtesy of the Westinghouse Electric and Manufacturing Co.*

from the mirror, will truthfully follow the variations of current through the strips. Alternating currents (currents which periodically reverse their

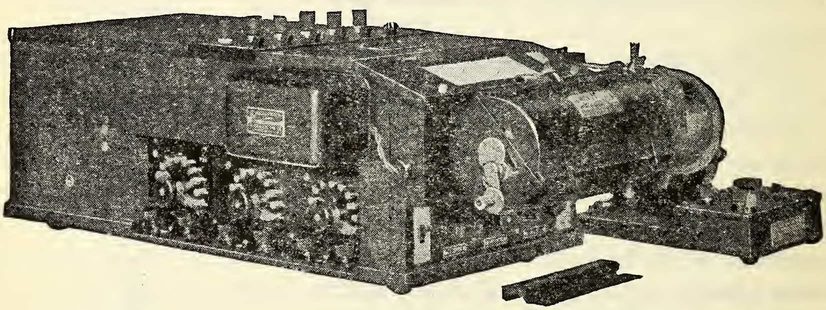


FIG. 9–19. A complete oscillograph. This model contains six vibrator systems, so that six records can be taken at the same time and in the proper time relation to each other. The revolving film holder fits on the front end; above it is a ground glass screen for viewing the picture to be taken. *Courtesy of the General Electric Co.*

direction of flow) may be accurately photographed if the frequency of reversal is not more than about a thousand per second.

A time variation is produced by drawing a photographic film across the beam or by reflecting it from a rocking mirror onto a screen.

To permit such rapid movements, the mirror is necessarily small; in the ordinary form of oscillograph, it measures about one-hundredth inch

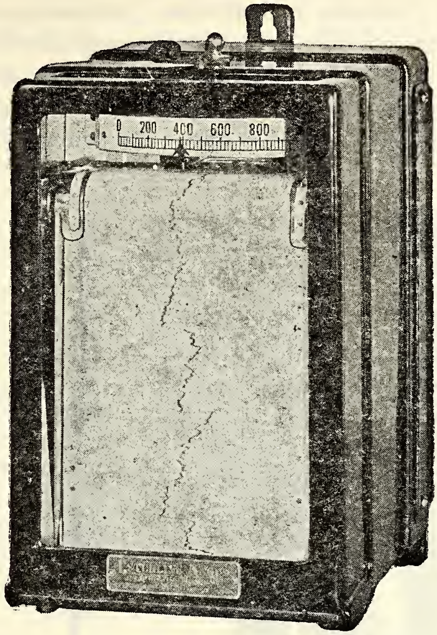


Fig. 9–20. A recording ammeter. The paper record winds from one drum on to another, being long enough for many days' use. *Courtesy of the Esterline-Angus Co.*

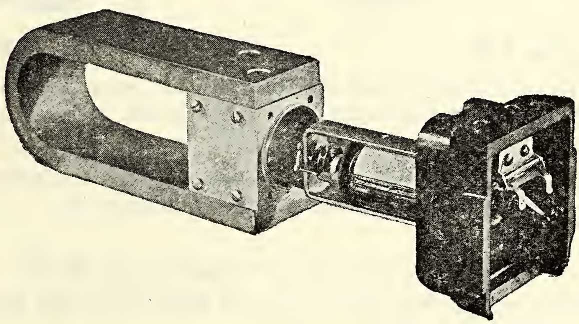


Fig. 9–21. Moving element, partly withdrawn from the permanent magnets, of the recording meter of Fig. 9–20. It is of the D'Arsonval principle. *Courtesy of the Esterline-Angus Co.*

wide and six-hundredths inch high. In Fig. 9–18 is shown a photograph of the vibrator of a Westinghouse oscillograph. From two to nine of these vibrators may be built into one oscillograph; the mirrors can all be oriented

so that their light beams come on the same film, and thus simultaneous records of two to nine voltages or currents may be obtained. In Fig. 9–19 is shown a general view of an oscillograph.

19. Recording or Graphic Instruments. Instruments intended to keep a record of instantaneous values are called recording instruments. Recording ammeters, voltmeters and wattmeters are much used in d-c station work. The moving element of a recording instrument carries a pen which draws a curve on a paper strip fed under the pen by suitable clockwork.

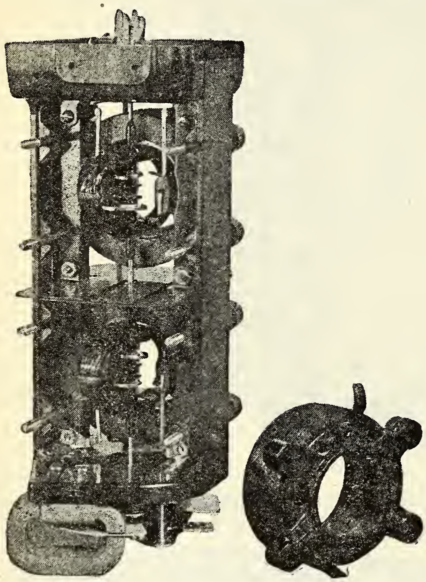


FIG. 9–22.

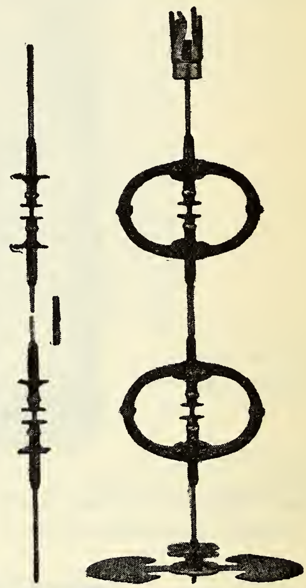


FIG. 9–23.

FIG. 9–22. Dynamometer-type assembly used in some recording meters. The two front field coils, one of which is shown, have been removed. *Courtesy of the Esterline-Angus Co.*

FIG. 9–23. Two views, front and side, of the moving element of assembly shown in Fig. 9–22. *Courtesy of the Esterline-Angus Co.*

A fine type of recording instrument is shown in Fig. 9–20. The moving element, which carries the pen back and forth over the paper strip as current through the instrument varies, is, in d-c recording instruments, usually of the D'Arsonval principle. Such a moving element, partly withdrawn from the permanent magnet, is shown in Fig. 9–21. For a-c recording instruments, the moving element is usually of the dynamometer type, but such dynamometer type may also be used for d-c instruments. In Fig. 9–22 is shown the assembly of an instrument of the dynamometer type, and in Fig. 9–23 are given front and side views of the moving element.

The records from such instruments are evidently valuable in showing the station manager how the load on his station varies, when peak load occurs and for how long, etc. The records of a recording voltmeter show how faithfully the voltage of a station is maintained at its rated value. In case of litigation involving the operation of the station, such records are, of course, invaluable.

CHAPTER X

AUXILIARY APPARATUS USED WITH DIRECT-CURRENT MACHINERY

1. Functions of Auxiliary Apparatus. The general purpose of auxiliary apparatus used with d-c machinery is for control, protection, and measurement. These functions may be combined; for example, a device that is used to control a d-c motor may incorporate features that would protect it in case of an accident.

Many devices used to control d-c machinery have already been described, such as the field rheostat, starting rheostat, etc. Instruments used in the measurement of electrical quantities were described in Chapter IX. There are, however, several important devices that should be described in more detail, as well as certain special uses of d-c machinery that should be discussed.

2. Switches. A switch is a device for opening and closing a circuit easily. The simplest type consists of a copper blade, hinged at one end and fitting between copper plates or jaws at the other. A hook or a handle of insulating material is fastened to the end of the copper blade by which the switch is operated. In this form it is known as a knife switch.

Switches are made in various sizes up to several thousand amperes. The larger ones use compound blades; they consist of a number of parallel bars that fit, upon closing, into a finger-like jaw. This is to obtain the necessary contact area to carry the large currents without overheating. The switches used with generators supplying power to lighting circuits are usually double-pole, one blade being in each side of the line, both blades being opened together by a common handle. In many cases, however, especially where one side of the line is *grounded* or connected to earth, the machine may be permanently connected to that line and a single-pole switch used to disconnect the machine. This is usually the case in railway systems.

The length of the blade, and the spacing between blades of a double-pole switch, depend upon the voltage for which the switch is designed, those to operate on a 600-volt circuit having considerably longer blades and greater spacing between blades than those for a 250-volt circuit. This is due to danger of a short switch not opening a 600-volt circuit, even though the blade has been pulled back as far as it will go. When the switch is opened,

an arc is formed, and the arc may run down the blade as the switch is opened and so hold over and burn from one post of the switch to the other.

3. Remote-control Switches. A switch that is opened directly by the operator is said to be a manually operated switch. Many switches, called relays or contactors, according to the service that they perform, are closed by electromagnets, the current supply of which is controlled by another relay, or directly by a small switch placed conveniently for the operator; the main contactor may be far from the controlling switchboard. In other types of remote-control switches the main switch is operated by small motors; in others, as used in railway installations, compressed air opens and closes the switches. It is the remote-control switch that makes the compact, remote-control switchboard possible.

The electromagnetically operated switch is widely used with d-c machinery, the starting and control often being effected by relays and contactors that operate automatically.

If a contactor must open circuits carrying considerable current it is often provided with some special means of extinguishing the arc that forms when the circuit is broken. If a circuit containing much inductance is broken (see Chapter IV) the decreasing current induces a high voltage of self-induction; this acts to keep the current flowing and may produce an arc through several inches of air between the opening contacts.

As long as the arc holds the circuit is not opened, the arc itself carrying the current through the air. The arcing will also badly burn the parts of the switch and cause the contact surface to become rough and uneven. This, in turn, would result in a reduced area of contact, further heating, and possibly more burning.

In order to lengthen the arc as quickly as possible *magnetic blowouts* are usually used on contactors. This consists of a coil of a few turns of heavy wire in series with the circuit being opened and so constructed that it produces a magnetic field *across* the arc. The arc, being a current, will move in the magnetic field, the direction of motion being made such that the arc is drawn out longer and extinguished.

Switches that must open large currents, called *circuit-breakers*, are specially constructed so that the burning of contact area is minimized. Circuit-breakers usually incorporate protection features as discussed later. Circuit-breakers for use with direct currents invariably open in air.

The circuit-breaker is designed so that the arc is formed between parts of the circuit-breaker that do not form the main current-carrying contact surfaces, and any burning or unevenness will not reduce this area. This is done by extending the contacts so that the arcing takes place between tips or by placing arcing electrodes or horns near the contacts. In the air circuit-breaker a series of parallel-connected contacts may open in sequence.

First the heavy copper main contacts open; these are carefully fitted and must not burn. As the breaker moves farther perhaps another smaller set of copper contacts opens, and finally the last set of contacts opens and an arc is formed. This last set is often a pair of carbon blocks; they do not burn easily and when badly damaged can be easily replaced.

4. Protection of Electrical Equipment. Fuses. When a machine or circuit is carrying more current than that for which it was designed, serious injury may result from overheating. The purpose of a fuse is to prevent such a possibility. A fuse consists of a piece of easily melted alloy, in the form of a wire or ribbon, connected in series with the machine or circuit to be protected. The size of a fuse is so selected that when a dangerous current is being carried by the circuit it is designed to protect, the heat generated by the I^2R loss in the fuse is sufficient to melt it and so the circuit is automatically opened.

5. Types of Fuses. There are several types of fuses in common use. The earlier type was the *string fuse*, which consisted merely of a piece of fuse wire inserted in the circuit by suitable clamps and screws. The disadvantage of this kind is that there is some danger of starting a fire when the fuse blows and throws melted lead.

The *plug fuse* is designed to overcome this possibility; it consists of a short string fuse mounted in a porcelain plug fitted with a screw base like that of an incandescent lamp. The cover to the plug is made of mica so that it may be seen whether or not the fuse has blown.

The National Board of Fire Underwriters do not permit the use of plug fuses for currents greater than 30 amperes. For larger sizes the *cartridge fuse* must be used. This consists of a tube made of fiber, sometimes filled with borax, infusorial earth, or similar substances, through the center of which the fuse ribbon passes. The two ends of the fiber tube are fitted with copper caps, to which the ends of the fuse are fastened. Short copper blades are fastened to these copper caps in the larger sizes and these fit into copper clips on the fuse block. When such a fuse blows, the arc is confined and smothered by the substance with which the fiber tube is filled.

In order to detect whether or not such a fuse is blown, a *tell-tale* is often provided. This consists of a very small fuse, soldered to the copper terminals so that it is in parallel with the main fuse. This small fuse, however, for a short way passes on the outside of the fiber tube, so that it can be seen. When the main fuse blows, the little one immediately melts and so gives evidence of the blowing of the main fuse.

6. Circuit-breakers. Circuit-breakers are usually used as protective devices when the currents are high or where the frequency of overload would make the replacement of fuses troublesome. The circuit-breaker, when used as a protective device, or as a combined switch and protective device, may be operated automatically whenever the current through it

exceeds a certain value. Such a circuit-breaker is called a simple *overload* circuit-breaker; it protects solely against excessive currents. Such a circuit-breaker is shown in Fig. 2-57; the operation is evident from Fig. 2–56.

It is often desirable to have a circuit-breaker with an *inverse-time* characteristic. In the discussion of rating, Chapter VIII, it was pointed out that a machine could withstand an overload for a limited time; the greater the overload, the shorter the period the overload could be supplied without injury to the machine. A manufacturing company will generally guarantee large generators to carry a 25 per cent overload for two hours.

In certain kinds of work, the load on an electric machine is intermittent, and for short periods of time there may be quite a heavy overload on the machine. If the duration of this overload is short, the machine will carry it safely and it is not desirable to have the circuit opened by a breaker or fuse. But if this overload should continue too long, the machine would be injured, and it is thus evident that a fuse or ordinary circuit-breaker could not properly take care of this kind of load.

In order to protect such a machine an *overload, time-limit* breaker is used. In one form it is essentially a circuit-breaker, to the plunger of which a dash-pot is attached. When an overload occurs, the plunger begins to move, but the damping is such that a considerable time is required for the plunger to move far enough to trip the breaker. The time elapsing from the moment the overload occurs to the tripping of the breaker is adjustable by a valve in the dash-pot, or similar device. It is evident from this description that such a piece of apparatus as the time-limit breaker just suits the needs of motors operating punch presses, rolls, hoists, etc. It is also evident the greater the overload the greater will be the pull on the plunger, and the quicker the breaker will open; this breaker has the inverse-time characteristic mentioned above.

Another widely used method of obtaining an inverse-time characteristic is by holding the breaker closed against a spring by means of a latch consisting of a bimetallic strip. The current passing through the breaker passes through a resistance element placed near the bimetallic strip. If the current exceeds a certain amount, the heat produced will cause the bimetallic strip to deflect, releasing the circuit-breaker. The greater the current the more rapidly the heat is produced and the more rapidly the breaker opens. A varying amount of time delay will be present, depending upon the magnitude of the current, as it will take time for the increased current to raise the temperature of the bimetallic strip. It is evident that an inverse-time characteristic is obtained.

Other types of circuit-breakers are made so that they will open if the direction of current flow through the breaker is reversed; such breakers are known as *reverse-power* or *reverse-current breakers*.

If two d-c generators are operating in parallel and the prime mover

of one failed, it would cease to generate power and run as a motor taking power from the other generator. A reverse-power breaker would protect the system by disconnecting the generator when its prime mover failed and the current to its armature reversed.

Still other types of breakers are designed to operate if the voltage rises too high, or falls too low, and for many other special applications.

7. Closing a Circuit-breaker; Replacing a Fuse. When correctly installed a circuit-breaker or fuse will have a switch placed in series with it. After opening due to an overload the switch should be opened, the circuit-breaker closed or fuse replaced, and then the switch closed. If the overload is still present the protective device is free to operate and will immediately open the circuit. If the switch were not opened, the fuse might act when being replaced with possibility of burning the operator, or the circuit-breaker might be held in while it was being closed. While many circuit-breakers are so constructed that they can open the circuit if an overload is present *even if the handle is held down,* most of them cannot open if the handle is held and are hence inoperative during the time that it takes to close them. If, however, the switch in series with it is opened first, then the circuit-breaker closed, and then the switch closed, the circuit-breaker is free to operate, and protection against overload is always obtained.

Many fused switches are constructed so that fuses may be inserted only when the switch is open, the fuse compartment being opened only when the switch is open.

There is always an adjustment on a circuit-breaker which fixes the current at which it opens the circuit. A certain breaker rated, for example, at 100 amperes may be set to trip at any current between 50 amperes and 150 amperes. The breaker should always be selected and adjusted to protect the circuit and apparatus.

The breaker or fuse should always be in the circuit in an operative condition; it should never be so large as to provide little protection and in no case should it be short-circuited, or, if a breaker, held closed. Such a practice leaves the machinery unprotected and, in case of an accidental overload, might result in costly damage.

8. Measurement of Electrical Quantities. Use of Instruments. A complete description of the most important types of instruments used in d-c circuits has already been given in Chapter IX; something will now be said of their use.

An *indicating instrument* has been defined as one in which the pointer deflects over a graduated scale, and so indicates at any instant the current or voltage in the circuit to which it is connected. It has no rotating parts and makes no record of the motion of its pointer. These instruments show the operator how much current a feeder is carrying at any instant, or what voltage a machine is generating. From their indications the

operator may properly adjust the voltage, re-distribute the load from one machine to another, etc.

A *recording instrument* keeps a time record on a suitable chart indicating by pen marks the magnitude of all fluctuations and the time at which they occurred; it must, therefore, include a time or clock mechanism within its parts.

Instruments are subdivided into *switchboard instruments* and *portable instruments.* The first are fastened permanently on a switchboard and can generally be used only for indication on the machine or feeder to which they are permanently attached. The portable instruments are smaller and more compact than the switchboard instruments, and are made for laboratory or test work, or for carrying out to different parts of a distributing system to read the current or voltage.

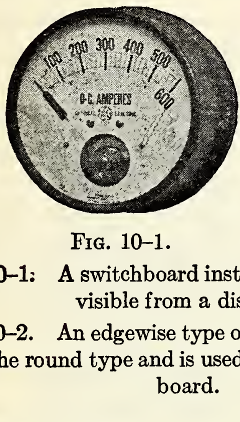


Fig. 10–1.

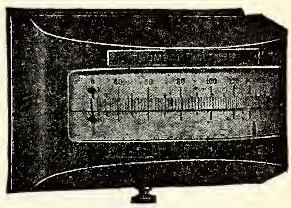


Fig. 10–2.

Fig. 10–1: A switchboard instrument of the round type. Its pointer is large so as to be visible from a distance. *Courtesy of the General Electric Co.*

Fig. 10–2. An edgewise type of switchboard instrument. It is somewhat more compact than the round type and is used when many instruments have to be assembled on a small board. *Courtesy of the General Electric Co.*

9. Switchboard Instruments. A switchboard instrument should be compact, have a large, well-marked scale of uniform graduations (except in some special cases), have a large, black pointer on the end of the indicating needle, and should be well damped. Of course, there are numerous other points to consider, such as permanency of calibration, freedom from temperature errors, etc., but only those mentioned above will be considered here.

On a large switchboard, having many generators connected to it, and supplying many feeders, the size of an instrument is of prime importance. There may be on one switchboard a hundred or more instruments, and it is evident that, if these instruments are not comparatively small, the switchboard must be very large and so difficult for one operator to manage. Also, the expense for bus-bars, copper feeders, etc., makes it advisable to keep the size of a switchboard as small as possible.

A round type of switchboard instrument is shown in Fig. 10–1 and in Fig. 10-2 is shown an edgewise type, having the scale horizontal. It will

be noticed that the design of the latter instrument has been carried out with the idea of getting compactness and still having a large, easily read scale.

10. Scale of an Instrument. The scale of an ammeter should be uniformly graduated, but sometimes it is advisable to have a voltmeter with a condensed scale on its lower ranges, for the purpose of getting a more open scale in the range where it is always used. For instance, an instrument to be used on a 600-volt circuit would probably have a total range of 750 volts; from zero to 400 volts the scale might well be condensed, as the instrument is practically never used on these ranges. From 400 volts to 750 volts, the scale could then be more open than if it were uniform throughout its range. The scale is often made to read from, say, 400 to 750 volts; below 400 volts the needle rests against the stop. Such a scale is known as a *depressed zero* scale. The necessity for a clearly marked scale and a large, easily seen indicating pointer is apparent when it is remembered that one operator may have to notice continually the indications of a hundred or more of these instruments.

11. Portable Instruments. The portable type of instrument differs from the switchboard type in that it is generally more accurately calibrated, has a more accurate and finely divided scale, and has a very thin indicating pointer. A switchboard instrument which indicates with an accuracy of 2 per cent is generally good enough; for laboratory tests, however, much higher accuracy is generally required, an accuracy of $\frac{1}{2}$ or $\frac{1}{4}$ per cent is usual. A common type of portable laboratory voltmeter is shown in Fig. 10–3.

12. Value and Importance of Recording Instrument Records. By inspection of the curve traced by a recording instrument, the station superintendent can tell at a glance just what the load on his station has been, what its maximum and minimum values were, and when they occurred. Or, if the record is from a voltmeter, it serves to show how well the operator or voltage regulator has maintained the voltage constant.

13. Maximum Demand Meters. In order to distribute the fixed charges of the power plant and distribution system of a power company more equitably among its customers, it is common, in charging for electric energy, to take into account the power demands of the customers as well as their energy demands. A customer who requires a large amount of energy at a uniform rate evidently ties up less generator and feeder capacity than one who takes the same amount of energy at widely varying rates. Thus, the unit charge for energy should take into account not only the total kilowatthours used by a customer during a given period, but also the maximum power demanded at any time during the same period.

It would not, however, be fair to consider as the maximum of a customer, the greatest instantaneous peak that occurred; it is better to take

the average of the power demanded over an appreciable time interval, so as not to include short circuits or other abnormal consumptions of energy that last for so short a time as not to affect the generator or feeder capacity of the system. The time interval over which the maximum demand is taken depends upon the relation the maximum demand bears to the connected capacity of the station, so that the interval may be fifteen, thirty, or sixty minutes.

If no time interval were to be considered, the simplest kind of demand meter for a d-c system, when the voltage at the customer's premises is practically constant, would be an ammeter, the needle of which pushes

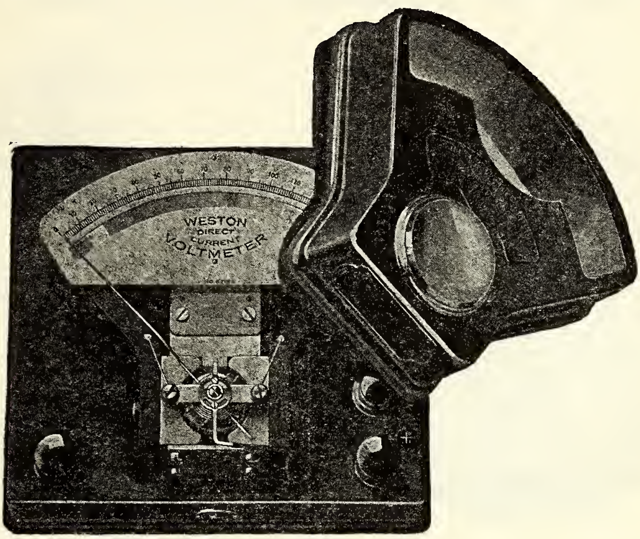


Fig. 10–3. A portable type of direct-current instrument. The pointer is very fine and can be very accurately read with the assistance of the mirror mounted in the scale to eliminate parallax. *Courtesy of the Weston Electrical Instrument Corp.*

another friction pointer before it up the scale and leaves it at the maximum indication.

Where the average power demand over a given interval is desired, without any record of the exact time at which it occurred, a thermal demand meter is sometimes used. One form operates by heat storage from an electrical heating element that includes a bimetallic spring system; the two metals of the spring, having different rates of expansion, cause a pointer to move over a scale. This pointer pushes the maximum demand pointer before it, and leaves it at the highest reading.

Where both the maximum power demand and the time of occurrence are desired, the demand meter works in conjunction with a watthour meter and a clock that times the interval over which the watt or kilowatt

demand is to be averaged. This is usually done by integrating the watts or kilowatts over the time interval by a register driven by the spindle of the watthour meter. By considering the length of the timed interval the scale can be made to read directly the average watts or kilowatts used during the interval.

This value may be printed on a tape, registered on a scale, or by means of a recording instrument, drawn as a line on a chart. In Fig. 10–4 such a recording demand meter is shown, together with a part of its chart. At the end of each time interval the pointer is reset to zero.

14. Switchboards. Originally, the switchboard was a very crude affair, a wooden rack on the front of which were mounted the switches and fuses

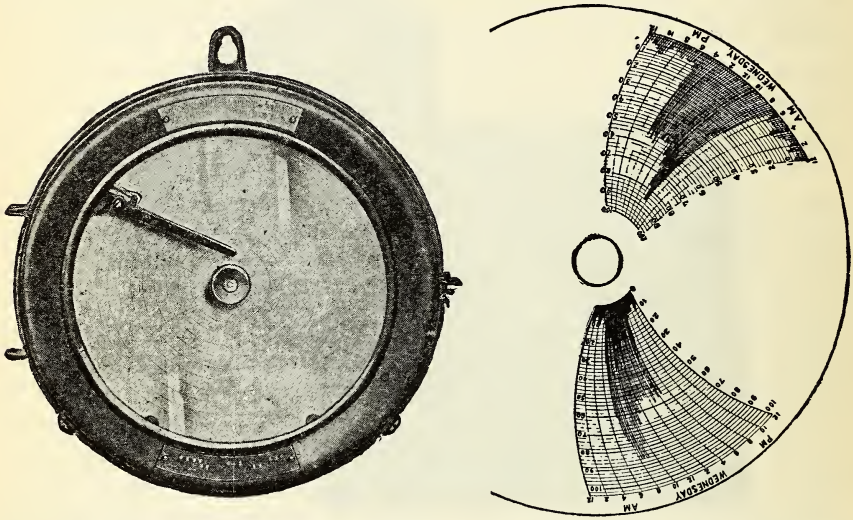


Fig. 10–4. A graphic demand meter and a sample chart. On the lower side of the chart the zero setting of the pen is at the center, on the upper side the zero is at the edge of the chart. *Courtesy of the General Electric Co.*

necessary for the operation of the plant. Today, the switchboard is probably the most important part of a generating plant; if an accident happens at the switchboard, the whole plant may be crippled.

The switchboard is the place to which the electric power from the generators is supplied and metered and thence distributed to the outside system through a group of feeders. On it are located all the switches, instruments, and protective devices of the plant, and from it the operation of the whole plant, both inside and outside the power house, is controlled.

15. Panels. A modern switchboard is divided into a number of *panels*; each panel serves for the control either of one generator or of a feeder or group of feeders. They are styled the *generator panels* and the *feeder panels*; in addition there may be panels for recording instruments, etc.

16. Construction of a Switchboard. The material of which a switchboard is made is often a good insulator, such as ebony asbestos, a dull black composition material. In many modern switchboards, however, the board is made of steel painted dull black. The equipment mounted on it is insulated by individual insulated mounts and is often mounted behind the board and operated by handles extending through to the front. There are often no current-carrying parts on the front of the switchboard; it is then known as a "dead-front" switchboard.

The switchboard is always designed for convenience of operating, for elimination of confusion in operating the devices mounted on it, and for a pleasing appearance. The panels are supported by a structural steel or pipe framework anchored to the floor and walls of the station house.

17. Bus-bars. Behind the whole length of the switchboard runs a set (two or three) of heavy copper bars, called *bus-bars*, or sometimes merely *buses*. All the generator panels are on one side and the feeder panels on the other; the bus-bars then convey the total power of the station lengthwise along the panels. For this reason they have a very large cross-section. They, as well as the rest of the connecting bars and wiring on the back of the board, are often supported by porcelain channels and cleats fastened to the steel framework of the board.

18. Arrangement of Panels. Each generator is connected through its respective circuit-breaker, ammeter, and switch to the bus-bars at a generator panel, and each feeder is connected to the bus-bars through its ammeter, circuit-breaker, and switch. By having all generator panels on one end of the board and all feeders on the other end, the addition of more generators or feeders is easily accomplished without disturbing the arrangement of the board; the proper number of panels may be added at either end of the board.

At the center of the board, between the generator and feeder panels, is located the *station output panel* on which are the recording and watthour meters that show the total power output of the station. By daily records of these meters and of the records of the customers' meters, the station manager may obtain an idea of the efficiency of his system, i.e., the ratio of the amount of energy sold to customers to the total energy sent out of the station. If this ratio is low he must improve it by better insulation of the outside lines, checking the accuracy of the customers' meters, etc.

In Figs. 10–5 and 10–6 are shown the front and rear views of a railway switchboard; in the front view the instruments, switches, and circuit-breakers are seen and in the rear view the general construction of the board, angle-iron supports, cable connections, etc., is shown.

19. The Voltage Regulator. When the load fluctuations on a plant are frequent and rapid, the automatic voltage regulation of a compound generator may not be sufficient to prevent momentary fluctuations in volt-

age. If a more nearly constant voltage is required than can be obtained from a compound generator by itself, an automatic voltage regulator is added.

Automatic voltage regulators as used with d-c generators are usually of two types, both of which effectively vary the shunt-field rheostat of the generator to regulate the voltage. One much-used type, known as the Tirrill regulator, is of the vibrating type.

With the vibrating type a sensitive relay opens and closes its contacts as the voltage rises above or falls below the desired value, respectively. It is so connected, usually by means of a second relay, that all or a portion

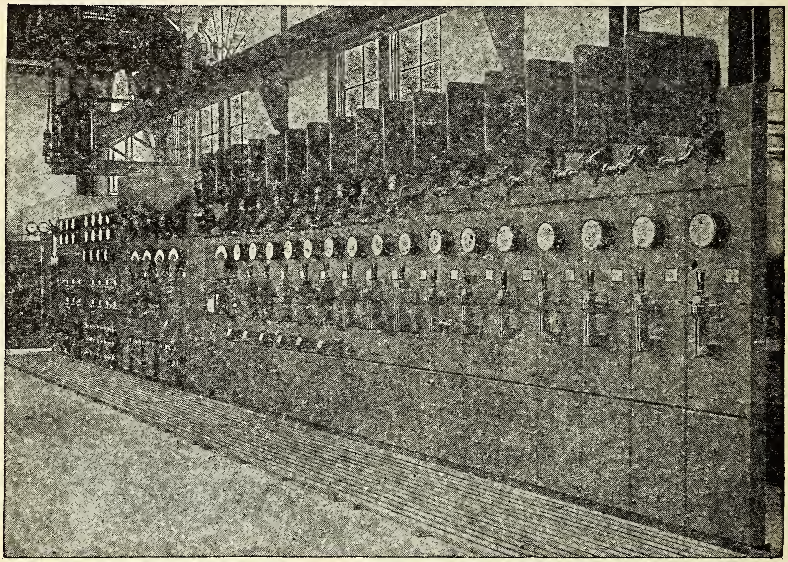


FIG. 10–5. This front view of a switchboard for a railway power plant shows the feeder panels, with their switches and circuit-breakers in the foreground, the station metering panel in the center of the board, and the generator panels in the background.

of the shunt-field rheostat is intermittently short-circuited. When the voltage rises, the relay closes and the short circuit is removed (by the second relay) from the field rheostat; the voltage will then fall because of the added field resistance. If the voltage falls too low, the relay acts, the field rheostat is short-circuited, and the voltage rises, the cycle of operations repeating itself.

This field rheostat is adjusted so that, when the relay contacts are open and the resistance is in the field circuit, the voltage of the generator will be about 35 per cent lower than rated; when the contacts close and short-circuit the field resistance, the generator voltage rises above rated value. The cycle of operations continues at a high rate, the contacts vibra-

ting 500 to 800 times a minute, depending on the tendency of the voltage to vary from normal. Suitable condensers are used across the relay contacts to reduce sparking.

By proper construction and adjustment, the action of the relay may be made very delicate, and the generator voltage maintained quite con-

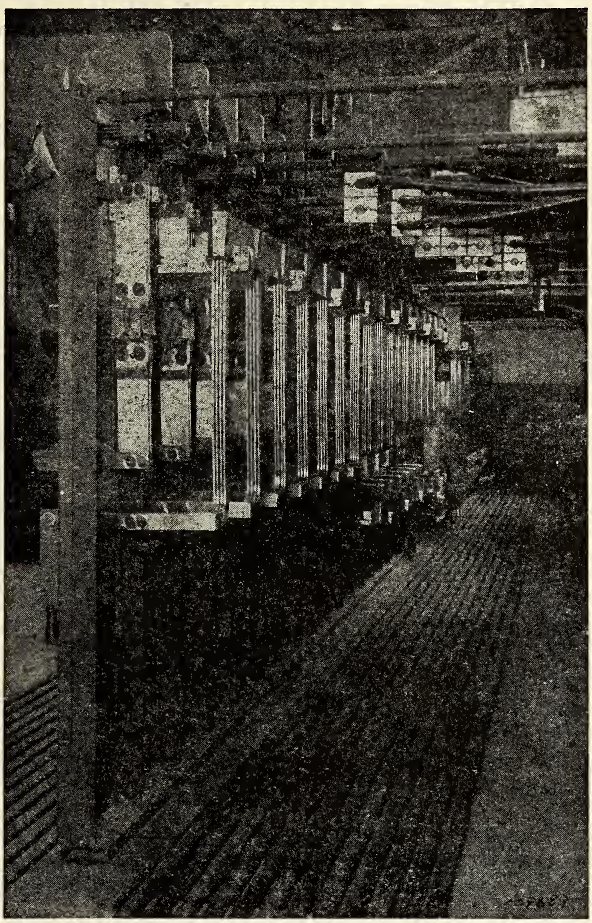


Fig. 10–6. Rear view of the switchboard illustrated in Fig. 10–5. This shows the general arrangement, method of supporting the bus-bars, ammeter shunts, etc. The panels themselves are supported by an iron pipe framework.

stant. The closeness of the voltage regulation depends upon the range and rapidity of the load fluctuations; but even with the most severe load fluctuations, the voltage can be held constant within 1 or 2 per cent.

The disadvantage of the vibrating type regulator is the number of moving parts and the trouble that is caused by wear and burning of the con-

stantly moving contacts. This type is largely replaced with small and medium size machines by the rheostatic type of voltage regulator in which the field rheostat is actually adjusted quickly to hold the voltage constant. This may be done in several ways.

One method used is to connect to the output terminals of the machine a small device exactly like a voltmeter in construction, except of larger size, called a "torque motor." Instead of a pointer the torque motor carries a lever and this lever is used to adjust the field rheostat. A special resistor is used that changes its resistance widely with a very small amount of motion. The torque motor, like a voltmeter, would come to a certain equilibrium position at a definite voltage, corresponding to the point at which the motor torque equals the resisting spring torque. The field rheostat would then be in a certain position. If the generator voltage rises the torque motor changes its position and adjusts the field rheostat so as to insert resistance in the shunt field, lowering the voltage and restoring the torque motor to its position of equilibrium. The device can be made very sensitive and quick-acting.

The same type of control and the same action can be obtained in a number of other ways. In some systems the torque motor is replaced by a coil and a moving iron mass or armature. When voltage is applied to the coil the iron mass is attracted. It is balanced by a spring, the equilibrium position being when the attraction of the magnet balances the spring pull. A rising voltage will increase the pull of the magnet and draw the iron mass toward it. The movement of the iron mass can be made to open contacts across resistance steps in the field circuit and hence lower the voltage, restoring the equilibrium condition.

20. Automobile Equipment. Electrical apparatus plays an important part in the operation of a modern automobile, the equipment serving to perform the functions of lighting the car, starting and igniting the engine, and keeping the battery charged. The various pieces of apparatus that carry out these functions will be discussed in a general way.

Lighting. To light the car, a storage battery is provided. This is of the lead type, giving either 6 or 12 volts, the former being almost universally used. The capacity of the battery is usually 80 to 120 ampere-hours.

A car will have the single-wire, or grounded-return, system of wiring, the various electrical circuits being completed by using the frame of the car as part of the circuit, each lamp or other piece of apparatus having one side grounded to the frame. Such ground connections must be perfect electrically and mechanically, so as not to be affected by corrosion and vibration.

Starting. To start the engine, a series motor is used, the motor being required to exert sufficient torque to start and turn the engine over at a

speed ranging usually between 100 and 200 rpm, a speed sufficient to cause the engine to ignite. The series motor is connected to the engine flywheel, through some type of sliding pinion and overrunning clutch, which enables the engine to run away from the motor after it has been ignited. The operator usually pushes a pedal against spring action, putting the motor into mesh with the engine flywheel by means of the sliding pinion, and also closing the electrical circuit of the starting motor. As soon as the engine is ignited, it speeds up, overrunning the clutch. Release of the starting pedal allows the spring to take the sliding pinion out of mesh with the engine, and also opens the electrical circuit of the motor. The above action may also be made automatic through the accelerating pedal.

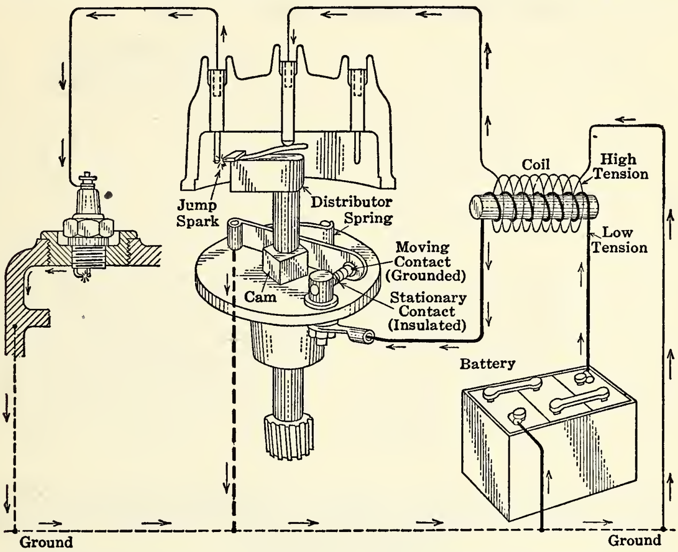


Fig. 10–7. A schematic diagram of the battery-and-coil scheme of ignition for an automobile engine.

Ignition. The primary object of ignition is to explode the compressed mixture of gasoline vapor and air in the engine cylinder, at the proper moment, by an electric spark. This is done by suddenly building up a voltage across a gap between the two terminals of the spark plug extending into the engine cylinder, the voltage being sufficiently high to cause a discharge across the gap.

Battery Ignition. In Fig. 10–7 the principle of battery ignition is shown, the principal parts indicated being a battery, an induction coil, an interrupter, a distributor, and a spark plug. The interrupter and the distributor are mounted on the same shaft, which is driven through gears from the engine. The interrupter is merely a contact which is momentarily made and broken by a cam, as many times per revolution as the

engine requires sparks; the distributor consists of an arm or brush which rotates over a surface of insulating material, in which are imbedded a number of metallic strips, each connected to the spark plug of one of the engine cylinders.

The induction coil has a low-tension winding of a small number of turns and a high-tension winding of a large number of turns. Each time the interrupter makes a momentary contact, current flows through the primary or low-tension winding of the induction coil, which, when broken by the interrupter, induces a high voltage in the high-tension winding. At the same instant that the high voltage is induced in the high-tension winding, the distributor arm is over one of the metal contacts; the high voltage is thus applied to a spark plug, causing a spark to pass.

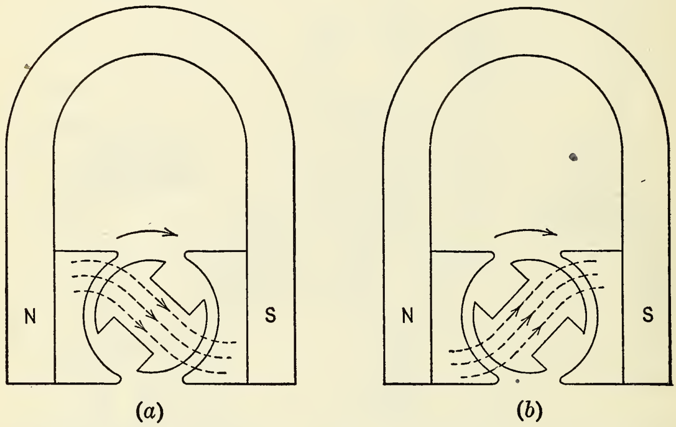


Fig. 10–8. Showing how the rapid change in flux interlinkages of the armature of a magneto occurs. From position *a* to position *b*, the flux through the armature has reversed completely.

In Fig. 10–7 there is a small gap between the distributor brush and the various contacts, and it is necessary that this gap be bridged by a spark in addition to the gap in the spark plug. Sometimes, no such gap is provided, the end of the distributor brush making actual wiping contact with the various contacts. Obviously, there must be as many contacts, evenly spaced, as there are engine cylinders.

Magneto Ignition. The ordinary magneto is a small generator having permanent magnets and a peculiarly shaped armature. Two successive positions of the armature within the field are shown in Fig. 10–8; as the armature moves from position (*a*) to position (*b*), there is a very rapid reversal of the lines of force through it.

The armature generally carries two windings placed on the central portion; one is a low-tension winding of few turns and the other is a high-

tension winding of many turns. The low-tension winding is closed on itself through an interrupter which breaks the circuit twice per revolution of the armature, while the flux through the armature is changing at its greatest rate. The collapse of the primary current is followed by the induction of a high voltage in the high-tension winding, this voltage being set up by the rapid reversal of the field. The high voltage is applied to each cylinder by a distributor, as before, one end of the high-tension winding being connected to the distributor arm through a slip ring, or similar connection, and the other end grounded.

A simple diagram of the magneto is given in Fig. 10–9. For a four-cylinder engine with two explosions per revolution, the magneto armature

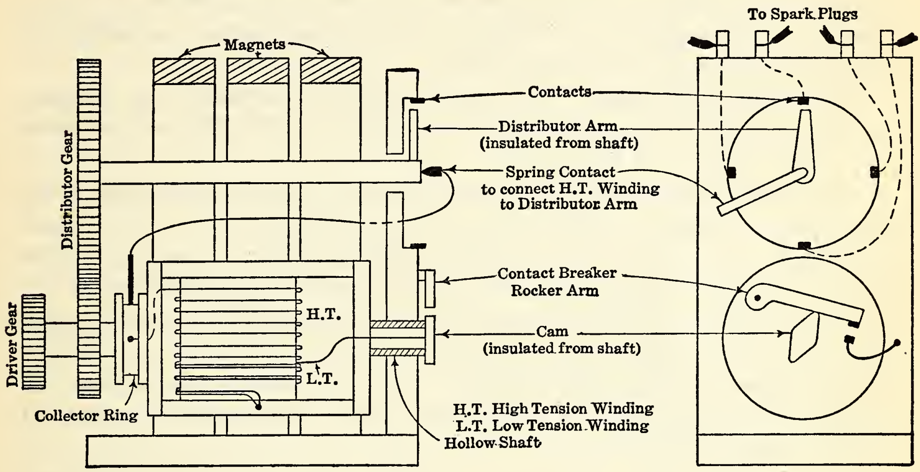


FIG. 10–9. Showing how the double-winding magneto is used for automobile ignition. The motion of the cam and rocker arm give one spark in the secondary when the primary circuit is opened, and this spark is distributed in turn to the various cylinders of the engine. The distributor is generally mounted integral with the magneto, as shown here.

is rotated at engine speed, since it gives two sparks per revolution of its armature; the distributor rotates at one-half engine speed.

The magneto described is known as a high-tension magneto, since it carries both a high- and a low-tension winding. If a magneto carries only a low-tension winding on its armature, it is called a low-tension magneto; it is used in connection with an external induction coil.

Charging the Battery. In order that the storage battery may remain more or less charged at all times, a generator for the purpose is usually provided. As this generator is driven from the engine at varying speeds, it may be of the type described in section 63, page 286, furnishing more or less constant current at all speeds. The generator is connected in parallel

with the battery through a cutout switch; the battery thus maintains the voltage of the generator fairly constant. Simple generators with automatic voltage regulators, similar to those described in section 19 above, may also be used. As the generator current tends to increase, the regulator inserts resistance in the field circuit, and vice versa.

The cutout switch was described in section 63, page 287; its function is to open the circuit whenever the voltage of the generator drops below that of the battery, so that the latter begins to discharge into the generator. While the generator is furnishing current, the cutout switch is held closed.

It is important, with all types of generator used on automobiles, that the generator never become disconnected from the battery. Should this happen, the voltage of the generator may rise to as much as 50 volts, burning out the lamps if connected, its own field coils, and probably the potential winding of the cutout switch.

21. Electric Welding. The electric arc can be utilized very successfully for making welds and for the deposition of metal, the process being used with metals and their alloys.

If a block of iron and a thin pencil of iron are connected to the two sides of a source of voltage, either direct or alternating current, and the pencil of iron touched to the block and then slightly withdrawn, an arc will be formed. If this arc is maintained, a spot on the block and the end of the pencil are heated to fusion temperature, and the metal of the pencil vaporizes, is carried across, and condenses on the block. The process is thus one of deposition of metal; a skillful operator is able by this method to deposit metal even overhead, filling up cracks and adding strengthening metal wherever desired.

For making welds or filling up cracks, the edges to be joined are cleaned and beveled, and metal is deposited to fill up the space. The pencil or rod of iron varies from about one-twelfth inch to one-eighth inch in diameter, depending upon the work; the rod is held in a suitable holder and is generally spoken of as the electrode. About 75 volts are needed to cut through scale and strike the arc, but once the arc is set up, an experienced welder is able to maintain the arc with 15 to 35 volts across the arc and uses a current of 50 to 175 amperes, both voltage and current values depending upon the nature of the work. The metal deposited by this process can readily be machined and has reasonable tensile strength.

The resistance of an electric arc varies inversely as the current it is taking; the greater the current the lower the resistance. This property results in an arc becoming fatter and drawing more and more current, if allowed to do so, as its resistance drops, resulting finally in a short circuit. It becomes necessary therefore to limit the current taken by an arc; this is done in one of two ways.

If welding is to be done, say, from a 110-volt constant-potential supply,

a certain amount of dead resistance, called ballast resistance, is put in series with the arc. Now, if the arc tends to draw more current, the drop in voltage across the ballast increases and cuts down the voltage across the arc, thus limiting the current flow. The use of dead resistance in this fashion is, however, not economical.

Most welding is done with special generators such as described in Chapter VI, page 290. These generators are usually mounted in sets, being driven by a gasoline engine or an electric motor. Although welding can be done with either direct or alternating currents, the special d-c welding generators, driven by electric motors where electricity is available, are usually used, as a better weld is obtained.

Electric welding is widely used to build up frames and parts of all kinds of machinery; to weld seams in boiler shells, tanks, ships' plates, piping, etc.; to fill cracks; replace corroded and worn-away metal; to repair broken parts of machines, etc. Recently it has frequently replaced riveting in structural construction. For many of the above usages, automatic welding machines have been developed.

With cast iron, electric welds are not reliable. However, if the material around the break is studded with steel plugs and considerable metal deposited over the break and studs, very satisfactory repairs may be effected.

Electric welding has materially changed the methods of fabrication of iron and steel machine parts and structures. In fact many of the advances in different fields, as for example in high-pressure steam-turbine practice, are partly due to the stronger construction made possible by electric welding. Desired amounts of carbon, vanadium, manganese, etc., may be introduced into a weld by coating the electrode with the proper salts. The special heat treatment that must often accompany welding is usually done electrically; in fact, it often cannot be done satisfactorily in any other way.

It is to be noted that an electric arc gives out a great deal of ultraviolet light, making it imperative that the eyes and every part of the body of the operator, and other persons in the vicinity, be protected. Exposure of the eyes results in severe pain and at least temporary blindness, and exposure of the skin results in severe burning, similar to sunburn. The skin is protected by ordinary clothing, gloves, and hoods or masks; sufficient protection for the eyes is provided by the use of suitable colored-glass windows set into a hood or mask, which must be used.

CHAPTER XI

PRINCIPLES OF ELECTRONICS

1. Free Electrons in Metals. It has been pointed out in Chapter I that there are many free electrons in metals, estimated at 10^{23} per cubic centimeter for copper, which can move with little constraint through the metal. The slow coordinated motion of this large number of charged particles under the influence of an electric field constitutes the electric current. This cloud of electrons has often been compared to a gas or vapor and it does act like a gas in many respects.

The number of free electrons and degree of "freedom" will be different in different metals; we might say by analogy that the gas pressure would be different. While electrons do not normally leave a metal to pass into surrounding space, they will pass from one metal surface to another in contact. If two dissimilar metals are joined, electrons will pass back and forth across the boundary, but there will be a greater diffusion of charge, or electrons, across the boundary in one direction than in the other. This net transfer of charge across the boundary results in a difference of potential at the boundary called the "contact difference of potential." It is usually of very small value, but must at times be considered in very accurate experiments.

If the dissimilar metals are bent to form a loop and a second junction made, no current will flow, the voltages at the two junctions being equal and opposite. If one of the junctions is made hotter than the other, the voltage across the hot junction will increase and be greater than that across the cold junction, and some current will flow. This combination of a hot and a cold junction of dissimilar metals is known as a *thermocouple*. While not a useful source of voltage, it is often used to measure temperature by measuring the resultant electric current.

2. Emission of Electrons. It has been shown that electrons are not free to leave the boundaries of metals, or other solids in contact with the metal. It is evident that as soon as an electron leaves the metal and passes into space, the metal in the vicinity of the emitted electron would be positively charged, so that the electron is drawn back into the metal. While the electrons are relatively free, the forces of attraction are usually great enough to hold the electron within the metal. This must be true, otherwise metals would lose electrons and become positively charged.

Experiment shows that metal objects normally are not charged but neutral.

Under certain conditions, however, the forces of attraction, tending to bring an electron back into the metal after it has emerged, may be overcome; an emission of electrons is thus possible.

3. Electron Volt. In measuring work quantities in electronics it is convenient to use as a unit the *electron volt.* It should be remembered that the original definition of the volt was that two points have a difference of potential of one volt if unit work is done in moving unit charge between these points. The *electron volt* is the energy that would be acquired by an electron if it were allowed to move freely between two points having a difference of potential of one volt.

Sometimes the energy of a moving particle is indicated by stating the potential through which an electron would have to move freely from rest to acquire the same energy. In this case energy is measured by a difference of potential in *volts.* This quantity is of course always numerically equal to the energy of the particle in electron volts.

The velocity that an electron acquires by moving in an electric field is simply related to the voltage difference through which it "falls." If an electron is situated at a point having a potential of $\overline{V}$ statvolts (1 statvolt $=300$ volts) *below,* or negative with respect to, a second point, the electron would have a potential energy of $\overline{V}\overline{e}$ ergs, with respect to the second point, where $\overline{e}$ is the electronic charge in statcoulombs (4.774×10^{-10}). This is the energy it would take to move the electron from the second to the first point. If the electron were released at the first point, it would move to the second or more positive point. Since it is assumed that the electron moves freely without collision, and there is no friction, the potential energy $\overline{V}\overline{e}$ would be transferred completely to kinetic energy. If the electron were to strike an electrode at the second point, the kinetic energy would be given to the electrode and appear as heat.

The velocity which an electron will have after it has fallen from rest through a given voltage may be calculated from the relation

$$\overline{V}\overline{e} = W \text{ (ergs)} = \tfrac{1}{2}mv^2 \tag{1}$$

where m is the mass of the electron in grams and v is the velocity in centimeters per second.

It should be noted that it is the voltage through which the electron moves that determines its velocity, not the distance. The product of force per unit charge and distance, i.e., work per unit charge, has already entered into the definition of the volt; it need only be multiplied by the charge to give the energy.

When an electron falls from rest through a potential of one volt, it acquires a kinetic energy of one electron volt, or, from Eq. (1), 1.59×10^{-12}

erg, and is moving at a velocity of 5.94×10^7 cm per sec. It can thus be seen that the electron volt is an extremely small unit of energy, and that the electron is a very light mobile particle; this velocity is 368 miles per second.

4. Mass and Energy. It is recognized by modern physicists that mass and energy under certain conditions are interchangeable. When the velocity of particles becomes extremely high, approaching the velocity of light, a portion of the energy of the particle goes into increasing the mass; hence as velocity greatly increases, the mass increases. The mass at any velocity is given by

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{2}$$

where m_0 is the "rest mass" of the particle, v is its velocity in centimeters per second, and c the velocity of light, 3×10^{10} cm per sec (approx.). Where mass change is appreciable, Eq. (1) cannot therefore be used; but in practically all electronic devices the velocities encountered are such that this mass change may be neglected.

5. Surface Work. To withdraw an electron from a metal against the attractive forces holding it, to a point far enough from the metal that the electron would not be drawn back, would require a certain amount of work. It has been shown that this work is all done in passing through an extremely small distance just as the electron leaves the metal, or passes through the surface of the metal. This work is called the "surface work"; for the usual materials it varies from about 1 to 6 electron volts.

There are four methods of overcoming surface work by which electrons may be removed from matter and made to pass across from a solid into the surrounding space. They are discussed in the following sections.

6. Removal of Electrons by Electric Fields. The application of an intense electric field in the proper direction will draw electrons out of a metal surface. The theory that an electric field ends abruptly at a metallic surface, and so does not extend inside the surface, is based upon the idea that the surface is a mathematical sheet, as smooth when highly magnified as it appears to the eye. However, when magnified to the proportions of electrons and atoms, this surface is highly irregular, with varying density, similar to the boundary of a cloud; there is no sharp demarcation from air to metal.

The field is thus able to reach into the metal to some extent, the depth of penetration depending upon the intensity of the field, and attract some electrons in the uneven outer fringes along the boundary. Since electron currents of only a few microamperes are obtained even with fields of very high intensity, the method finds little application in electrical engineering.

7. Removal of Electrons by Bombardment; Secondary Emission. A second method of causing electrons to pass from a solid, or to be emitted, is to knock or "splash" them out by bombarding the solid with high-speed particles. The bombarding particles are often electrons accelerated to high speed by an electric field. One electron, if moving fast enough, may knock out several secondary electrons from the solid. It is evident that the bombarding particle must have sufficient velocity, and therefore energy, to do the surface work required to remove the electrons from the solid. This process of removing electrons is called *secondary emission.* In general it is not a useful method, and may, in fact, greatly hinder the operation of a device, so that often it must be guarded against.

8. Removal of Electrons by Heating; Thermionic Emission. One of the most useful methods of producing the emission of electrons is by heating metals. The effect of heating a metal is to increase the thermal agitation of the particles. As a result, a few of the free electrons may, if the metal is sufficiently heated, acquire a component of velocity normal to the surface sufficient to perform the surface work and allow the electron to be emitted. The process may be regarded as similar to an evaporation or "boiling off" of the electrons. It is evident that the electrons, since they are the light, mobile particles, would "boil off" first, rather than the heavy remainder of the atoms.

There are only a few practical pure metal emitters of electrons, most metals melting before the temperature is high enough for practical emission. Tungsten, with a very high melting point (3543 K) operated at about 2500 K,* is one of the pure metals that can be used.

Certain combinations of materials are found to emit electrons at lower temperatures than tungsten or other pure metal emitters. There are two general types of composite emitters, the *thoriated tungsten* and the *oxide coated.* The thoriated tungsten emitter is prepared by mixing a very small amount of thoria or thorium oxide and some carbon with the tungsten. After shaping, the emitter is carefully heat-treated to form what is believed to be a monoatomic layer of thorium on the tungsten. This layer of thorium is stable even if heated somewhat above the melting point of pure thorium (2120 K). Thoriated tungsten emitters are usually operated at about 2000 K and give copious emission. Operation at 2250 K evaporates the thorium layer.

In the oxide-coated type of composite emitter, a support of tungsten or certain alloys, such as Konel metal (an alloy of cobalt, nickel, iron, and titanium), is coated with the oxides of the calcium, barium, strontium group. The coated type of emitter gives copious emission at a dull-red

* The Kelvin, or absolute, temperature scale has the same size degree as the Centigrade scale, 1/100 of the temperature difference between melting ice and boiling water, but starts at absolute zero, 273 degrees below the Centigrade scale.

heat and is often operated at a temperature as low as 1000 K. It is, however, a rather expensive type of emitter, and this offsets to some extent the saving in heating the emitter.

The mixtures and coatings in composite emitters must be very carefully prepared, since even a trace of many other materials will reduce the emission to a very small amount. Even the pure materials must be carefully compounded and the heat treatment may be quite involved.

The mechanism of emission from these composite surfaces is not fully understood; it is usually assumed that the added materials form monoatomic layers on the surface of the tungsten and the emission takes place from this layer.

9. Thermionic Emission as a Function of Temperature. The emission of electrons from pure metals has been shown by Richardson and Dushman to be given by

$$I = AT^2\epsilon^{-\frac{b_0}{T}} \text{ amperes per sq cm} \tag{3}$$

where I, the current in amperes per square centimeter of emitter surface, is a measure of the electrons per second, T is the temperature in degrees Kelvin, A and b_0 are constants, depending upon the emitting material. Values of A and b_0 for common emitters are given in Table XI of the Appendix. Constant A has the value 60.2 for all pure metals, but the equation gives also the approximate currents for composite surfaces if constant A is given empirical values.

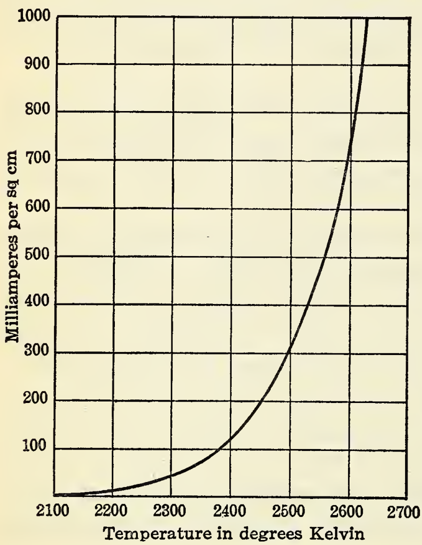


FIG. 11–1. Theoretical emission from a pure tungsten emitter.

The equation has been plotted for a pure, clean tungsten emitter in Fig. 11–1. The emission rises extremely rapidly with increasing temperature; at 2500 K a change of 1 per cent in temperature produces a 23 per cent change in emission.

Most electronic devices do not make use of this property, the temperature being adjusted and held constant at a value high enough to give a copious emission, but to insure a relatively long life to the emitter.

10. Removal of Electrons by Light; Photoelectric Emission. Another useful method of causing the emission of electrons is by allowing light to fall on a metal. Certain metals when illuminated under certain conditions

will emit electrons. The exact mechanism of photoelectric emission is as yet unknown, but experiment has established many facts about the process, and a combination wave and corpuscular theory is usually used to explain it.

It is assumed that light is made up of "units" of some sort, called *photons*, each photon being a group of waves, or wave train. It is also assumed that each photon has associated with it an indivisible block of energy or *quantum*, and that this energy is proportional to the frequency of the light.

It may then be written that the energy of a photon or quantum is $h\nu$ ergs, where h is a constant, known as Planck's constant (6.55×10^{-27} erg-second), and ν is the frequency of the light in cycles per second.

These explanations are based on the observed facts that the velocity of the emitted electrons varies directly with the frequency but is not dependent upon the light intensity. An increase in light intensity of a beam which causes emission is observed to increase the number of electrons emitted, but does not affect their velocity. A simple wave theory will not explain these facts, so that a composite theory is now used.

Assuming that the electron in the metal completely absorbed the whole quantum of energy of the photon, and that the electron performed the surface work and escaped as a *photoelectron*, Einstein postulated the highest possible velocity of the photoelectron to be given by

$$\tfrac{1}{2}mv^2 = h\nu - p \tag{4}$$

where m is the mass of the electron, v is its velocity as it emerges from the metal, and p is the surface work of the metal.

The equation assumes that the electron had no energy of its own to help perform the surface work and that it lost no energy in collision after it absorbed the quantum.

Emission must cease if the energy per quantum is less than the surface work. It is thus evident from this equation that there is a lower limit to the frequency of light that will cause the electrons to be emitted for a given material with a given surface work. This lower limit will be given by Eq. (4); calling this frequency ν_0,

$$h\nu_0 = p \quad \text{or} \quad \nu_0 = \frac{p}{h} \tag{5}$$

This value of frequency ν_0 is known as the *threshold frequency* and corresponds to the *long wavelength limit.*

The wavelength and frequency of light are related by the usual relations of wave motion; the wavelength, multiplied by frequency, gives the velocity. Thus $\lambda\nu = 3 \times 10^{10}$ cm per sec where λ is wavelength in centimeters. If the wavelength is given in Ångström units, represented by the symbol Å (1 Å = 10^{-8} cm), the expression becomes $\lambda'\nu = 3 \times 10^{18}$.

These equations and the threshold frequency have both been checked by careful experiment and found to be true.

The metals used for photoemission are those having low values of surface work (in contrast to thermionic emission we need not consider melting point for this application), for when the surface work is high, the threshold frequency is outside the visible spectrum. The pure metals used are usually lithium, sodium, potassium, rubidium, cesium, calcium, strontium, and barium. It will be noticed that these elements are the ones having one or two electrons in an outer shell, and there is reason to believe that it is these electrons that are emitted. The threshold frequency will be dif-

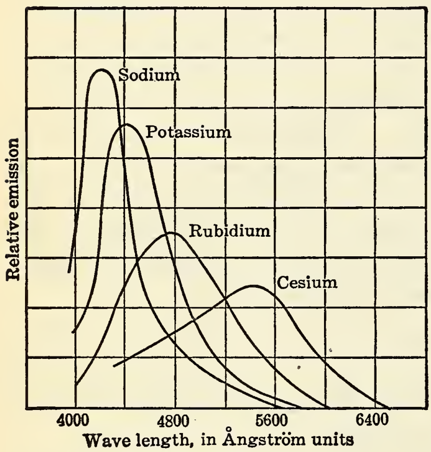


FIG. 11–2.

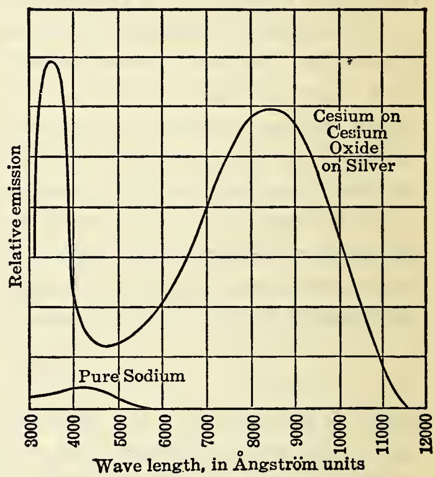


FIG. 11–3.

FIG. 11–2. Photoelectric response of the alkali metals.

FIG. 11–3. Photoelectric response of a composite surface. The maximum response is much greater than for a pure metal and occurs at a different wave length, as may be seen by comparison with the curve for pure sodium.

ferent for each of the materials and the relative number of emitted electrons is also different. This is shown in Fig. 11–2.

As in the case of thermionic emitters, it is possible to change greatly the characteristics of photoemitters by forming composite surfaces. With a composite surface, the threshold frequency may be lower and in many cases below the visible spectrum. There will also be certain irregularities in the response curve, as shown in Fig. 11–3.

11. High-vacuum Thermionic Electronic Devices. Some electronic devices are evacuated as highly as commercially feasible, while in others a small amount of vapor or gas is placed.

The necessity for a thermionic emitter to operate in a vacuum, or at least in the absence of a substance that would react chemically with it, is

evident. The emitter is hot, usually very hot, and even a trace of certain substances would react and destroy it or greatly change its emission. In addition, the presence of a small amount of gas will completely change the characteristics of some of the devices, as is described later.

12. High-vacuum Diode or Two-electrode Tube. The simplest electronic device or *tube*, and one of the earliest discovered, is the *diode*. It consists essentially of an emitter to release electrons and a *collector* or *plate*, to receive them. Since it is to be a high-vacuum device, it must be enclosed in an evacuated glass or metal envelope.

The emitter must be heated to produce emission; this can be accom-

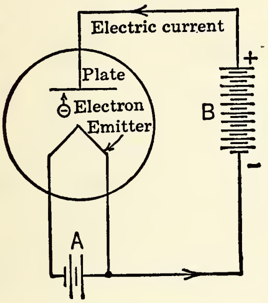


Fig. 11–4.

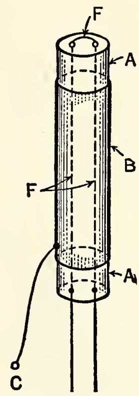


Fig. 11–5.

Fig. 11–4. A simple high-vacuum diode. Note that the direction of current is *opposite* to the direction of electron flow.

Fig. 11–5. The modern vacuum tube often uses an equipotential cathode or emitter. This illustration suggests one construction; the emitter, *B*, is wrapped about the ceramic cylinder, *A*, and is heated by conduction through the cylinder from the heater filament *FF*.

plished by arranging the emitter in a loop or *filament*, and passing an electric current through it. Such an arrangement is suggested by Fig. 11–4. A second arrangement widely used in modern devices is made by wrapping a thin sheet of the emitting material about a small cylinder of ceramic material. The heat-producing filament is threaded through holes in the cylinder and is not in electrical contact with the emitter. The emitter is thus heated indirectly by heat conducted from the filament through the cylinder. This construction is shown in Fig. 11–5.

The plate is usually made in the form of a cylinder or tube surrounding the emitter. When connected in an electrical circuit the plate is made positive with respect to the emitter so as to attract the electrons emitted; hence it is often called the *anode*. This makes the emitter negative with

respect to the plate, so that the emitter is often called the *cathode*. When a battery is used to produce these polarities, as in Fig. 11–4, it is usually designated by the letter *B*.

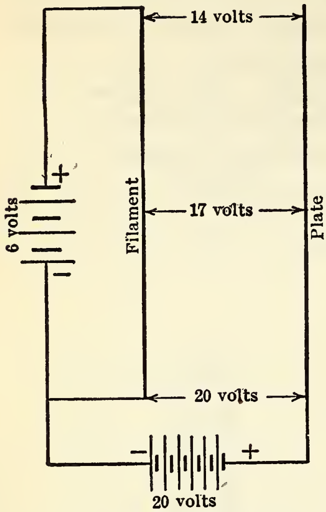


Fig. 11–6. In a diode with filamentary cathode, the potential between anode and cathode is different at different parts of the cathode. This indicates why the emitter shown in Fig. 11–5 is called "equipotential."

A diode with a filamentary cathode is represented in Fig. 11–4. The filament is heated by current from battery *A* to a temperature at which emission takes place. The positively charged plate, made positive by battery *B*, will immediately attract the emitted electrons. These electrons will then make the passage from cathode to plate and return through the battery and connected circuits to the cathode. This passage of electrons constitutes an electric current, flowing, conventionally, in the opposite direction, as shown by the arrows in Fig. 11–4. This current will not flow if emission were to stop owing to a decrease in temperature of the emitter or if the plate were not made positive to draw the electrons to the plate. *This current will flow only when the plate is positive.*

13. Space Charge. If it is assumed that the diode of Fig. 11–6 is operating with its plate positive, electrons will be passing rapidly from cathode to plate. It is to be observed that, while the free electrons move slowly inside the metal parts of the circuit to form the electric current, being hindered in their passage by constant collision, yet *in their passage from cathode to plate they move very rapidly and in smaller number*, thus providing the same rate of charge transfer and hence the same current as in the connected metallic parts of the circuit.

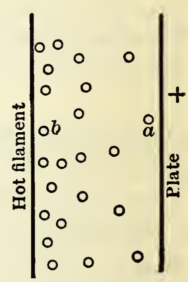


Fig. 11–7. Elementary representation of the space charge in a vacuum tube.

As the electrons move from cathode to plate, we could look upon them as a cloud or vapor in motion, constantly emerging from the cathode and being absorbed into the plate. This group of electrons in flight is called the *space charge*. Since all electrons are negatively charged and all will repel each other, the electrons near the plate, as at *a* in Fig. 11–7, will be urged on by those behind near the cathode, while those near the cathode, as at *b*, will be retarded by those ahead near the plate. The electrons near the plate thus act as a screen and tend to neutralize

the effect of the positive charge of the plate on the electrons near the cathode.

The space charge will cause the electrons near the plate to have a higher velocity than those near the cathode and result in a greater space-charge density near the cathode. Once an electron starts across to the plate it will reach the plate if the voltage between plate and cathode remains constant. Also the electrons will adjust themselves so that the product of velocity and number, and hence the current, will be the same at all points in the space between filament and plate. Figure 11–7 suggests the distribution of electrons at an instant in their passage.

Assuming that the temperature of the emitter is high enough to cause the emission of an ample number of electrons, in the equilibrium condition the electrons of the space charge will just neutralize the positive charge of the plate up to the cathode. The emitted electrons may be looked upon as a reservoir from which the plate charges attract electrons, and additional electrons will be drawn from this reservoir until the equilibrium condition is realized and no more are attracted. Since the space charge is in motion toward the plate, electrons will constantly enter the plate and disappear from the space charge; electrons must then be drawn from the reservoir of emitted electrons at exactly the same rate to maintain equilibrium. If the plate charge is raised, by raising the plate voltage, a greater space charge must build up, which requires a greater number of electrons in the space. The greater voltage, through which the electron moves in passing from cathode to plate, requires a higher velocity at all points. There are then more electrons at higher speed, representing a greater current, passing to the plate.

Thus for a given positive charge on the plate corresponding to a given plate voltage, the passage of a definite number of electrons from emitter to space charge and from the latter to the plate is required, so that the current is fixed. An increase in plate voltage would increase the plate current.

It is to be observed that the plate current might have been limited by the number of electrons emitted as controlled by the temperature of the emitter; certainly no more electrons could enter the space charge than are emitted. Such a limit is known as a *temperature-limited current*, but is usually not effective with electronic devices; they are usually operated at such an emitter temperature that the plate voltage determines the current flowing. The current is then said to be *space-charge* limited.

14. Plate Currents of a Diode. The variation of plate current, I_p, with plate voltage, E_p, has been shown theoretically to be of the form

$$I_p = BE_p^{3/2} \tag{6}$$

where B is a constant. Many factors affect this formula, and practically it is only approximate.

In Fig. 11–8 is shown the variation of plate current as the plate voltage is varied, with the emitter operated at three different temperatures, as controlled by the filament current, I_f. Curve 3 shows the variation following approximately the law of Eq. (6), the emission being ample to supply all the necessary electrons. Curves 1 and 2 show the effect of operating at lower emitter temperatures; even though the voltage is increased, the

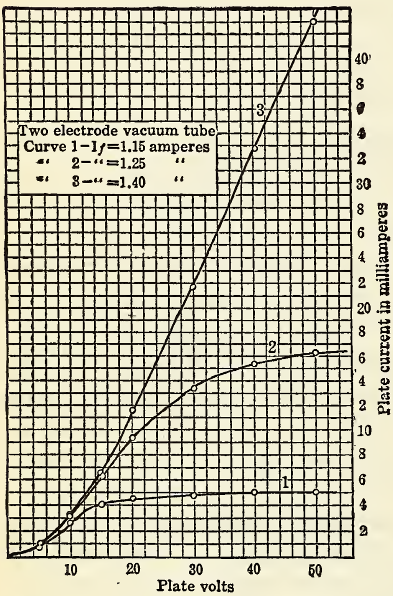


Fig. 11–8.

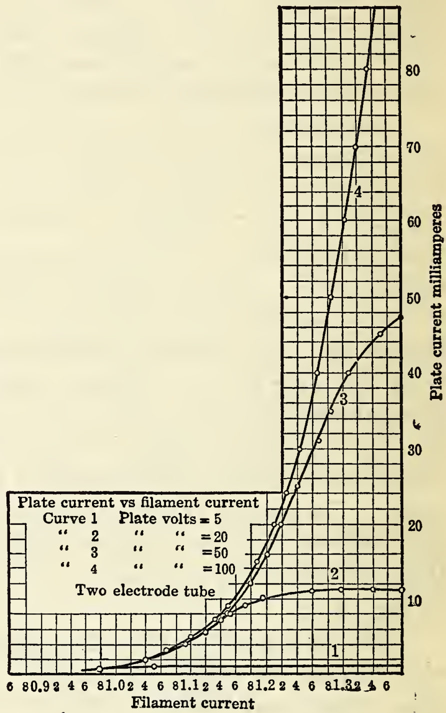


Fig. 11–9.

Fig. 11–8. The variation of the anode current of a small diode with the anode voltage, for three different cathode temperatures.

Fig. 11–9. The variation of the anode current of a small diode with the cathode temperature, as indicated by the filament current, for four different anode voltages.

current does not increase beyond a certain point, since all the emitted electrons are being drawn across to the plate. Such a condition is known as *temperature saturation* and the current is temperature-limited.

In Fig. 11–9 is shown the variation of plate current with filament current, the latter a measure of temperature. Curve 4 shows the variation following closely Eq. (3), the plate voltage being high enough to draw over all the emitted electrons. Curves 1, 2, and 3 show the effect of operating at lower plate voltages, the current being limited by space charge, and not

all the emitted electrons are drawn across. Such a condition is known as *space-charge saturation.*

Two such families of curves constitute the characteristics of any given diode and are used to determine the proper tube for a given application.

15. Diode Characteristics. The usual application of diodes is one in which the voltage applied to the plate controls the current flowing; the diode acts as a current throttling device, the plate voltage controlling the current. It is to be noticed that it is a uni-directional current-carrying device. Current will flow into the plate from the external circuit when the plate is positive; no current at all can flow when the plate is negative.

The maximum average current that can flow through the device is, as in all electrical devices, limited by its ability to dissipate the heat produced in its operation. Electrons strike the plate with an energy determined by the plate voltage, and heat the plate. In small tubes the plate is cooled almost entirely by radiation; in larger tubes the plate may be water-cooled. Tubes are rated in watts that the plate can dissipate, the product of the plate voltage which determines the energy of an electron impact, and the plate current which determines the number of electrons and hence the number of impacts.

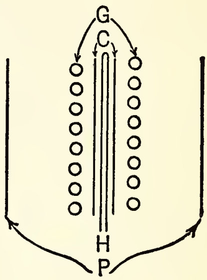


FIG. 11–10. Cross-section through a triode. The cathode, C, is heated indirectly by the heater H. The grid, G, is a loosely wound spiral. The plate is represented by P.

The maximum instantaneous current is limited by the emission of the filament and is always higher than the maximum average current.

16. High-vacuum Triode or Three-electrode Tube. This device and those derived from it are among the most important electronic devices that have been developed and are widely used. The triode has the usual two elements of the diode and adds a third element between the emitter and plate. This third element, called the *grid*, is usually in the form of a loosely wound wire spiral, as is shown in section in Fig. 11–10. It must be constructed so that it remains cool and does not emit electrons.

Since the grid is placed between emitter and plate, it is in the space charge; yet, by its open construction it does not greatly hinder the flow of electrons. However, its location in the space charge makes it possible to reinforce greatly or to neutralize the effect of space charge by giving the grid the proper charge. If the grid is made positive it neutralizes the space charge and thus causes more plate current to flow. If the grid is made negative it will reinforce the space charge and reduce the flow of current to the plate; in fact, if it is made sufficiently negative, the flow of current to the plate may be stopped altogether and we have the condition

known as *cutoff*. Usually the grid is made somewhat negative, but not enough to produce cutoff.

Because of its location right in the space charge, the grid is very effective in controlling the plate current; a change of one volt in the grid voltage may produce many times the change in plate current that a one-volt change in plate voltage would produce. Also, if the grid is negative, it will attract no electrons and no current will flow in the grid circuit. The control is thus effected with no expenditure of power.

17. Static Characteristics of a Triode. The plate current in a triode is a function of the temperature, plate voltage, and grid voltage. Almost invariably the temperature is held constant at a value high enough to

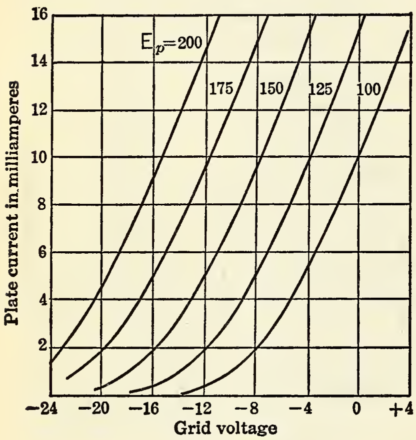


FIG. 11–11.

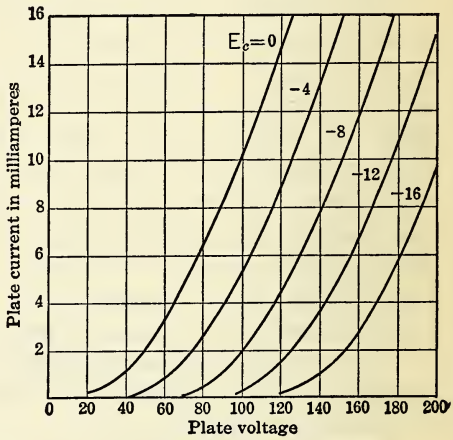


FIG. 11–12.

FIG. 11–11. Transfer characteristic curves of a typical triode. (Type 101F, Western Electric Co.)

FIG. 11–12. Plate characteristic curves of the same triode as in Fig. 11–11. (Type 101F, Western Electric Co.)

produce ample emission, and any variation due to temperature is not realized. The plate current may then be expressed as a function of plate and grid voltages alone. By applying different values of grid and plate voltages and measuring the resultant currents, curves, known as the *static characteristic* curves of a triode, are obtained.

The curves showing the relation of plate current to grid voltage, at a series of different plate voltages, are known as the *mutual* or *transfer* characteristics; a set for a typical triode is shown in Fig. 11–11. The curves showing the relation of plate current to the plate voltage for a series of different grid voltages are known as the *plate characteristics*; a set for the same triode is shown in Fig. 11–12. It is evident that these two sets of curves represent the same thing, the variation of plate current with grid and plate voltages, in two different ways.

18. Tube Factors of a Triode. Certain *tube constants*, or more appropriately *tube factors*, are usually defined for the triode. The *amplification factor*, represented by μ, is defined as the negative of the ratio of change in plate voltage to change in grid voltage required to hold the plate current constant. Thus, if the plate voltage were increased by an amount ΔE_p and at the same time the grid voltage were decreased by an amount ΔE_g, in such a way that the plate current does not change, then

$$\mu = -\frac{\Delta E_p}{\Delta E_g} \tag{7}$$

It is to be noted that ΔE_g is here a negative number, since it represents a decrease, so that the amplification factor is always positive.

The *plate resistance* is defined as the ratio of the change in plate voltage to the change in plate current at constant grid voltage. It is seen that this is not a simple resistance obtained by dividing voltage by current as in the case of metallic conductors.

The *transconductance* of the device is defined as the ratio of the change in plate current to the change in grid voltage at a constant plate voltage.

A more rigorous definition of these quantities may be made by considering

$$I_p = f(E_g, E_p)$$

Taking the complete derivative

$$dI_p = \frac{\partial I_p}{\partial E_g}\,dE_g + \frac{\partial I_p}{\partial E_p}\,dE_p$$

since the partial derivatives have the appearance of conductances, let us set

$$\frac{\partial I_p}{\partial E_g} = S_m \tag{8}$$

and call it the transconductance; and set

$$\frac{1}{\dfrac{\partial I_p}{\partial E_p}} = R_p \tag{9}$$

and call it the plate resistance. If we hold I_p constant, $dI_p = 0$, and we have

$$0 = S_m dE_g + \frac{1}{R_p}\,dE_p$$

$$\mu = -\frac{dE_p}{dE_g} = S_m R_p = \frac{\left|\dfrac{\partial I_p}{\partial E_g}\right|_{Ep=\text{const}}}{\left|\dfrac{\partial I_p}{\partial E_p}\right|_{Eg=\text{const}}} \tag{10}$$

defining μ in terms of S_m and R_p. It can be seen that the previous definitions are approximations of these quantities. Actually, since the relation of the various voltages and currents can be expressed only graphically, the derivatives must be replaced by the approximate ratio of increments as previously given.

From these definitions the tube factors may be determined from the static characteristic curves. The ratio of increments is seen to be given by the slopes of the various curves. It will also be evident that these tube factors or "constants" are not constant, since the slope of the curves is not constant. Curves showing the variation in these factors are shown in Fig. 11–13. Usually average values are used to describe the tube.

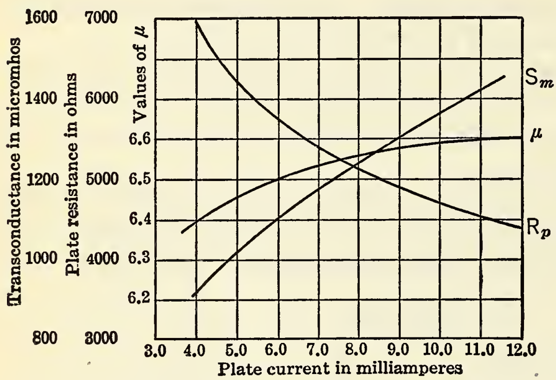


FIG. 11–13. Tube factors plotted to show variations with respect to the plate current. These factors are usually assumed constant for small variations in plate current. (Type 101F, Western Electric Co.)

19. Dynamic Characteristics of a Triode. The static characteristic curves of a triode do not represent its action when connected to a circuit.

One of the simplest circuits for a triode, an amplifier, is shown in Fig. 11–14. The battery C is used to maintain the grid negative and to produce variations in the grid voltage. The battery B maintains the plate positive and, as will be seen later, supplies the operating power to the device. The battery A supplies the power to heat the cathode and produce the emission. In actual circuits these batteries would be replaced probably with other sources of voltage, but the principle would be unchanged. The constant resistance R_L is the load on the device in which the output is absorbed. It might be any type of load; a resistor is used in this example.

This amplifier could be used to measure a very small voltage variation introduced into the grid in series with the constant grid biasing battery C. Let us assume the voltage variation was in such a direction as partly to neutralize the battery C and make the grid slightly less negative. This, in turn, would cause a greater plate current to flow and increase both the current and voltage of the load. Actually, as pointed out in the next paragraph, the voltage E_p will also slightly decrease; the plate or load current and the load voltage will increase even with this plate voltage decrease. The increase in the voltage across the load E_L may be many

times the small voltage variation introduced into the grid circuit; in fact, by using several amplifiers in tandem, taking the output of one, and feeding it into a following amplifier, this increase may be made thousands of times as large.

If it is assumed that the voltages of the various batteries (of Fig. 11–14) are known and are constant, and that the tube used is one of the type for which characteristics are given in Figs. 11–11 to 11–13, the device will then be operating at some point on the static characteristic curves. This point, however, cannot be found directly since the plate current flowing will produce a voltage drop in the load resistance and the plate voltage will be less than the voltage of battery B. Since the plate current depends upon the plate voltage, this point could be found only by trying values of plate current until one was found that would give the correct voltage drop and plate voltage. We know, however, that

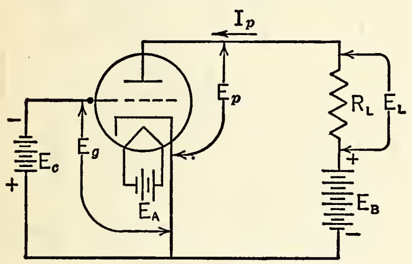


FIG. 11–14.

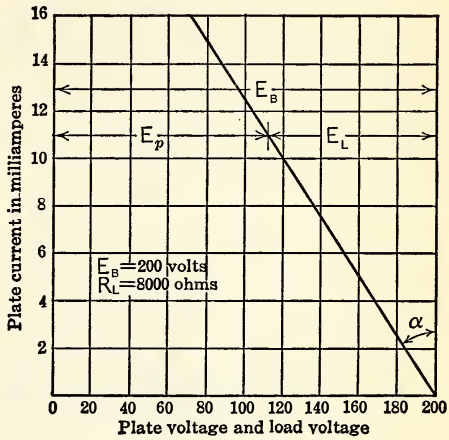


FIG. 11–15.

FIG. 11–14. A simple amplifier circuit using a triode.

FIG. 11–15. The load line of an amplifier. This variation is necessary to satisfy Ohm's law at the load.

$$E_p = E_B - I_pR_L = E_B - E_L \tag{11}$$

and that E_L is directly proportional to I_p, since R_L is constant. We can thus draw a curve showing the relation of I_p to E_L and hence to E_p, as is shown in Fig. 11–15, using the values of E_B and R_L shown in the figure. The straight line shown is known as the *load line,* and it is evident that its slope can be determined from R_L; the tangent of the angle α is R_L.

An inspection of Fig. 11–15 shows that it has the same coordinates as the plate characteristic, Fig. 11–12, and the two curves may be superimposed. Such a combination for the tube and load being used, is shown in Fig. 11–16. The point at which the tube is operating can be located on

the superimposed curve of Fig. 11–16, since the grid voltage is known; the point is called the *quiescent point* and is designated as Q. Since every point on the plate characteristic curves corresponds to some point on the mutual characteristic curves, this point may be transferred back to the mutual characteristic curves of Fig. 11–11, as shown at Q in Fig. 11–17.

If now a change of 4 volts is introduced into the grid circuit, in such a manner as to change the grid voltage from the value at Q of -8 volts to a value of -4 volts, the operating point will change. Since variations in the plate circuit will follow the load line of Figs. 11–15 and 11–16, as long as E_B and R_L remain fixed, the new operating point must lie at A in Fig. 11–16,

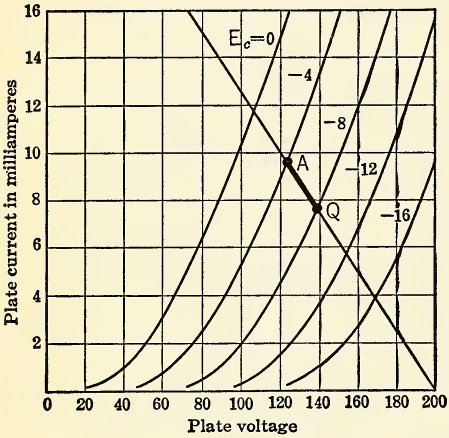


Fig. 11–16.

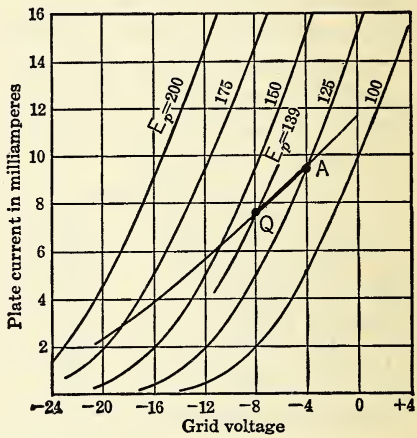


Fig. 11–17.

Fig. 11–16. The load line superimposed upon the plate characteristics. If the voltage E_c of Fig. 11–14 were −8 volts, the point Q would be located as shown.

Fig. 11–17. The load line has been transferred to the mutual characteristic curves. In this case it is known as the *dynamic characteristic* curve.

determined by the intersection of the load line and the curve for a grid voltage of -4 volts.

By remembering that E_L is represented by the horizontal distance from the load line to the voltage corresponding to E_B (see Fig. 11–15), it is evident that the change of 4 volts in the grid circuit has resulted in a change of 14 volts in the value of E_L, when the operating point moved from Q to A. The *amplification* of the circuit and tube is then $14/4 = 3.5$, since the voltage change has been increased in that ratio.

The point A of Fig. 11–16 may also be transferred to the mutual characteristic curves of Fig. 11–11 and is shown at A in Fig. 11–17. By considering other points along the load line of Figs. 11–15 and 11–16, corresponding points may be determined on the mutual characteristic curves of Figs. 11–11 and 11–17. The curve found by joining a series of such points,

as shown in Fig. 11–17, describes the operating path or *dynamic characteristic* of the tube and circuit.

All amplifier behavior may be analyzed by means of such curves. Complications are added when the load R_L is a variable resistance such as a tungsten lamp, or contains inductance or capacitance. An analysis in that case is beyond the scope of this book.

It is to be noted that the amplification in the case considered is not as large as the amplification factor, indicated as about 6.56 for the value of plate current used in this example, as may be seen from Fig. 11–13. The latter is a theoretical upper limit to the amplification which can be realized only if R_L were infinite. This is an impractical value, although in practice, R_L is often made high to obtain high amplification. The objection to making R_L too high is the large value of voltage, E_B, required to overcome the drop across R_L and still provide the plate voltage, E_p.

It will also be noted that the amplification is not as high as it would be if the voltage E_p did not fall off as I_p increases. This has led to the design of tubes in which the plate current is more nearly independent of changes in plate voltage.

20. High-vacuum Tetrodes and Pentodes. By placing screens, usually in the form of grids or loosely wound wire spirals, between the grid and plate and maintaining them at constant voltages, it is possible to minimize the effect of the plate voltage on the plate current. Also by controlling more closely the passage of the electrons by such screens, other desirable characteristics may be obtained. This has led to the development of four- and five-electrode tubes. The general principle of operation is the same as for a triode, but the shape of the characteristic curves is quite different. The same analysis could be made for a pentode as was made for a triode and the load line and the dynamic characteristic drawn. Tetrodes are now not widely used.

The static characteristic curves for a typical pentode are shown in Fig. 11–18. It will be noticed that the plate current is largely independent of plate voltage. Many more applications of these devices are described in connection with their use with alternating currents.

21. Conduction of Electricity through Gases. The conduction of electricity is, as we have seen from an earlier chapter, the transfer of charge, and any study of conduction becomes a study of the mechanism of charge transfer. In the case of a solid conductor the free electrons transfer the charge and, being present in large numbers, their relatively slow motion results in the transfer of even large charges and thus the passage of large currents.

In the case of a gas the mechanism is entirely different. There are no free electrons, the molecules of gas are widely separated, and the electrons are all closely held by their individual atoms. The atoms are all

neutral and no current carriers exist.* If an electric field is impressed, no charge would move and no current would flow.

22. Excitation and Ionization. If there were electrons or positively charged particles present among the neutral molecules of a gas and an electric field were impressed, these particles would move under the influence of that field. If the charged particles could travel under the influence of the field and not lose energy by collision in the meantime, they would acquire a velocity high enough to represent considerable energy. If they can travel uninterruptedly far enough they will acquire enough energy to disturb or disrupt any neutral gas atom they finally impinge upon.

The disturbance of the atom takes one of two forms according to the energy with which the particle hits the atom. In the first form one of the

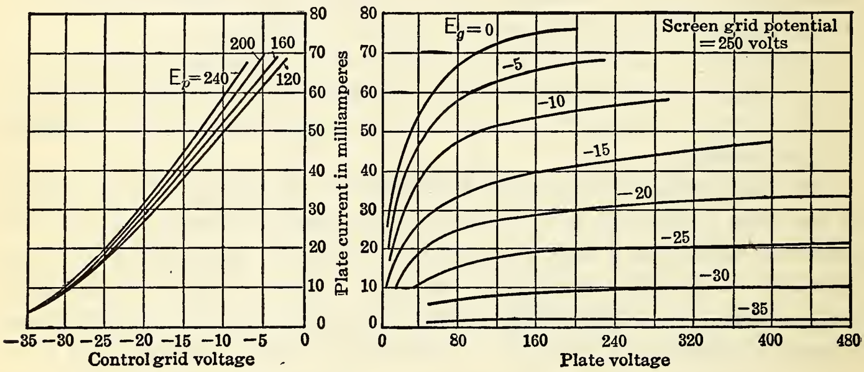


FIG. 11–18. Static characteristic curves for a typical pentode. These are for a type RCA–59 tube. (RCA Manufacturing Co.)

electrons is knocked or moved from one shell to another *within* the atom, the transferred electron acquiring some energy from the impinging particle. The atom is left neutral, and usually the electron soon drops back to its stable shell and the energy it acquired is returned as radiation. This radiation may be of visible light; in many cases it is of invisible light. This process is called *excitation* or *resonance* and requires impinging electrons having energies of 5 to 20 electron volts, depending upon the kind of gas atoms.

A common example of the radiation coming from excited atoms is in the gas discharge tube, such as is used in neon signs. The exact frequency of the radiation, distinguishing its color, depends upon the kind of gas and the particular shells between which the electron is transferred.

If, however, the impinging particle has enough energy and the collision with an atom is right, an electron may be knocked completely out of the

* This statement is not quite true as will be shown in a later paragraph.

atom. This supplies another electron and leaves the remainder of the atom as a positively charged particle. This process is called *ionization*, and the positive particle so formed is called a *positive ion* or simply an *ion*. Both these particles will start to move under the influence of the electric field toward the electrodes having the opposite sign to their own, and may in turn produce still further charged particles by collision with the atoms. Thus the process may be cumulative and lead to the formation of a great number of ions and free electrons.

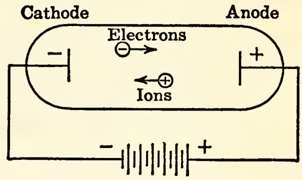


Fig. 11–19. In a gas-filled tube there will be both electrons and positive ions after ionization takes place.

Ionization may also be produced by bombardment with X-rays, ultra-violet light, and cosmic rays. There are always a few ionized particles in any volume of gas, probably due to the ever-present cosmic rays, and it is the presence of these free particles which starts ionization if the conditions are correct.

If a stream of electrons is passing through a gas, the pressure of the gas will have a great effect upon ionization. If the electrons are moving between the electrodes, as shown in Fig. 11–19, and the gas pressure is high enough, there will be a great many atoms and the electrons will collide with atoms before they acquire enough energy to ionize. If the voltage between the electrodes were raised, more energy would be given to the electrons in traveling the same distance and ionization might take place.

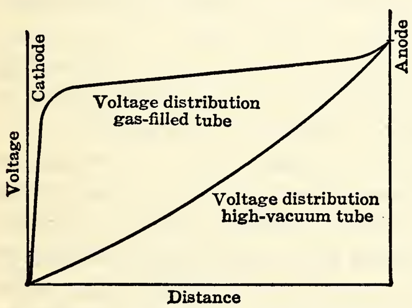


Fig. 11–20. The voltage distribution in a gas-filled tube after ionization takes place. For comparison, the voltage distribution in a high-vacuum tube is also shown.

If, however, the gas pressure were too low, there would be but few atoms, and the electrons could make the whole passage between electrodes without collision; they would acquire enough energy to ionize but would not strike any atoms because of the small number present. Evidently there is an optimum pressure for a given electrode spacing and voltage that will produce maximum ionization.

The conduction of electricity through gases is thus dependent upon the production of electrons and ions by ionization. It depends upon such things as gas pressure, electrode spacing and voltage, and does not obey Ohm's law or the usual circuit relations. In fact most of the ionization takes place near the electrodes, and the voltage distribution between electrodes is of the form shown in Fig. 11–20.

If electricity is conducted through a gas-filled device, such as a neon tube, the current will be carried by particles produced by ionization. The arc formed when a switch is opened, or the arc in an arc lamp, depends upon ionization to supply the current carriers. Ionization is always accompanied by excitation, many of the collisions producing excited atoms.

Typical values of the energy necessary for ionization are: helium 24.6, neon 21.5, argon 15.7, and mercury vapor 10.39 electron volts. The energy required for excitation is somewhat lower, having, for example, a minimum value of 4.86 for mercury vapor.

23. Deionization. The recombination of electrons and ions to form neutral atoms is called *deionization*; it takes place principally at the boundaries of the discharge. Ionization must produce carriers enough to carry the current and allow for a certain amount of deionization at the same time. If the voltage is removed from the electrodes of Fig. 11–19, complete deionization takes place in an extremely short time, usually an interval of ten to several hundred microseconds.

24. Gas-filled Diode. If a small amount of gas, often argon, or more commonly mercury vapor, is introduced into the diode previously described, its behavior is completely changed. If the cathode is heated, electrons will be emitted; and, if a voltage is applied from cathode to plate, making the plate positive, electrons will pass from cathode to plate. If this voltage is low, only a few electrons will pass and their passage will be impeded by collision with the gas particles. As the voltage is raised, however, a value is finally reached at which ionization starts. Once started the effect is cumulative, and many ions and electrons are very rapidly formed to provide many carriers. The device is then said to be *ignited* or *firing* and the current-carrying stream of particles is called an *arc.*

The behavior of the device might be represented by a curve such as Fig. 11–21. The voltage at which the curve bends sharply upward corresponds to that required to give the electrons sufficient energy to produce ionization; for most tubes using the usual gases this will be between 10 and 30 volts. The voltage across the tube will not rise above this value; if higher voltages are applied the current will increase and be limited by the resistance of the connected circuit. In Fig. 11–22 is shown a circuit including a gas-filled diode which is assumed to be of a type that ionizes when 20 volts are applied to the plate. The current that will flow will be

$$I_p = \frac{E_B - 20}{R} \tag{12}$$

and it is at once evident that voltages in excess of 20 volts should not be applied directly to the tube without some resistance in the plate circuit.

If the voltage is lowered the current will follow exactly the same curve

shown in Fig. 11–21; when the plate voltage drops below the ionizing value the current stops abruptly and the device is *extinguished.* Ignition and extinction occur for the same plate voltage.

The gas-filled diode therefore acts as a switch and not like a current-throttling device. It is, however, a uni-directional conducting device, carrying current only in the direction into the plate from the external circuit, when the plate is positive. It will not carry current in the opposite direction when the plate is negative.

Both ions and electrons, moving in their respective directions, tend to act as current carriers. But since the mass of the ions is so much greater than that of the electrons (mass of a mercury ion is about 400,000 times as

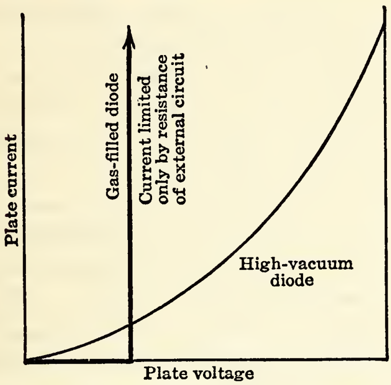


Fig. 11–21.

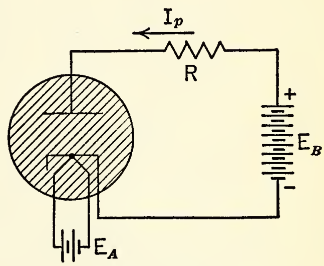


Fig. 11–22.

Fig. 11–21. The variation of current with voltage in a gas-filled diode. For comparison this is also shown for a high-vacuum diode. Once ionization takes place there are very many current carriers and the current can rise to very high values.

Fig. 11–22. The circuit used with a gas-filled diode. A gas-filled tube is indicated by this hatched symbol.

much as the mass of an electron), their acceleration is inappreciable. The electrons available consist not only of those emitted but also of those produced by ionization. The result is a very large number of charged particles. Higher currents may thus be carried by gas-filled devices having the same cathode as high-vacuum devices, and the voltage across the device will be much less. The space charge is completely neutralized by the presence of positive ions, and the space-charge effect does not exist in the gas-filled tube. In general, gas-filled devices are very much more efficient than high-vacuum devices.

25. Gas-filled Triode; Thyratron. The gas-filled triode or *thyratron* is similar in construction to the high-vacuum triode, having the usual cathode, grid, and plate. The plate voltages at which ignition and extinction occur in the thyratron may be vastly different.

If the cathode of an un-ionized gas-filled triode is heated, electrons are emitted, but whether these are drawn to the plate depends upon the voltage of the plate *and the voltage of the grid.*

The effect of the positively charged plate is to cause the passage of electrons, but, if the grid is made sufficiently negative, as at cutoff in a high-vacuum device, this passage may be prevented even if the plate is quite highly positive. As has been seen, the higher the positive voltage on the plate the higher must be the negative voltage on the grid to produce cutoff. This is shown graphically in Fig. 11–23, where the negative grid voltage just necessary to prevent the flow of electrons is plotted against the corresponding positive plate voltage.

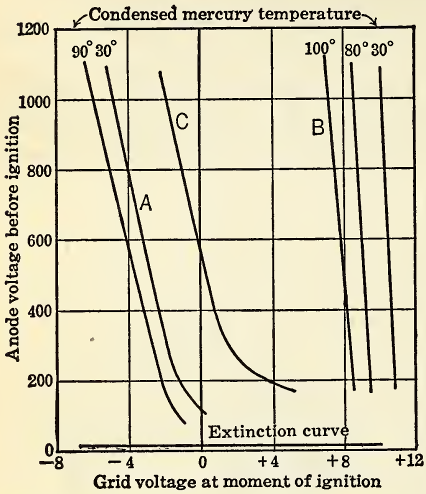


FIG. 11–23. Ignition characteristics of three types of thyratrons; type B, a positive control type, requires a positive grid for breakdown, while type A breaks down with a negative grid. Type C has characteristics intermediate to the other two. The condensed mercury temperature, since it determines the gas pressure, is an important factor in the ignition of the device.

If the plate is made more positive or the grid less negative than that represented by a point on this curve, electrons will pass. If the plate voltage is greater than the ionization value, as soon as electrons pass, ionization results. Current now flows, and the voltage across the tube becomes the 20 volts or so necessary to produce ionization. This drop remains constant, and change in plate voltage, so long as it remains above the ionizing value, causes current change according to the equation

$$I_p = \frac{E_B - 20}{R}$$

where R is the resistance of the connected circuit as before, and E_B is the applied voltage.

In some thyratrons the grid must be positive, when the plate is positive, to cause the tube to fire; such types are called *positive control* types. The ignition curves for such a tube are shown at B in Fig. 11–23. In other types the grid must be negative to prevent ignition, when the plate is positive, as shown at A, Fig. 11–23; such types are called *negative control* types. At C, Fig. 11–23, is also shown a tube having characteristics intermediate to these two types.

As mentioned previously, section 22, there is an optimum pressure at which a maximum amount of ionization will take place for a given voltage. The gas pressure in the case of a vapor-filled device, such as the ones for which the curves of Fig. 11–23 were made, depends upon the temperature of the condensed mercury in the device, and hence the characteristics change with the temperature of the condensed mercury. This temperature, in turn, depends upon the ventilation and construction of the device. Thyratrons, in common with all vapor-filled electronic devices, are very sensitive to temperature changes, and should always be used at the temperatures recommended by the manufacturer. Often special heating and cooling systems are used in conjunction with them to regulate the temperature at which they operate.

The interesting fact is that the grid voltage in the usual type of thyratron is powerless to stop the current after ionization has begun. The grid, if it is made negative, will attract to itself the positive ions, and these sluggish particles will form a blanket or *sheath* about the grid, neutralizing its charge and its effect on the arc stream. Making the grid more highly negative results only in a thicker sheath about the grid; its effect on the current stream is still neutralized.

The only way to stop the current flow through the device is to reduce the plate voltage below that required for ionization. The instant this is done the device deionizes in a few microseconds and will not again become conductive until the proper combination of grid and plate voltages is applied. The *extinction* curve is also shown in Fig. 11–23; as was indicated above, it is independent of grid voltage.

26. Positive-ion Bombardment; Cathode Hot Spot. The positive ions in an ionized gas are drawn to the cathode, or emitter, because of its negative charge. If the voltage is fairly high, these heavy particles will strike with considerable energy and produce heating of the cathode. In some devices this heating, due to positive-ion bombardment, may be sufficient to produce the emission. But if the voltage is still higher, these particles may bombard the emitter with enough energy to destroy it. The voltages at which gas-filled devices can be used is often limited by this phenomenon. It does not exist in the case of a high-vacuum tube, although sometimes the accidental presence of a small amount of gas leads to the destruction of the emitter. This is the reason for the high vacuum required in tubes operating at very high voltage.

Positive-ion bombardment is cumulative in its effect, the bombardment of a small spot raising the temperature of that spot and increasing the emission from it. This greater number of electrons attracts a greater number of positive ions to the spot, in turn raising the temperature by increased bombardment. The ultimate result is the production of an intensely hot spot called the *cathode hot spot*, since it will always be on the cathode. The

large voltage drop is always at the cathode (see Fig. 11–20), and the positive ions, moving through this voltage, strike with considerable energy. The voltage drop at the anode is quite small and the electrons, being light particles and falling through a small voltage, do not strike the anode with enough energy to heat it hot enough for emission. This hot spot becomes the source of most of the emitted electrons and is usually vital to the maintaining of the arc if no other means of heating the emitter is used.

In many cases the temperature of the cathode hot spot will be below the value at which emission ordinarily occurs from a heated cathode; since emission does occur from the cathode hot spot, the mechanism of the emission is not fully understood. Perhaps the electric field in the vicinity of the hot spot takes on a form that leads to the extraction of electrons by the field. In any case the cathode hot spot is the source of the primary electrons.

This hot spot is very hot, usually hot enough to vaporize the material of the electrode. It is obvious how this would lead to the destruction of the cathode if the energy of bombardment and the temperature were high enough. The hot spot is, however, extremely small in area and the volume of material vaporized is very small. It does account for the burning and pitting of such electrodes as switch blades.

In some types of gas-filled electronic devices the cathode consists of a pool of mercury. Not only does the vapor from the pool supply the gas, but also a cathode hot spot on its surface serves as a primary source of electrons. This spot is initiated usually by dipping an electrode into the mercury and withdrawing it. If the polarity is correct the arc that forms when the rod is withdrawn will lead to a hot spot on the mercury surface. Other positively charged electrodes can then attract the emitted electrons and carry current.

In other pool-type devices such as the *ignitron* the arc is started by passing current into a high-resistance igniter dipping in the mercury; a hot spot develops where the igniter touches the mercury. In still others the device is moved or tipped so that the mercury flows away from an electrode, or the electrode draws out of the mercury, causing an arc to form as the mercury breaks contact with the electrode.

This type of device has an indestructible cathode; the mercury vaporized by the hot spot condenses on the cooler portions of the device and drains back to the cathode. There is a constant vaporization which causes the hot spot to move rapidly to and fro over the surface of the mercury.

The phenomenon of ion bombardment is to be seen when a switch in air is opened or a circuit is broken in any gas, in that a cathode hot spot is always formed to support the arc. As the circuit is broken the last point of contact, because of its small area, becomes very hot. The hot area on

the cathode starts to emit, and a cathode hot spot develops. If the polarity of the electrodes is reversed and the arc is maintained, the hot spot appears on the other electrode, the new cathode.

If the arc is formed between separating contacts, such as on a switch or circuit-breaker, the arc finally becomes so long that there is a much more rapid deionization and at the same time the voltage across the contacts cannot produce the necessary ionization.

27. Light-sensitive Devices. There are three general types of light-sensitive devices that may be grouped under the general headings of *photoemissive* devices, *photoconductive* devices, and *photovoltaic* devices. While all are electronic in principle, only the first group functions directly by emitted photoelectrons. Photoemissive devices may be either of the high-vacuum or gas-filled type.

28. High-vacuum Phototubes. As was mentioned previously, certain metals or composite surfaces, when illuminated with light, emit photoelectrons. A common *photoelectric cell, photocell,* or *phototube* is constructed by incorporating an electrode coated with the emitting material, and another electrode to collect the electrons, in an evacuated glass envelope. The photosensitive electrode will be the cathode and consists of a layer of the emitting material deposited either on the inside glass wall of the envelope, or on a metal surface. Usually the cathode is in the shape of a half cylinder of metal with the photoemissive coating on the inside surface. It is mounted so that light can pass through the glass envelope and fall on this inner surface.

The collector plate, or anode, is made of a straight wire mounted in the axis of the cylinder and so proportioned that it does not shade the plate any more than necessary.

If such a cell is connected to a battery, as shown in Fig. 11–24, making the anode or plate positive and the cathode negative, no current will flow if the cathode is not illuminated, in so much as no electrons are being emitted. If, however, light falls upon the cathode, electrons are emitted and are drawn to the positively charged anode or collector. The electrons pass on through the connected battery and a current flows as indicated by the arrow.

The current that flows will be proportional to the intensity of the light that falls upon the cathode; a curve showing this relation is plotted in Fig. 11–25. The current will necessarily be small since the low energy content of the light falling upon the cathode will cause the emission of only a very few electrons. The voltage applied to the cell should be large enough to draw to the anode all the emitted electrons.

As in other high-vacuum devices, the cloud of electrons will constitute a space charge, but the relatively small number of electrons in it will require only a nominal voltage between cathode and anode. The usual voltage

is about 90 volts; this value was used in obtaining the curve of Fig. 11–25. Providing the voltage is high enough to draw over all the emitted electrons, the device is practically independent of the voltage used, and hence is independent of voltage variations.

The sensitivity of the cell shown is about 12 microamperes per lumen. It had a composite emitter surface made by depositing a layer of silver, oxidizing this surface, then depositing a very thin layer of cesium on the silver. The cesium takes up the oxygen, forming cesium oxide. The composite surface then will be cesium on cesium oxide on silver. A still more sensitive surface may be formed by depositing a thin layer of silver on the cesium. Cells have been devised having sensitivities up to 60 microamperes per lumen. By proper control of the forming process these cells may be made responsive even to infrared light.

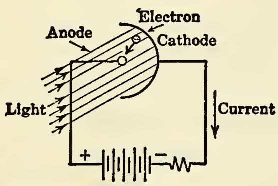


FIG. 11–24.

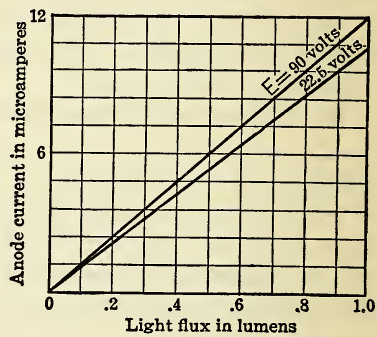


FIG. 11–25.

FIG. 11–24. Circuit for a photocell. Current will flow when the cell is illuminated.

FIG. 11–25. Response of a high-vacuum photocell. The current is directly proportional to the illumination. When the candle power is known, the amount of light flux, in *lumens*, upon a given area is calculated as follows:

$$L = \frac{\text{C.P.} \times \text{Area}}{(\text{distance})^2}$$

area and distance being in corresponding units. This assumes a point source and area measured perpendicular to the light flux. This cell has an area of 0.9 square inch. (Type PJ–22, General Electric Co.)

29. Application of High-vacuum Phototubes. The feeble currents of high-vacuum phototubes are usually not sufficient to operate electrical devices directly so that amplifiers are usually used in connection with them. A circuit, using a triode as amplifier, is shown in Fig. 11–26.

When the cell is illuminated, current flows through the photocell and the high resistance R, so that the grid of the triode is made negative with respect to the cathode by the battery C. With negative grid potential no current will flow in the plate circuit of the triode. When the photocell is not illuminated it no longer conducts, but acts as an open circuit, so that

the negative voltage of battery C is removed from the grid of the triode; the latter is then essentially at the same potential as the plate of the triode. Plate current will then flow in the triode, operating the relay, the necessary power coming from battery B. Such a device can be used for automatically turning on lights when it becomes dark or operating any device when a light beam falling on the cell is broken.

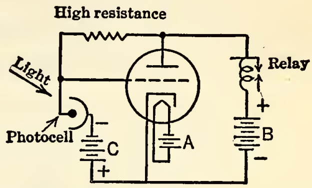


FIG. 11–26. A typical circuit using a photocell. When the photocell is not illuminated, the grid of the triode is made positive and the plate current is high. The relay is energized and closes its contacts, which in turn can be used to turn on lights, etc. When the photocell is illuminated, it carries current, the grid of the triode is made negative by battery C, and the plate current is reduced. The relay will then open its contacts.

Since the response of the photocell to changes in light intensity is instantaneous and since the cell current varies directly with the light intensity, it is useful in sound reproduction such as in sound moving pictures. When a beam of light passes through a pattern on the moving film and falls on a photocell, the pattern of light and dark areas causes a similar variation of current in the photocell. After amplification the current operates a speaker and reproduces sound.

The photocell is a necessary part of any television or picture transmitter. It converts variation in light intensity, as a beam rapidly passes across the subject or the image of the subject on a screen, into exactly proportional variations in electric current and without any appreciable time lag. These variations in current may be reconverted at a receiver into variations of light.

30. Gas-filled Phototubes. Some amplification can be obtained by filling the photocell with an inert gas, or one that will not react chemically with the material of the cathode. The electrons that are emitted from the cathode when it is illuminated will then ionize gas particles and increase the number of current carriers. The gas pressure will be adjusted so that the cell, when operated at a certain voltage, will produce ionization. This voltage must not be too high or the discharge will pass over into a self-maintaining discharge independent of illumination and the tube may be destroyed. Usually this voltage is approximately 100 volts.

The *gas amplification ratio* of a gas-filled cell is the ratio of current for a given illumination to the current for a similar high-vacuum cell and the same illumination. It is usually approximately ten.

The response of a gas-filled cell to light is shown in Fig. 11–27. As would be expected, variations in voltage will cause different amounts of ionization, and differences in the current for a given illumination. There is also a slight time lag with this cell in that a variation in light intensity is not immediately reflected in a current variation. Although the change

in emitted electrons is instantaneous, as with a high-vacuum tube, it takes a small time interval to produce the ions. This time lag is, however, very small and is not ordinarily of any significance, and the cell can be used for all but a very few applications.

As can be seen from the curves of Fig. 11–27, the current is almost proportional to light intensity. Each emitted electron, on the average, leads to the production of the same number of ions and electrons. Since the emitted electrons are proportional to the illumination, the final ions and electrons will also be proportional.

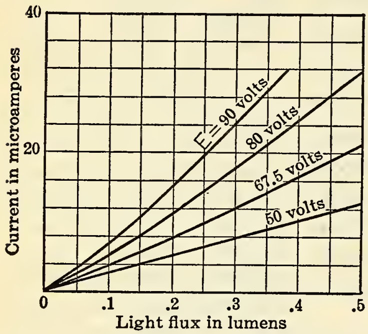


FIG. 11–27. Response of a gas-filled photocell. The voltage has a marked effect on the response since it effects ionization. This cell has an area of 0.9 square inch. (Type PJ–23, General Electric Co.)

31. Photoconductive Devices. These devices, the oldest type of photosensitive devices, depend upon the fact that certain materials change their resistance when illuminated. If connected to a battery the current in the circuit will change when the device is illuminated.

One of the most widely used materials is selenium. Since it is a very high-resistance material, it is desirable to form a conductor having a short length and great area if appreciable currents are to flow. As light cannot penetrate into the selenium the selenium must be in a thin layer, and hence must have great length in a direction at right angles to current flow in order to have as large a cross-sectional area as possible.

One form of practical construction is making a thin metallic plating on a glass plate and then separating the metal into two electrodes by means of a very narrow cut (Fig. 11–28). A thin layer of selenium is placed over the plating, bridging the cut between the electrodes, and annealed to give the desired properties. The plate is mounted inside of an evacuated bulb for protection.

A selenium cell will not have infinite resistance when dark, and some current, called the *dark current,* will flow when the cell is connected to a battery. When illuminated a greater current will flow because of the decreased resistance. The current change is proportional to the square root of light intensity, as shown in Fig. 11–29.

The cause of the change in resistance with light intensity is not understood; one theory is that the light causes an "internal emission" that increases the number of free electrons. As with the gas-filled photocell, there is a time lag in change of current behind a change in illumination. This lag is quite marked with this cell and may amount to minutes. In

general, photoconductive cells are not as stable or as reliable as photo-emissive tubes and are not widely used.

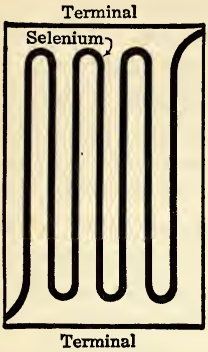


FIG. 11–28.

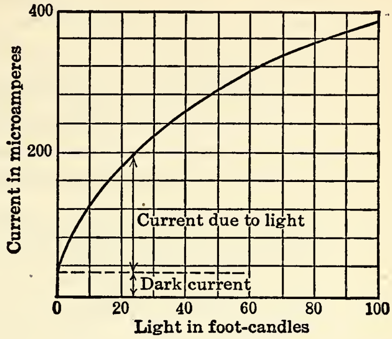


FIG. 11–29.

FIG. 11–28. The plate upon which selenium is placed to form a selenium cell. A zig-zag cut is made in the conducting material deposited on the glass plate; the selenium bridges the gap as shown.

FIG. 11–29. Response of a photoconductive cell. The dark current flows when no light falls on the cell.

32. Photovoltaic Devices. When the junction between certain materials is illuminated, electrons are transferred across the common boundary between them and a voltage difference thus appears between the two materials. One widely used cell of this type consists of a layer of cuprous oxide on copper. The cuprous-oxide layer is made thin enough to allow light to penetrate and illuminate the common boundary or barrier plane. A very thin transparent layer of silver is often placed on the cuprous-oxide layer to make electrical contact with it. Another type of photovoltaic cell consists of a layer of iron selenide on an iron plate with a thin translucent plate of silver covering the selenide.

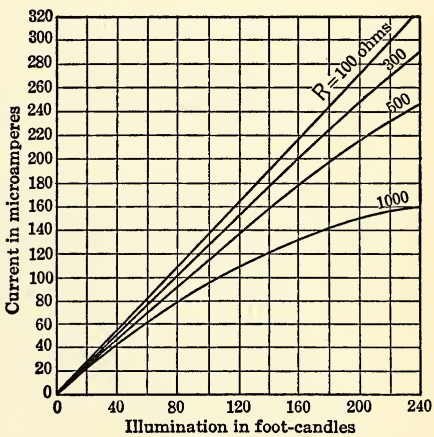


FIG. 11–30. Illumination response curves for the Photronic cell, a photovoltaic cell. The response varies somewhat with the resistance of the external circuit, as shown by the curves. (Weston Electrical Instrument Corp.)

This type of cell develops no voltage when dark and the open-circuit voltage increases with the illumination. It is a generator of voltage and as such does not require a

battery or other source of voltage for its operation. Under the names of Photox* and Photronic,† this type of cell has found wide application.

When connected directly to a sensitive microammeter, this cell may be used to measure light intensity as an illuminometer. If the internal resistance of the microammeter is low, the current will be directly proportional to the illumination; but, as the instrument resistance is made higher, the current follows values as shown in Fig. 11–30 for the Photronic cell.

The current that a photovoltaic cell can supply, while only of the order of 100 to 200 microamperes, is still sufficient to operate very light, sensitive relays, and the device may then be used without an amplifier. The sensitive relay in turn may energize a heavier relay and so operate any device.

Like several of the other devices, there is a time lag between the application of the illumination and the change in the current; sometimes as long as a minute before a steady current value is reached, although very much less in most cells.

* Trademark, Westinghouse Electric and Manufacturing Company.

† Trademark, Weston Electrical Instrument Corporation.

PROBLEMS

11–1. In a solid copper conductor it is esimated that there are 10^{23} free electrons per cc of conductor. What average velocity of charge transfer in a direction along the conductor will represent a flow of 1 ampere in a No. 18 copper wire?

11–2. In a certain high-vacuum diode the voltage between cathode and plate is 180 volts. What energy in ergs will an electron acquire when it moves from cathode to plate?

11–3. In the above problem, with what velocity will it arrive at the plate, if it started from the cathode from rest? Compare this with the velocity of an electron *within* a solid conductor as found in problem 11–1.

11–4. If a current of 10 milliamperes is flowing in the above tube what will be the total energy delivered to the plate per second, in ergs? Determine by summing the energy of impact of the electrons. What will be this power in watts? Compare with the product of $E \times I$ where E is the cathode-to-plate voltage, I the plate current.

11–5. The plate-to-cathode voltage in a certain high-vacuum tube is 90 volts. If a current of 50 milliamperes flows, what will be the power that must be radiated from the plate due to the energy of electron impact?

11–6. What current will flow in the diode of problems 11–2 to 11–4 if the plate voltage is raised to 200 volts? Assume the cathode temperature high enough to prevent temperature saturation.

11–7. If a certain diode carries a current of 15 milliamperes with 150 volts between its cathode and plate, what current will flow if 200 volts are impressed between cathode and plate?

11–8. To what value would the plate voltage be raised in problem 11–7 to cause a current of 25 milliamperes to flow?

11–9. If a current of 15 milliamperes flows from a certain tungsten filament at 2500 K, what current will flow if the temperature is raised to 2550 K?

11–10. What will be the current from a tungsten filament of 0.05 cm radius and 4.0 cm long at 2550 K?

11–11. Using the curve of Fig. 11–1, determine the temperature at which a filament 5 cm long and 0.0032 cm in diameter must operate to give a current of 15 milliamperes.

11–12. If a current of 15 milliamperes flows from a certain tungsten filament at 2400 K, to what temperature must the filament be raised to double the emission?

11–13. What will be the emission from a thoriated tungsten filament of unit area at 1500 K? At 1600 K? At 1700 K? At 1800 K? Plot a curve of the currents at these temperatures against temperature on semi-logarithmic cross-section paper.

11–14. Using the curve plotted in problem 11–13, determine the temperature at which a thoriated tungsten filament 2.5 cm long and 0.005 cm in diameter must be operated to give 15 milliamperes.

11–15. If the filament of problem 11–13 were pure tungsten instead of thoriated tungsten, what current would it give at 1500 K?

11–16. If a high-vacuum tube is rated at 6 watts, what current can it carry if the plate voltage is 180? If the plate voltage is 220?

11–17. Determine the tube factors of a high-vacuum triode at a plate voltage of 250 and a grid voltage of -8, for the RCA Type 6C5 tube for which the curves of Fig. 11–31 apply.

11–18. Determine the tube factors for the tube of Fig. 11–31 for a plate voltage of 200 and a grid voltage of -5.

11–19. Using the tube for which the curves of Fig. 11–31 were made, connected as an amplifier as in Fig. 11–14, determine the quiescent or Q point, when $E_B = 400$ volts, $E_C = -8$ volts, and the load resistor is 20,000 ohms.

11–20. If a voltage of 1.0 volt were introduced into the grid circuit of the tube of problem 11–19 in series with E_C so as to make the grid voltage less negative, by what amount would the voltage across the load be changed? What is the amplification in this case?

11–21. The RCA 6C5 tube, for which the curves of Fig. 11–31 were drawn, is arranged as an amplifier as in Fig. 11–14, with a load resistor of 15,000 ohms. The battery voltages are: $E_B = 300$ volts, $E_C = -4$ volts. Draw the load line and determine the operating point, Q. If a voltage of 2.5 volts is introduced into the grid circuit in series with E_C, so as to make the grid more negative, what will be the change in the voltage across the load? What will be the amplification?

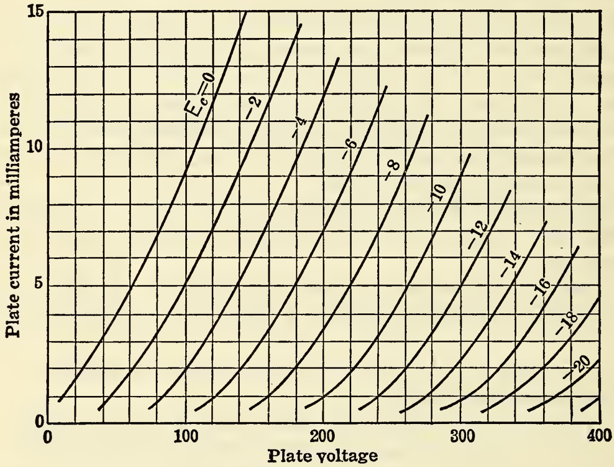


FIG. 11–31. Plate characteristics for a RCA–6C5 triode. This tube has a metal envelope. (RCA Manufacturing Co.)

11–22. Plot the mutual or transfer characteristic curves for the tube of Fig. 11–31.

11–23. Plot on the curves of problem 11–22 the dynamic characteristic curve for the amplifier of problem 11–19.

11–24. Using a plate battery of 480 volts, a grid biasing battery of -15 volts, and a load resistor of 6000 ohms, draw the load line for the tube of Fig. 11–18, and determine the quiescent point, if it is arranged in a circuit as an amplifier.

11–25. If a voltage of 5.0 volts were introduced into the grid circuit of the tube of problem 11–24 in series with the biasing battery so as to make the grid less negative, what will be the increase in voltage across the load resistor?

11–26. What will be the actual amplification of the device in problem 11–25?

11–27. Plot the dynamic characteristic for the tube and circuit of problem 11–24.

11–28. If the load resistor of problem 11–24 were made 8000 ohms, what would be the amplification of the 5.0 volts of problem 11–25?

11–29. It is desired to have the same quiescent point with the 8000 ohm resistor of

problem 11–28 that was obtained with the 6000 ohm resistor of problem 11–24. By how much need the battery voltage, E_B, of problem 11–24 be increased?

11–30. Neglecting the power necessary to heat the filament, what is the efficiency of the circuit of Figs. 11–14 and 11–15? (*Hint:* the input is the power put into the circuit by the battery, the output is the power appearing in the load due to the voltage injected into the grid circuit.)

11–31. A voltage that increases uniformly from 0 to 100 volts in 0.1 second, and then decreases uniformly to 0 in 0.1 second, is impressed on a circuit containing a 5-ohm resistor and a gas-filled diode. Plot the current that flows, against time. Assume that the voltage necessary to produce ionization in the tube is 20 volts.

11–32. What is the average value of the current in the circuit of problem 11–31 if the voltage wave passes through the above variation continuously?

11–33. Plot the current-against-time curve, for the thyratron of Fig. 11–23 for which the left-hand curve applies, in series with a 5-ohm resistor. The voltage applied to the plate varies uniformly from 0 to 800 volts in 0.25 second and then decreases uniformly to 0 in 0.25 second. The grid voltage is -4 volts. The voltage necessary to produce ionization is 20 volts.

11–34. What is the average value of the current wave in problem 11–33?

11–35. If the grid voltage in problem 11–33 is made -2 volts, what will be the shape of the current curve plotted against time, the same voltage being applied?

11–36. What is the average value of the current wave in problem 11–35?

11–37. If the photocell of Fig. 11–25 were connected in series with a 90-volt battery and a 10,000-ohm resistor, what would be the change in the voltage across the resistor when 0.25 lumen falls on the phototube?

11–38. What is the power output in problem 11–37?

11–39. How much of an increase would there be if the phototube of problem 11–37 were replaced by the gas-filled phototube of Fig. 11–27? What is the gas amplification ratio in this case?

11–40. What would be the current that would flow in a load resistor of 100 ohms if the Photronic cell of Fig. 11–30 were illuminated by an illumination of 200 foot-candles? What power is being developed in the load? If the load resistor were made 500 ohms, what would be the answers?

CHAPTER XII

BATTERIES

1. Classification of Batteries. A battery is essentially nothing but two dissimilar metals or other conductors, dipping into a conducting solution of some kind. Two metals, such as copper and zinc, dipping into a solution of sulphuric acid, will show a difference of potential of about one volt; and if an external circuit is connected to the two metals, sufficient current will be delivered to ring a door-bell, operate a telegraph sounder, etc. As current flows, chemical action takes place, the zinc dissolving in the acid as zinc sulphate.

Batteries in which these chemical changes cannot be economically reversed, by forcing current to flow through the battery in the reverse direction, are called *primary batteries*; when they have supplied current for a certain length of time, their materials become exhausted. The old solution must be taken out and new supplied, and new electrodes must be furnished periodically for the ones which go into solution.

There are certain other combinations of metals or metallic salts which form, in the proper electrolyte, efficient batteries; these batteries, when exhausted, may be entirely recuperated by the passage of a current in the reverse direction for a sufficient time. Such combinations evidently constitute reservoirs for electric energy, and are hence called *storage batteries.*

2. Primary Batteries. There are various combinations of metals and solutions which form efficient primary batteries, but at present only one or two types have any appreciable application. For telegraph lines the gravity battery is used at wayside stations for local sounders; it consists of copper and zinc for the electrodes with a solution of zinc sulphate for the electrolyte. Its emf is about one volt, and the internal resistance varies from 0.1 ohm to a few ohms, depending upon the size and proximity of the electrodes. The Leclanché type of primary battery is important, because in a slightly modified form it constitutes the modern dry cell, of which many millions are used each year. In this battery the electrodes are carbon and zinc, and the electrolyte is a solution of sal ammoniac (NH_4Cl). The Leclanché cell gives an emf of about 1.5 volts and generally has a resistance of 1 to perhaps 5 ohms.

3. Polarization and Depolarizers. As current flows from a cell, changes take place at the electrodes which reduce very materially the available

emf of the cell. These changes may be merely an alteration in the density of the electrolyte near the electrodes, or may involve the production of new substances, such as hydrogen, at the surfaces of the electrodes. This blanket of hydrogen bubbles increases the internal resistance of the cell and materially reduces the active area of the electrodes. This effect is of sufficient importance to require the use of another element in the cell, in addition to the electrodes and electrolyte. This extra substance is generally an oxidizing agent, to oxidize the hydrogen set free at the positive pole of the battery, forming water; it is called a *depolarizer*. In the gravity battery, the depolarizer is a solution of copper sulphate, and in the Leclanché cell it is a mass of manganese dioxide which surrounds the carbon electrode.

4. Dry Battery. The dry battery, so much used for flashlights, radio sets, ignition, etc., is a modification of the Leclanché cell; the ingredients are the same, but they are put together in such a manner that the cell may be used in any position. The cell is not actually dry, but is filled with a moist paste or mass of some inert material containing the electrolyte, and is covered with a water-tight seal of wax or pitch; it can thus operate perfectly well even if turned upside down.

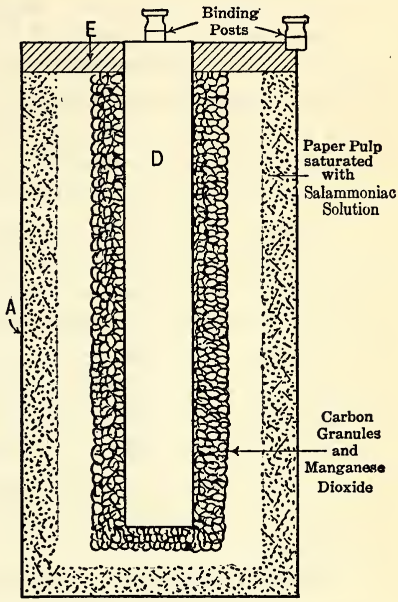


FIG. 12–1. General construction of the ordinary dry cell. The outside zinc cup serves as one element of the cell as well as container for the electrolyte and carbon element.

In the form generally used, it consists of a tall cylindrical cup of zinc which serves as one electrode and also as a container for the other ingredients, about as indicated in Fig. 12–1, which shows a cross-section of the ordinary cell. Next to the zinc is a layer of pulp, blotting paper, or starch paste, which is saturated with sal ammoniac and zinc chloride. The carbon rod is centrally placed, as shown at D, and is surrounded by a mass of carbon granules and manganese dioxide. A seal of pitch, applied hot, serves to hold the carbon in place and to make the cell tight.

5. EMF and Short-circuit Current of a Dry Cell. The emf of a new cell on open circuit is between 1.5 and 1.6 volts, but the terminal voltage is considerably lower, especially after being used a short time. In normal use, the average terminal voltage of the cell during its useful life is about 1.1 volts. A new cell may have an actual internal resistance of only 0.05

ohm, but this increases as the cell ages, and with an old cell, may be several tenths of an ohm. The current which the cell delivers on short circuit (say, external resistance less than 0.01 ohm) is 20 to 30 amperes, with a new cell $2\frac{1}{2}$ inches by 6 inches, the ordinary size for a dry cell. As the cell ages, this short-circuit current rapidly decreases, even though the cell may not be in use. The shelf life of a 6-inch cell is only about one year; after the cell has stood idle for this length of time, the short-circuit current will generally be not more than 10 amperes. The smaller sizes of dry cells have a shelf life of much less than a year, unless especially well prepared.

6. Ampere-hour Capacity of a Dry Cell. The ordinary 6-inch cell has a capacity of 20 to 40 ampere-hours, when discharged through a resistance of 16 ohms continuously until the terminal voltage falls to 0.5 volt. Actually, a cell becomes useless for ordinary purposes long before this quantity of electricity has been taken from it; when the terminal voltage has fallen to one volt, the cell is generally unserviceable. The more rapidly the cell is discharged the less is its capacity; discharged through 40 ohms, a certain cell had 1200 hours life (to terminal voltage of 1.0), and when discharged through 2 ohms it had only 9 hours life. The ampere-hours delivered were only about one-sixth as much with the lower resistance as with the higher. The ordinary 6-inch dry cell should not be used for a circuit requiring more than 0.1 ampere; under such a load, the cell will last about 300 hours. For intermittent service the cell will give a greater ampere-hour output than for continuous service; it seems to recover somewhat during the idle periods. The cell, as ordinarily constructed, makes rather poor use of the materials; when the useful life of the cell is ended, much zinc and depolarizer are still unused.

7. Storage Batteries or Accumulators. Of the reversible, or storage, batteries, two types are in use: the lead-acid battery, and the alkaline battery, of which the Edison battery is the best-known type. In the lead battery, the electrodes are of lead and lead peroxide, both of which change to lead sulphate as the battery discharges. The electrolyte is a solution of sulphuric acid, generally contained in a glass or rubber jar. The Edison battery has one electrode of high nickel oxide and the other of powdered iron, the electrolyte being a solution of potassium hydroxide. By far the greater percentage of storage batteries used today are of the lead-acid type.

8. The Lead Storage Battery. There are two methods of preparing the electrodes for a lead battery. In one, the Planté type, a sheet of pure lead, properly corrugated to expose a very large surface to the electrolyte, serves as the original material for both electrodes. Two of these plates are used as the poles of an electrolytic bath, and by repeated charging and discharging cycles one plate becomes covered with lead peroxide (PbO_2) and the other with spongy lead. In the other type, this long forming process is practically done away with; the plates are formed by

using a flat metal grid as container and filling it with a paste of low lead oxide, such as PbO or Pb_3O_4. Such plates are called pasted, or Faure, plates. These plates are made ready for service by one forming charge.

9. Action of the Lead Cell. Although authorities differ as to the exact sequence of actions taking place in a lead cell, it is agreed that the ultimate action may be represented by the theoretical equation

$$\overbrace{\underset{\text{Positive plate}}{PbO_2} + 2H_2SO_4 + \underset{\text{Negative plate}}{Pb}}^{\text{Battery charged}} \rightleftarrows \overbrace{\underset{\text{Positive plate}}{PbSO_4} + 2H_2O + \underset{\text{Negative plate}}{PbSO_4}}^{\text{Battery discharged}} \quad (1)$$

This equation is to be read from left to right for discharge, and from right to left for charge. During discharge, the sulphuric acid is decomposed to form lead sulphate at both electrodes and the density of the electrolyte therefore decreases materially as the cell discharges; a corresponding increase in specific gravity of the electrolyte occurs, of course, during the charging period.

In actual practice, however, more active material and sulphuric acid are used than are theoretically required in a battery of a certain capacity. As a battery discharges, lead sulphate forms on both plates, tending to cover up the active materials, resulting in decreased porosity. A lead-acid battery is said to be discharged, not because all the active material is exhausted, but because the formation of lead sulphate has sealed it up. The practical equation may be written

$$2PbO_2 + nH_2SO_4 + mH_2O + 2Pb \rightleftarrows$$
$$PbO_2 \cdot PbSO_4 + (n-2)H_2SO_4 + (m+2)H_2O + Pb \cdot PbSO_4 \quad (2)$$

10. Plates of the Lead Cell. Most of the lead storage cells in use today are of the pasted variety; wherever it is necessary to get the greatest possible capacity from a cell of minimum weight and volume, the pasted is chosen in preference to the older type of plate, the Planté. Thus, for automobile ignition and for the propulsion of electric vehicles (if lead cells are used), the pasted plates are always used.

The grid which serves to hold the active material is generally of a stiff lead-antimony alloy; its exact form varies with the various manufacturers, but all the forms are designed to hold the active material as firmly as possible, to have sufficient mechanical strength to prevent breaking and buckling, to have sufficient cross-section to carry the current without appreciable drop, and to bring the metal of the grid into as intimate contact with the active material as possible. This latter consideration is important, in view of the relatively poor conductivity of the active material.

These grids have their interstices filled with a paste of yellow lead oxide (PbO) for the negative plates, and red lead (Pb_3O_4) for the positives.

These lead oxides are made up into a paste of about four parts of the lead oxide, one part dilute sulphuric acid or ammonium sulphate, and 0.1 to 0.3 per cent of graphite, powdered wood pulp, and the like, as porosity

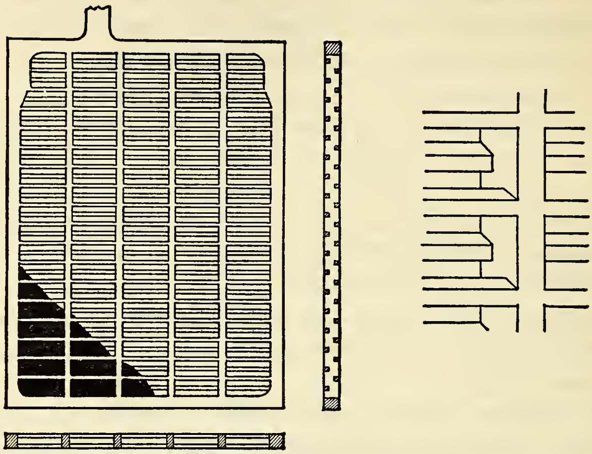


Fig. 12–2. Standard lattice grid. Active material has been placed in the lower left corner; an enlarged detail at one of the center stiffeners is shown at the right.

agents. Figure 12–2 shows one type of grid used, and the appearance of one of these pasted plates is shown in Fig. 12–3.

After drying, these lead oxide plates are assembled in a tank of dilute sulphuric acid for forming. As current is passed through the bath, the yellow oxide is changed to spongy lead, and the red lead is changed to lead peroxide; in case plates of only one kind are to be formed, pure lead plates are used for the other electrode of the bath, anode or cathode according as negative or positive plates are being formed. When charged the positive plate of a storage battery (the one from which current flows during discharge) has a dark chocolate color and is quite hard; the negative plate is of slate-gray, spongy lead and is comparatively soft. The active material of this plate can be quite easily scraped off.

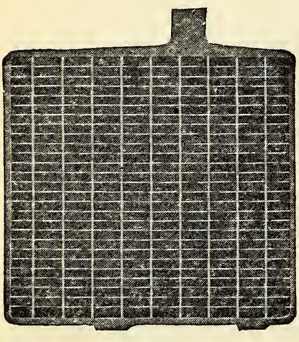


Fig. 12–3. This shows the appearance of such a grid as that of Fig. 12–2, after it has been pasted full of the active material.

The Planté plates have a longer life than the pasted plates, and should be used in cases where the weight is not of so much importance and where discharges at excessive rates are not likely to be demanded. They have a higher conductivity than is possible with

the grid construction of the pasted plates but must be made thicker for strength as they are prone to continue formation and thus buckle.

11. Separators. If a positive and a negative plate of a battery touch, it is short-circuited and so soon loses all its charge and also is not rechargeable. With charging and discharging cycles, the active material works and bulges, so that plates once straight may become warped and crooked, especially if the charging and discharging take place at rapid rates. To prevent the plates from coming in contact with each other, which they would naturally do when sufficiently warped, insulating separators of some kind are generally used; these are especially necessary when the battery must be made compact and hence with plates very close together. Thin sheets of wood ($\frac{1}{16}$ inch to $\frac{3}{32}$ inch thick) are frequently used for separators; the wood must be treated chemically to remove all injurious substances, before being used in a battery. Thin, hard-rubber sheets, slotted or perforated with small holes, and sometimes made with imbedded threads, are also used alone or together with wooden separators, the rubber sheet being placed next to the positive plate. The separator must be very porous, so that acid may readily diffuse through it; otherwise the resistance of the battery is increased and its capacity and watthour efficiency decreased.

12. Electrolyte. The electrolyte used in a lead battery is always a dilute solution of sulphuric acid; the proper strength of the solution varies somewhat with the use to which the battery is to be put, but is generally of specific gravity about 1.280 for automobile batteries, and 1.230 for batteries where more space for electrolyte is allowed, as station stand-by batteries. These figures for density are for the battery in fully charged condition.

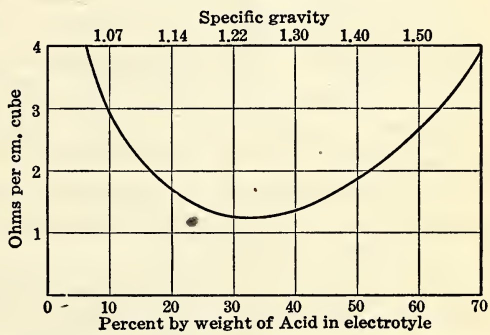


FIG. 12–4. The resistance of the sulphuric acid solution used as the electrolyte in lead cells varies according to the specific gravity of acid used, as shown by this curve.

The resistance of the electrolyte varies considerably with the density, about as shown in Fig. 12–4; it is seen that the densities specified above give the lower values for resistance. Pure water should be used in making up the electrolyte, as even small amounts of certain mineral salts are very injurious to a battery, materially shortening its life. In getting the right density, the acid should be added to the water slowly, as the heat of solution may crack the jar or even sputter the acid around; this is especially likely to occur if the water is poured into the acid, which should never be

done. It is also of interest to note that the lowest freezing point of sulphuric acid electrolyte, that of -96 F, obtains close to a specific gravity of 1.280.

The resistance varies appreciably with temperature, increasing about 1.5 per cent for every degree Fahrenheit decrease in temperature; it is twice as much at zero as it is at 70 F. This effect very materially reduces the amount of current which can be drawn from a battery, as illustrated by the fact that a battery which operates the starting motor of an automobile easily in the summer may not be able to turn the engine over in the winter, not only because of the increased viscosity of the lubricating oil, but also because the increase in battery resistance prevents the starting motor from getting its proper current.

13. Assembly of Battery. For station batteries, the plates are generally assembled in large glass jars, these being placed on sand trays or similar supports. The plates are separated from each other by perhaps one-half inch, and no separator is required; a badly buckled plate, if such occurs, can be seen and replaced before giving trouble. The jars are sometimes covered with glass plates to prevent too rapid evaporation of the water from the electrolyte. Cells of the pasted type, enclosed in rubber jars, are also used in central-station practice.

For automobile batteries, the plates per cell are held perhaps $\frac{1}{8}$ inch apart by suitable separators, are squeezed tightly together, and forced into the compartments of a mono-block jar of rubber composition, each cell being sealed with a cover containing a filling plug. The plates do not touch the bottom of the jars, but rest on several high ridges cast integral with the bottom of the jar; a deep sediment space is thus formed below the plates. As the battery charges and discharges, the active material gradually falls off; if the plates rested on the bottom of the jar, this material would soon short-circuit the plates. The sediment space is made deep enough to care for all the active material which is likely to fall off during the useful life of the battery.

For batteries of both types, the containers are made so that there is room for about one inch of electrolyte above the top of the plates; the plates should never be allowed to project above the electrolyte, as the projecting part of the plates would spoil if this occurred. In the case of the sealed type of automobile battery, provision must be made for the escape of the gas formed during charge; this is generally taken care of by small holes through the plug provided for filling the battery with electrolyte and adding water.

A sectional view of a cell of an automobile battery, used for starting, lighting, and ignition, is shown in Fig. 12–5. In order, from the front, will be seen a negative plate, a wooden separator, and a positive plate. The negative and positive plates rest upon different ribs, which also provide the sediment space.

One very prominent form of lead-acid cell, used for motive power work, train-lighting, marine work, submarine under-water propulsion, etc., is known as the Exide-Ironclad.* The positive plates of this type of cell consists of a series of parallel vertical core rods, joined at each end to horizontal and vertical bars, all of a lead-antimony alloy. The active material surrounds each core rod, being held in place by a finely slotted rubber tube. The effective surface of the active material exposed to electrolyte is greater than in flat plates of equal overall dimensions. The rubber tube acts as a separator and effectively retains the active material, so that the cells have a longer life. The negative plates are the same as in auto-

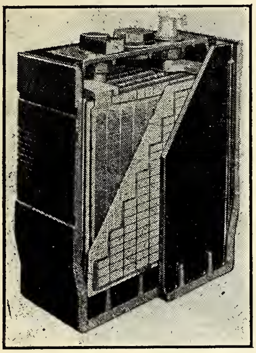


Fig. 12–5.

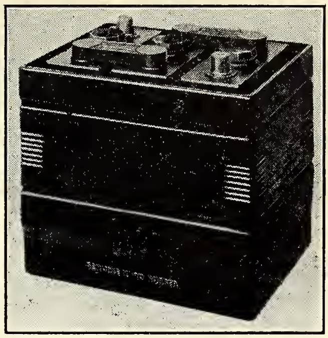


Fig. 12–6.

Fig. 12–5. Sectional view of one cell of an automobile battery. In order from the front may be seen a negative plate, a separator, and a positive plate. *Courtesy of the Electric Storage Battery Co.*

Fig. 12–6. A 6-volt automobile battery. *Courtesy of the Electric Storage Battery Co.*

mobile batteries. A positive and a negative plate of the Ironclad type are shown in Fig. 12–7, and a sectional view of a cell in Fig. 12–8.

14. Voltage of the Lead Cell. The voltage of a lead cell may be taken roughly as 2 volts, but, of course, this varies with the condition of charge, whether the battery is being charged or discharged, and the amount of current passing through it. The exact form of the voltage curve of a cell, for discharge and charge, varies with the type and age of the plates, temperature, and similar factors, but is generally of the shape given in Fig. 12–9. The charge curve is naturally considerably higher than the discharge curve, because of the effect of the internal drop of the battery; this increases the terminal voltage of the battery on charge and decreases it on discharge.

During charge at constant current, the voltage rises rapidly at first, then holds nearly constant for several hours, to rise again as the active

* Electric Storage Battery Company.

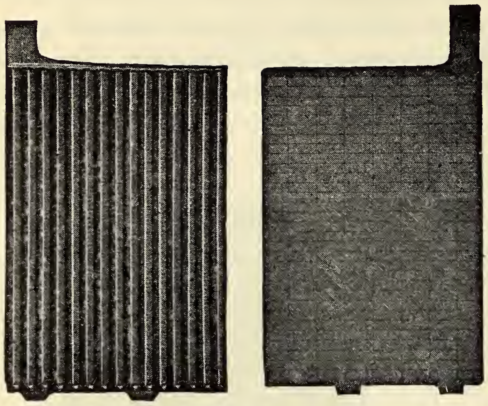


Fig. 12–7.

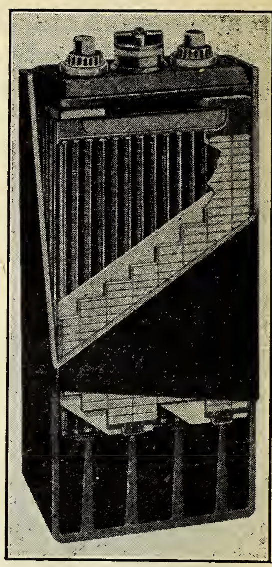


Fig. 12–8.

Fig. 12–7. A positive and a negative plate used in Ironclad cells. *Courtesy of the Electric Storage Battery Co.*

Fig. 12–8. Sectional view of an Ironclad cell. In order from the front are seen a negative plate, a separator, and a positive plate. *Courtesy of the Electric Storage Battery Co.*

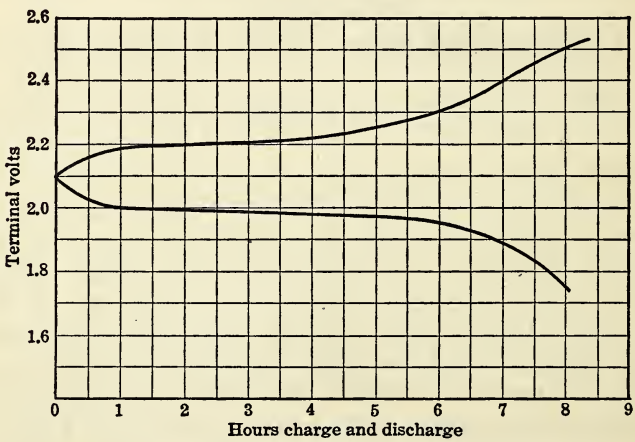


Fig. 12–9. Representative charge and discharge curves of a lead-acid cell at normal current. These curves vary with the type and condition of the cell.

material is nearly all changed. At this time the battery begins to give off gas; when nearly all the active material has been transformed according to Eq. (1), the current begins to electrolyze the water. The gas which is given off is highly explosive, and because of this the battery room should be well ventilated; in certain installations on board ship, for example, blowers must be provided which continually renew the air in the battery chamber and carry off the old air in closed ducts to keep it away from commutator sparks, etc.

The normal rate of discharge of a lead-acid cell is that which causes its terminal voltage to fall to a value of 1.8 to 1.75 volts, in 8 hours, at the end of which time whatever active material is left is made inoperative. When the battery is discharged at this or a lower rate, the voltage falls rapidly for a short time, then holds fairly constant for several hours, and then, at the end of the discharge period, falls rapidly again. The discharge should not be carried below certain limits depending upon the rate of discharge (sections 16 and 17); if this is done repeatedly, the ampere-hour capacity of the battery is seriously diminished, and its internal resistance increased by the formation on the plates of excessive sulphate of a hard, insoluble nature.

As the battery is charged, the electrolyte increases in density; the continual release of the SO_4 ions at both plates during charge increases the acid density correspondingly. In station batteries the specific gravity may increase from perhaps 1.160 to 1.230, and in the automobile battery from 1.200 to 1.300, the greater change being due to the correspondingly smaller amount of water present in the latter type.

15. Ampere-hour Capacity of a Cell. The capacity of a storage cell is given in either ampere-hours or watthours; the latter takes into account the IR drop in the cell, and so is the quantity from which the efficiency of a cell must be reckoned. The ampere-hour rating has nothing directly to do with the resistance of the cell. In ordinary practice the ampere-hour rating is always used. The theoretical ampere-hour capacity can, of course, be figured from the known electrochemical equivalents of the constituents; thus, per ampere-hour capacity, there are required 4.42 grams of lead peroxide and 3.88 grams of spongy lead. It is customary with battery manufacturers to allow two to three times this theoretical quantity to allow for lack of porosity (preventing all the active material from being used) and also to allow for the scaling off of the active material with age. The negative plates, being soft, do not retain their active materials as well as the positives, so that in the assemblage of cells it is customary to have the number of negatives greater by one than the number of positives.

The ampere-hour capacity of an ordinary cell can be approximated by assuming that one ampere can be supplied for 8 hours (8 ampere-hours) for each 25 square inches of surface of positive plate. In calculating the

area, both sides of the plate are to be used, thus, a positive plate 4 inches by 6 inches could deliver 16 ampere-hours. These figures hold for plates about one-eighth inch thick; thicker plates have more capacity, the number of ampere-hours obtainable increasing about as the square root of the plate thickness.

16. Variation of Capacity with Rate of Discharge. The ordinary lead cell is rated by the number of ampere-hours obtainable when the cell is discharged at the 8-hour rate; thus, an 80 ampere-hour battery is one that will deliver 10 amperes for 8 hours. Both the ampere-hour and the watthour capacities vary greatly with the time taken for discharge; if the above battery were discharged at one-half the normal rate, that is, with a discharge current of 5 amperes, about 95 ampere-hours capacity could be obtained. This change of capacity with rate is due principally to the non-porosity of the active material. The discharge at first changes the active material at the surface of the plates; this can be accomplished very effectively, but the material below the surface can change only as the electrolyte penetrates. As the rate of penetration is slow, it is evident that for rapid discharge rates the inner material will not be sufficiently supplied with fresh electrolyte, and so the ampere-hour capacity must be correspondingly less. A certain battery having an 8-hour rating of 100 ampere-hours had a capacity of 110 ampere-hours at the 12-hour rate, 80 ampere-hours at the 4-hour rate, and 55 ampere-hours at the 1-hour rate.

17. Limiting Terminal Voltage. A battery is regarded as discharged when its terminal voltage falls below a certain value, *this to be measured while the battery is delivering current.* As the terminal voltage will evidently depend upon the internal IR drop, as well as the condition of discharge, it is necessary to state the limiting voltage in terms of the current being delivered. In Fig. 12–10 is given a curve showing the final voltages to which a typical lead cell may be taken with safety, when discharged at various constant rates. Thus an 80-ampere-hour cell, discharging at the rate of 10 amperes, should not be discharged below a voltage of 1.75 volts (read with current flowing) or below 1.70 when discharging at the two-hour rate.

18. Indications of Charge. In determining whether a battery is charged, it is of no avail to read the open-circuit voltage; neither is it always safe to depend upon the specific gravity of the electrolyte unless the specific gravity in the discharged condition is known. If the specific gravity of the electrolyte of a cell, after a charge, is 0.100 above that of the discharged condition, it is safe to assume the cell to be fully charged. If, at the moment when discharging at normal rate begins, the terminal voltage is 2.05 volts or more, the battery is probably fully charged. The best criterion, however, is its behavior after it has been put on charge; if, when charging at normal rate, the battery gasses freely, with voltage and specific gravity

of electrolyte constant for one hour, it may be assumed to be completely charged.

19. Sulphation of Plates. Whenever a battery discharges, active material on both plates is changed to lead sulphate; this sulphate is fairly porous, and readily changes back to lead or lead peroxide on charge. If, however, a cell is left in the discharged condition for some time, or if too dense an electrolyte is used, or if the battery gets too hot, a deposition of hard dense sulphate forms, and this may seriously injure the battery. In a bad case of sulphation of this kind, light gray patches can be seen on the plates, and the surface of these patches is very hard. Excessive internal resistance is thus caused by the isolation of the active material below the impervious sulphate layer; the difference between charge and discharge

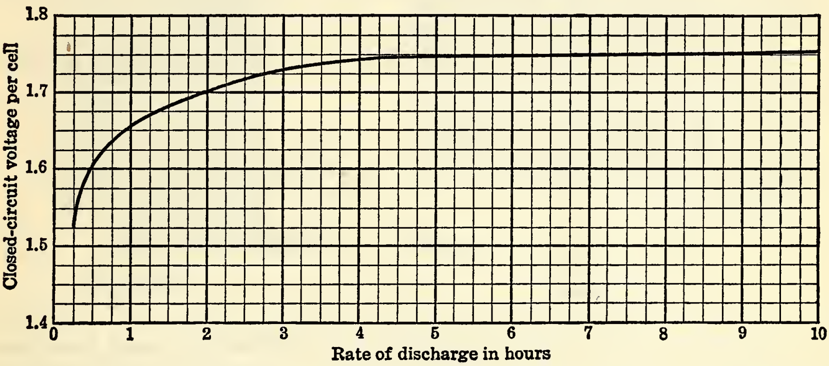


Fig. 12–10. Curve showing the approximate final voltages to which lead-acid cells may be carried when discharged at various constant rates. Values apply with current flowing.

voltage is thus increased, and the electrolyte density is lowered, owing to the permanent loss of the sulphate ions.

Sulphation can be remedied, if it is not too severe, by giving the battery several overcharges at low rates; the gassing resulting from overcharge tends to convert and to crack off the hard sulphate layer. In case of bad sulphation, the plate must be either discarded or given some special treatment, such as the Glauber's salt treatment, for an explanation of which a book on storage batteries should be consulted.

20. Charging a Battery. For charging a depleted storage battery, it is advisable to use the 8-hour rate for about 6 hours, or until gassing occurs, and the voltage per cell (measured with charging current flowing) is about 2.35 volts. The rate should then be diminished to about one-half the normal value, and this low value maintained until the voltage per cell is about 2.40 and remains at this value for about an hour. A battery should not be allowed to gas violently for any length of time, as gassing tends to

crack off the active material, with consequent loss of capacity for the battery. Severe gassing also causes loss of water, objectionable heating, and in open-type cells causes loss of electrolyte due to spray.

21. The Alkaline Storage Battery. The Edison is the only commercial battery of this type now available, although a similar battery, the "Junger," is manufactured and used in Europe. The Edison battery depends for its action on the oxidation from a lower to a higher oxide of nickel in the positive plate, and the reduction of ferrous oxide to metallic iron in the negative plate. The electrolyte has a peculiar action, in that whatever change is caused by the electrolytic action at one plate is at once neutralized by the reverse action taking place at the other. The electrolyte (a mixture of solutions of potassium and lithium hydrates) is not converted at all during the charging operation, and hence but little of it is required. The plates can thus be put very close together.

The action of the Edison cell during charge or discharge is probably represented by

$$\underset{\substack{\text{Positive plate}\\ \text{charged}}}{6NiO_2} + 8KOH + (H_2O) + \underset{\substack{\text{Negative plate}\\ \text{charged}}}{3Fe}$$

$$\rightleftarrows \underset{\substack{\text{Positive plate}\\ \text{discharged}}}{2Ni_3O_4} + 8KOH + (H_2O) + \underset{\substack{\text{Negative plate}\\ \text{discharged}}}{Fe_3O_4} \qquad (3)$$

The equation read from left to right represents discharge, and from right to left gives the cycle of charge. After charge or discharge further reactions are known to occur.

22. Plates of the Edison Cell. The elements of the positive, or nickel oxide, plate, consist of perforated, nickel-plated, steel tubes, packed tight with nickel hydroxide, alternated with layers of flake nickel, the latter being mixed with the hydroxide to increase the effective conductivity of the active material. The nickel flakes come in contact with the steel container, and then, reaching through the body of the hydroxide, serve to conduct the current throughout the mass of the active material.

The negative plate is made up of perforated, nickel-plated, steel containers, filled with powdered iron oxide. The elements of both plates, of which several are required to make one plate of a battery, are fastened, by pressing, into nickel-plated steel grids which serve as the plates of the cell. Figure 12–11 shows the construction of the negative and positive plates of an Edison cell.

23. Assembly of an Edison Cell. The steel grids containing the active elements are assembled and held together by through-bolts, hard rubber being used to keep the plates a short distance apart; as in the lead cell, the number of negative plates is one greater than that of the positives.

The containing jar is of thin nickel-steel, having corrugated sides to increase its rigidity. In the cover of this steel tank are three holes, two of which are fitted with insulating bushings for bringing out the battery terminals; the third hole, for filling with electrolyte and for adding water from time to time, is fitted with a cap and pop valve which lifts to let out gas but prevents entrance of extraneous matter. The cells are assembled in wooden trays, so built as properly to separate adjacent tanks; these are conductors and they must not touch one another, as large leakage currents

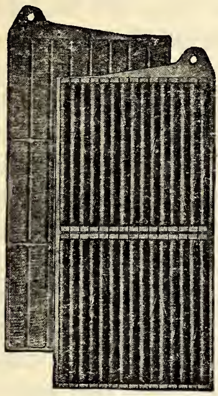


Fig. 12–11.

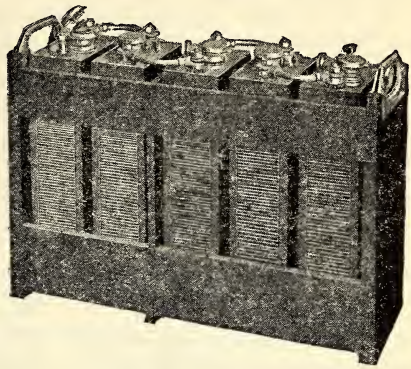


Fig. 12–12.

Fig. 12–11. In the Edison cell the active material is held in small steel containers, these being fastened into steel plates about as shown here. The size of the steel plates, and the number of the small elements used in building up the positive and negative plates, depend upon the capacity the battery is intended to have. All the steel is heavily nickel-plated. *Courtesy of the Edison Storage Battery Div., Thomas A. Edison, Inc.*

Fig. 12–12. As the Edison cell is held in a steel container, an assembled battery must be such that the various cells are held from coming into contact with each other. This shows a typical assembly of five cells, in a wooden rack, in which the different cells are held apart by wooden spacers. *Courtesy of the Edison Storage Battery Div., Thomas A. Edison, Inc.*

would otherwise flow. A completely assembled Edison battery is shown in Fig. 12–12.

24. Voltage of Edison Cell. The emf of the Edison cell is much less than that of the lead cell; on discharge at a normal rate of five hours, the terminal voltage starts at 1.40, falls rapidly to about 1.20, has an average value during discharge of about 1.10, and a useful limiting terminal voltage of from 1.0 to 0.9 volt. For a given battery installation using Edison cells, it is therefore necessary to put in about 70 per cent more cells than would be required if lead cells were used. . Owing to the higher internal resistance of the Edison cell, there is a greater difference between the charge and discharge voltages than is the case with the lead

cell. Typical charging and discharging curves for an Edison cell are given in Fig. 12–13.

25. Charging Rate for the Edison Cell. It is customary to rate an Edison cell at the five-hour rate; thus, a 75 ampere-hour cell will deliver 15 amperes for 5 hours. To charge such a cell completely requires a current of 15 amperes for about 7 hours. The cell is regarded as charged when the charging voltage has remained constant at about 1.8 volts for half an hour at ordinary temperatures.

The alkaline cell will stand much more abuse than will the lead cell; it can be repeatedly discharged to complete short circuit, with no harm, whereas the plates of a lead cell may buckle and sulphate under such treatment. It will yield about the same ampere-hour output, after being fully charged, at any discharge rate from one-quarter normal to four times the normal rate. It can be left in any condition of charge or discharge, with no permanent deleterious effects, whereas a lead cell might be badly sulphated under the same treatment.

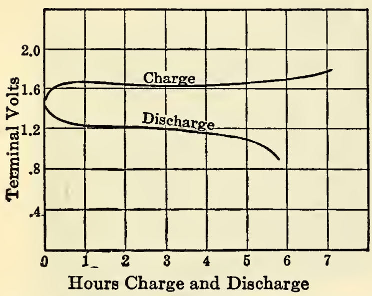


FIG. 12–13. Typical charge and discharge curves of an Edison cell.

26. Effect of Temperature on Edison Cell. Any storage battery shows a loss of capacity with temperatures lower than 70 F, but the effect is more marked with the Edison. The discharge capacity decreases somewhat more than 1 per cent per degree fall in temperature, until, at about 40 F, practically nothing can be obtained from the battery, though it be fully charged. However, if the cell is warmed up, its normal output is available, the cooling not being permanently injurious. If Edison cells are used for vehicles in cold weather, they should be given a warming charge just before starting a run, and put in a covered box; the I^2R loss during discharge will be sufficient to keep them warm even in very cold weather.

During charge, an Edison cell should not be allowed to rise above 115 F, but during discharge a temperature of even 160 F may obtain without doing the cells any injury. Lead cells should not be operated at such high temperatures, 135 F, being the upper limit if the life of the cell is not to be seriously diminished.

27. Resistance of Storage Batteries. The internal resistance of a lead cell is extremely low, and varies approximately inversely with the ampere-hour capacity of the cell; it also varies with the rate and state of discharge. A typical 60-ampere-hour lead cell will have an average internal resistance of about 0.0025 ohm at 80 F. At 70 F the short-circuit current of a lead

cell is about fifty times its rated current; this excessive current would be maintained for a short time only, because of the polarization of the cell. The Edison cell has comparatively more resistance than the lead cell; when short-circuited, it gives a current about ten times its rated value, showing the resistance of the alkaline battery to be about five times as much as that of a lead battery of the same rating. As mentioned before, the resistance of both types rises rapidly as the temperature falls.

28. Efficiency of Storage Batteries. The efficiency of a cell may be measured by the ratio of ampere-hours output to ampere-hours input, or by the ratio of watthours output to watthours input. The ampere-hour output is measured by the ampere-hours delivered by a completely charged cell until its terminal voltage has fallen to the value given, for a lead cell, in Fig. 12–10, and the corresponding input by the ampere-hours required to charge a cell from this condition until the charging voltage remains constant, at the upper limit, for an hour.

The ampere-hour efficiency of a lead cell depends somewhat upon the rate of discharge and also upon the rate of recharge; for the eight-hour rate, it is 80 per cent to 95 per cent. The latter value may be obtained by a modified current recharge, the finishing rate being low. The corresponding figure for the Edison cell is approximately 80 per cent. The watthour efficiency shows up the lead cell much more favorably, because of its lower internal resistance; for the lead cell it is generally 80 to 85 per cent, whereas the Edison cell on normal recharge rate shows 50 to 60 per cent watthour efficiency. For both cells, this figure decreases for currents greater than the normal rated value.

29. Life of Cells. With normal care, a lead cell using thin pasted plates should, according to certain authorities, last for at least 300 complete cycles of discharge and charge before showing appreciable loss of capacity; for Planté plates 50 per cent more life can reasonably be expected. The life depends upon so many factors, such as purity of materials, regularity of charging and discharging, rates of discharge, amount of gassing permitted, variation of temperature allowed, etc., that much variation may be expected. The useful life of a traction cell, for example, is defined as the number of complete cycles of charge and discharge obtainable before its ampere-hour capacity falls to 80 per cent of its rated value. In automobile service, a good lead cell should last two years at least, unless it is continually overcharged, or allowed to freeze; in this service the battery seldom is completely discharged, but generally suffers from excessive gassing.

The Edison cell is so free from troubles that its life is much longer than that of the lead cell; the manufacturers generally guarantee their cells to show their rated capacity at the end of five years' use.

30. Comparison of Lead and Edison Batteries. The weight and space required per kilowatthour capacity are nearly the same for both types.

The Edison evaporates its water much faster than the lead cell, and so requires more attention for this reason. If the cells are in a confined space, so that artificial cooling is required to prevent overheating, the Edison requires much more blower capacity to keep the temperature within advisable limits. The watthour efficiency of the lead cell is much better than that of the Edison cell, especially at high rates of discharge; and at low temperatures the lead cell is incomparably superior to the Edison. The initial cost is greater for the Edison, but this is offset by the longer life guarantee.

31. Uses of Storage Batteries. By far the greatest number of storage batteries in use today are in automobile lighting and ignition equipments; the total capacity of cells in such service, in the United States alone, is over three million kilowatthours. For farm-lighting and other small, isolated plants, about one million kilowatthours capacity is used, while for electric trucks, locomotives, and stand-by service in large stations, another million kilowatthours is installed.

For central-station work, the cells have two distinct uses. In small stations they are used to carry the peak load, helping the generators to carry the load when the station is most heavily loaded, and then being charged when the generators would otherwise be underloaded. Their use in this service very materially reduces the amount of generating equipment required. In the large stations, connected principally to lighting loads, they are used "floating" on the line, continually charged; their function here is to care for the entire load of the system for a short time, in case the generating equipment fails, thus insuring continuity of service. The installed capacity in such stations is sufficient only to carry the load for perhaps twenty minutes, but in this time the generating apparatus would presumably be again ready to operate. In large a-c power plants the storage battery is extensively used as a stand-by for control energization.

APPENDIX

GREEK ALPHABET

A	α	alpha	I	ι	iota	P	ρ	rho
B	β	beta	K	κ	kappa	Σ	σ	sigma
Γ	γ	gamma	Λ	λ	lambda	T	τ	tau
Δ	δ	delta	M	μ	mu	Υ	υ	upsilon
E	ϵ	epsilon	N	ν	nu	Φ	ϕ	phi
Z	ζ	zeta	Ξ	ξ	xi	X	χ	chi
H	η	eta	O	o	omicron	Ψ	ψ	psi
Θ	θ	theta	Π	π	pi	Ω	ω	omega

MULTIPLES AND SUBMULTIPLES OF UNITS

micro	=	$\frac{1}{1,000,000}$	or	10^{-6}	deka	=	10	or	10^{1}
milli	=	$\frac{1}{1000}$	or	10^{-3}	hecto	=	100	or	10^{2}
centi	=	$\frac{1}{100}$	or	10^{-2}	kilo	=	1000	or	10^{3}
deci	=	$\frac{1}{10}$	or	10^{-1}	mega	=	1,000,000	or	10^{6}

CONVERSION FACTORS

Length:	1 inch	=	2.540 centimeters
Area:	1 square inch	=	6.452 square centimeters
Volume:	1 cubic inch	=	16.39 cubic centimeters
	1 gallon	=	231 cubic inches
Mass and Weight:	1 pound	=	453.6 grams
Force:	1 gram	=	980.7 dynes
	1 pound	=	453.6 grams
Energy and Work:	1 Btu	=	252.0 gram-calories
		=	777.9 foot-pounds
		=	1054.8 joules
		=	2.930×10^{-4} kilowatthour
	1 foot-pound	=	1.356 joules
		=	3.766×10^{-7} kilowatthour
	1 kilowatthour	=	3413 Btu
		=	2.655×10^{6} foot-pounds
		=	3.6×10^{6} joules
Power:	1 kilowatt	=	4.426×10^{4} foot-pounds per minute
		=	737.6 foot-pounds per second
		=	1.341 horsepower
		=	56.89 Btu per minute

SYSTEMS OF UNITS

The unit of electrostatic charge can be defined by Eq. (1), page 5, and from this definition of charge a whole system of units may be built up for all electrical quantities.

Again, the unit of magnetic pole strength can be defined by Eq. (1), page 18, and a second system of units built up from this definition, for all electrical quantities.

Early experimenters, failing to recognize that electrostatic and magnetic phenomena were merely different manifestations of one general phenomenon, devised the two systems to describe the two different types of phenomena. Later experimentation showed that either system could be extended to describe completely all electrical phenomena.

It is, however, often convenient to use the electrostatic system to measure and describe phenomena in electrostatics and the electromagnetic system to measure and describe phenomena in electromagnetics. These two systems are known as "absolute" systems and the units are often written as esu and emu respectively. More properly, they should be written cgsemu and cgsesu respectively, since the units of length, mass, and time are always taken in the cgs (centimeter, gram, second) or metric system when using either of the absolute systems.

It is evident that, since the two systems overlap and describe and measure the same thing, it is possible always to express a measured quantity in one system as a quantity in the other system. The conversion factors to change from one system to the other are given in Table I.

When electricity came into wide use it was discovered that the units of both systems were of such sizes that the expression of the usual electrical magnitudes was often awkward, and a system of units of more convenient size was arbitrarily devised. This system is called the "practical" system and the units are designated by pu. The units are usually taken as decimal multiples of the units of the electromagnetic system. The conversion factors necessary to change to either of the absolute systems is also given in Table I.

The units of the practical system, since they are widely used, have been given names, usually the names of famous electrical experimenters. The units of the other systems do not have names, but the practice has grown up of designating the corresponding unit by taking the name of the practical unit and using the prefix "stat" for the electrostatic system unit, and the prefix "ab" for the electromagnetic system unit. This practice is followed in this book.

It was recognized several years ago that, if the meter, kilogram, and second (mks) were used as a basis for deriving the absolute systems, rather than the centimeter, gram, and second (cgs), the new absolute units could be made identical with the practical units. The obvious advantage would be the elimination of all the conversion factors.

The International Electrotechnical Commission in 1935 adopted the mks system, replacing the three systems mentioned above. There are, however, certain outstanding difficulties that must be settled before all the units stand in a one-to-one relation. One difficulty is the problem of the 4π in all the fundamental magnetic relations. Modified or "rationalized" mks units are proposed to overcome this difficulty but are not fully accepted. The "unrationalized" mks system of units leaves 4π in all magnetic relations.

While the mks system is already in use it is doubtful that it will be fully accepted by physicists. It does offer some simplification and may be more widely used in the future. The sizes and names of the units of the mks system are the same as in the practical system.

TABLE I

Interrelation of Units of Different Systems for the Measurement of Electrical Quantities

	Symbol	esu Electrostatic units	emu Electromagnetic units	pu or mks Practical units
Charge	q, Q	3×10^9 statcoulombs	$= 0.1$ abcoulomb	$= 1$ coulomb
Current	i, I	3×10^9 statampere	$= 0.1$ abampere	$= 1$ ampere
Voltage	v, V, e, E	$\frac{1}{300}$ statvolt	$= 10^8$ abvolts	$= 1$ volt
Resistance	r, R	$\frac{1}{9 \times 10^{11}}$ statohm	$= 10^9$ abohms	$= 1$ ohm
Capacitance	C	9×10^{11} statfarad	$= 10^{-9}$ abfarad	$= 1$ farad
Inductance	L	$\frac{1}{9 \times 10^{11}}$ stathenry	$= 10^9$ abhenry	$= 1$ henry
Power	p, P	10^7 ergs/sec	$= 10^7$ ergs/sec	$= 1$ watt
Energy	w, W	10^7 ergs	$= 10^7$ ergs	$= 1$ joule

TABLE II

Resistivity and Temperature Coefficient

Resistivity in ohms per mil-foot = 6.015 × resistivity per centimeter cube	Resistivity				Temperature Range, °C		Temperature Coefficient	
	Microhms per cm cube		Ohms per mil-foot					
	at 0 C	at 20 C	at 0 C	at 20 C	From	To	Referred to 0 C	Referred to 20 C
Aluminum (hard-drawn)......	2.600	2.828	15.64	17.01	0	100	0.00438	0.00403
Copper, standard............	1.589	1.724	9.56	10.37	0	100	0.00427	0.00393
Copper, hard-drawn..........	1.60	1.737	9.62	10.44	0	100	0.00427	0.00393
Nickel, commercial wire......	8.98	9.98	54	60	0	100	0.00555	0.0050
Platinum, drawn............	11.0	11.8	66.7	71.2	0	100	0.00367	0.00342
Silver......................	1.517	1.641	9.12	9.87	0	100	0.0041	0.0038
Steel, soft..................	11.8	12.80	70.98	76.98	0	35	0.00423	0.00390
Steel, hard..................	45.6	47.1	274.3	283.1	0	35	0.00161	0.00156
Tungsten, wire..............	5.05	5.52	30.38	33.20	0	100	0.0047	0.0043
Alloys*:								
Advance..................	48.9	48.9	294	294	0	100	±0.00002	±0.00002
Manganin †..............	48.2	48.2	290	290	15	35	±0.000015	±0.000015
Nichrome.................	112	112	672	675	0	100	0.00017	0.00017
Nichrome V ‡............	108	108	648	650	0	100	0.00013	0.00013
Ohmax....................	169	167	1007	1000	0	500	−0.00035	−0.00035
Radiohm.................	131	133	789	800	0	500	0.0007	0.0007

* All alloys mentioned, except manganin, are trademark names of the Driver-Harris Co.

† See curves of Fig. 3–17.

‡ See curves of Fig. 3–16.

TABLE III

TEMPERATURE COEFFICIENT OF COPPER AT DIFFERENT TEMPERATURES

$$\left(\text{From equation } \alpha_t = \frac{1}{234.5 + t}\right)$$

Initial Temperature	Increase in Resistance per 1 C	Initial Temperature	Increase in Resistance per 1 C
0	0.00427	30	0.00378
5	0.00418	35	0.00371
10	0.00409	40	0.00364
15	0.00401	45	0.00358
20	0.00393	50	0.00351
25	0.00385		

TABLE IV*

Solid Copper Wire—American Wire Gage

100 Per Cent Conductivity; Density 8.89 at 20 C

Gage No.	Diameter in Mils	Cross-section		Resistance at 20 C or 68 F †		Weight in Pounds		Feet per Pound
		Circular Mils	Square Inch	Ohms per 1000 Feet	Ohms Per Mile	Per 1000 Feet	Per Mile	
0000	460.0	211,600	0.1662	0.04901	0.259	640.5	3380	1.561
000	409.6	167,800	0.1318	0.06180	0.326	507.9	2680	1.968
00	364.8	133,100	0.1045	0.07793	0.411	402.8	2130	2.482
0	324.9	105,500	0.08289	0.09827	0.519	319.5	1680	3.130
1	289.3	83,690	0.06573	0.1239	0.654	253.3	1340	3.947
2	257.6	66,370	0.05213	0.1563	0.825	200.9	1060	4.977
3	229.4	52,640	0.04134	0.1970	1.04	159.3	841	6.276
4	204.3	41,740	0.03278	0.2485	1.31	126.4	667	7.914
5	181.9	33,100	0.02600	0.3133	1.65	100.2	529	9.980
6	162.0	26,250	0.02062	0.3951	2.09	79.46	420	12.58
7	144.3	20,820	0.01635	0.4982	2.63	63.02	333	15.87
8	128.5	16,510	0.01297	0.6282	3.32	49.98	264	20.01
9	114.4	13,090	0.01028	0.7921	4.18	39.63	209	25.23
10	101.9	10,380	0.008155	0.9989	5.28	31.43	166	31.82
11	90.74	8,234	0.006467	1.260	6.65	24.92	132	40.12
12	80.81	6,530	0.005129	1.588	8.38	19.77	104	50.59
13	71.96	5,178	0.004067	2.003	10.58	15.68	83	63.80
14	64.08	4,107	0.003225	2.525	13.3	12.43	63.3	80.44
15	57.07	3,257	0.002558	3.184	16.8	9.858	52.0	101.4
16	50.82	2,583	0.002028	4.015	21.2	7.818	41.3	127.9
17	45.26	2,048	0.001609	5.064	26.7	6.200	32.7	161.3
18	40.30	1,624	0.001276	6.385	33.7	4.917	26.0	203.4
19	35.89	1,288	0.001012	8.051	42.5	3.899	20.6	256.5
20	31.96	1,022	0.0008023	10.15	53.6	3.092	16.3	323.4
21	28.46	810.1	0.0006363	12.80	67.6	2.452	12.9	407.8
22	25.35	642.4	0.0005046	16.14	85.2	1.945	10.3	514.2
23	22.57	509.5	0.0004002	20.36	108	1.542	8.14	648.4
24	20.10	404.0	0.0003173	25.67	135	1.223	6.46	817.7
25	17.90	320.4	0.0002517	32.37	171	0.9699	5.12	1,031
26	15.94	254.1	0.0001996	40.82	216	0.7692	4.06	1,300
27	14.20	201.5	0.0001583	51.46	272	0.6100	3.22	1,639
28	12.64	159.8	0.0001255	64.90	343	0.4837	2.55	2,067
29	11.26	126.7	0.00009953	81.84	432	0.3836	2.03	2,607
30	10.03	100.5	0.00007894	103.2	545	0.3042	1.61	3,287
31	8.928	79.70	0.00006260	130.1	687	0.2413	1.27	4,145
32	7.950	63.21	0.00004964	164.1	866	0.1913	1.01	5,227
33	7.080	50.13	0.00003937	206.9	1,090	0.1517	0.814	6,591
34	6.305	39.75	0.00003122	260.9	1,380	0.1203	0.635	8,310
35	5.615	31.52	0.00002476	329.0	1,740	0.09542	0.504	10,480
36	5.000	25.00	0.00001964	414.8	2,190	0.07568	0.400	13,210
38	3.965	15.72	0.00001235	659.6	3,480	0.04759	0.251	21,010
40	3.145	9.888	0.000007766	1049	5,540	0.02993	0.158	33,410
41	2.800	7.842	0.000006159	1323	6,983	0.02374	0.125	42,130
42	2.494	6.219	0.000004884	1668	8,806	0.01882	0.0994	53,120
43	2.221	4.932	0.000003873	2103	11,100	0.01493	0.0788	66,990
44	1.978	3.911	0.000003072	2652	14,000	0.01184	0.0625	84,470

* Reproduced from Pender-Del Mar, *Electrical Engineers' Handbook*, John Wiley and Sons, 1936.

† Let C = per cent conductivity, R_{20} = resistance of 100 per cent conductivity wire at 20 C (from table), R_t = resistance of wire of conductivity C at any temperature t C, then

$$R_t = R_{20}\left[\frac{100}{C} + 0.00393(t - 20)\right]$$

TABLE V*

Copper Cables, Concentric-lay

Circular Mils and American Wire Gage

100 Per Cent Conductivity; Density 8.89 at 20 C

Circular Mils and AWG	Resistance at 25 C or 77 F †		Weight in Pounds, Bare		Class B ‡ Stranding			Class C ‡ Stranding		
	Ohms per 1000 Feet	Ohms per Mile	Per 1000 Feet	Per Mile	Number of Wires	Diameter of Wires in Mils	Outside Diameter, in Mils	Number of Wires	Diameter of Wires in Mils	Outside Diameter in Mils
2,000,000	0.00539	0.0285	6180	32,600	127	125.5	1631	169	108.8	1632
1,900,000	0.00568	0.0300	5870	31,000	127	122.3	1590	169	106.0	1590
1,800,000	0.00599	0.0316	5560	29,300	127	119.1	1548	169	103.2	1548
1,700,000	0.00634	0.0335	5250	27,700	127	115.7	1504	169	100.3	1504
1,600,000	0.00674	0.0356	4940	26,100	127	112.2	1459	169	97.3	1460
1,500,000	0.00719	0.0380	4630	24,500	91	128.4	1412	127	108.7	1413
1,400,000	0.00770	0.0407	4320	22,800	91	124.0	1364	127	105.0	1365
1,300,000	0.00830	0.0438	4010	21,200	91	119.5	1315	127	101.2	1315
1,200,000	0.00899	0.0475	3710	19,600	91	114.8	1263	127	97.2	1264
1,100,000	0.00981	0.0518	3400	17,900	91	109.9	1209	127	93.1	1210
1,000,000	0.0108	0.0570	3090	16,300	61	128.0	1152	91	104.8	1153
950 000	0.0114	0.0600	2930	15,490	61	124.8	1123	91	102.2	1124
900,000	0.0120	0.0633	2780	14,670	61	121.5	1093	91	99.4	1094
850,000	0.0127	0.0670	2620	13,860	61	118.0	1062	91	96.6	1063
800,000	0.0135	0.0712	2470	13,040	61	114.5	1031	91	93.8	1031
750,000	0.0144	0.0759	2320	12,230	61	110.9	998	91	90.8	999
700,000	0.0154	0.0814	2160	11,410	61	107.1	964	91	87.7	965
650,000	0.0166	0.0876	2010	10,600	61	103.2	929	91	84.5	930
600,000	0.0180	0.0949	1850	9,780	61	99.2	893	91	81.2	893
550,000	0.0196	0.1036	1700	8,970	61	95.0	855	91	77.7	855
500,000	0.0216	0.1139	1540	8,150	37	116.2	814	61	90.5	815
450,000	0.0240	0.1266	1390	7,340	37	110.3	772	61	85.9	773
400,000	0.0270	0.1424	1240	6,520	37	104.0	728	61	81.0	729
350,000	0.0308	0.1627	1080	5,710	37	97.3	681	61	75.7	682
300,000	0.0360	0.1899	926	4,890	37	90.0	630	61	70.1	631
250,000	0.0431	0.228	772	4,080	37	82.2	575	61	64.0	576
0000	0.0509	0.269	653	3,450	19	105.5	528	37	75.6	533
000	0.0642	0.339	518	2,735	19	94.0	470	37	67.3	471
00	0.0811	0.428	411	2,170	19	83.7	418	37	60.0	420
0	0.102	0.540	326	1,720	19	74.5	373	37	53.4	374
1	0.129	0.681	258	1,364	19	66.4	332	37	47.6	333
2	0.162	0.858	205	1,082	7	97.4	292	19	59.1	296
3	0.205	1.082	163	858	7	86.7	260	19	52.6	263
4	0.259	1.365	129	680	7	77.2	232	19	46.9	234
5	0.326	1.721	102	540	7	68.8	206	19	41.7	209
6	0.410	2.170	81.0	428	7	61.2	184	19	37.2	186
7	0.519	2.74	64.3	339	7	54.5	164	19	33.1	166
8	0.654	3.45	51.0	269	7	48.6	146	19	29.5	147

* Reproduced from Pender-Del Mar, *Electrical Engineers' Handbook*, John Wiley and Sons, 1936.

† Let C = per cent conductivity, R_{25} = resistance of 100 per cent conductivity cable at 25 C (from table), R_t = resistance of cable of conductivity C at any temperature t C, then

$$R_t = R_{25}\left[\frac{100}{C} + 0.00385(t - 25)\right]$$

‡ Concentric stranding. Class A for bare, weather-proof, and slow-burning, cables. Class B for rubber, varnished cambric, and paper cables. Classes C and D, for use where greater degrees of flexibility are desired.

TABLE VI

Allowable Current-carrying Capacities of Conductors in Amperes*

For Not More Than Three Conductors in Raceway or Cable

(Based on Room Temperature of 30° C or 86° F)

Size AWG or MCM	Rubber Type RW Type R	Synthetic Type SN Type RU Rubber Type RPT Type RP	Rubber Type RHT Type RH	Paper Synthetic Type SNA Asbestos Var-Cam Type AVB Var-Cam Type V	Asbestos Var-Cam Type AVA Type AVL	Impregnated Asbestos Type AI	Asbestos Type A
14	15	18	22	23	28	29	32
12	20	23	27	29	36	38	42
10	25	31	37	38	47	49	54
8	35	41	49	50	60	63	71
6	45	54	65	68	80	85	95
5	52	63	75	78	94	99	110
4	60	72	86	88	107	114	122
3	69	83	99	104	121	131	145
2	80	96	115	118	137	147	163
1	91	110	131	138	161	172	188
0	105	127	151	157	190	202	223
00	120	145	173	184	217	230	249
000	138	166	199	209	243	265	284
0000	160	193	230	237	275	308	340
250	177	213	255	272	315	334	372
300	198	238	285	299	347	380	415
350	216	260	311	325	392	419	462
400	233	281	336	361	418	450	488
500	265	319	382	404	468	498	554
600	293	353	422	453	525	543	612
700	320	385	461	488	562	598	668
750	330	398	475	502	582	621	690
800	340	410	490	514	600	641	720
900	360	434	519	556	......	.....	.....
1000	377	455	543	583	681	730	811
1250	409	493	589	643	......	.....	.....
1500	434	522	625	698	784	.....	.....
1750	451	544	650	733	......	.....	.....
2000	463	558	666	774	839	.....	.....

Correction Factor for Room Temperatures Over 30° C.

C	F							
40	104	.71	.82	.88	.90	.94	.95	.....
45	113	.50	.71	.82	......	......	.....	.....
50	122	.00	.58	.75	.80	.87	.89	.....
55	131	......	.41	.67	......	......	.....	.....
60	140	......	.00	.58	.67	.79	.83	.97
70	158	......	......	.35	.52	.71	.76	.93
75	167	......	......	.00	......	......	.....	.....
80	176	......	......	......	.30	.61	.69	.89
90	194	......	......	......	......	.50	.61	.86
100	212	......	......	......	......	......	.51	.82
120	248	......	......	......	......	......	.....	.72
140	284	......	......	......	......	......	.....	.63

* Taken, by permission, from the National Electrical Code, 1940, as adopted by the National Fire Protection Association.

For explanation of Type Letters, see Table VIII.

TABLE VII

Allowable Current-carrying Capacities of Conductors in Amperes*

Single Conductor in Free Air

(Based on Room Temperature of 30° C, 80° F)

Size AWG MCM	Rubber Type R	Rubber Type RP	Rubber Type RHT Type RH	Synthetic Type SNA / Asbestos Var-Cam Type AVB / Var-Cam Type V	Asbestos Var-Cam Type AVA	Impregnated Asbestos Type AI	Asbestos Type A	Slow-burning Type SB / Weatherproof Type W Type SBW / Paper
14	20	24	29	30	39	40	43	20
12	26	31	37	40	51	52	57	30
10	35	42	50	54	65	69	75	35
8	48	58	69	71	85	91	100	50
6	65	78	94	99	119	126	134	70
5	76	92	110	115	136	145	158	80
4	87	105	125	133	158	169	180	90
3	101	122	146	155	182	194	211	100
2	118	142	170	179	211	226	241	125
1	136	164	196	211	247	264	280	150
0	160	193	230	245	287	306	325	200
00	185	223	267	284	331	354	372	225
000	215	259	310	330	384	410	429	275
0000	248	298	358	383	446	476	510	325
250	280	338	403	427	495	528	562	350
300	310	373	446	480	555	592	632	400
350	350	421	504	529	612	653	698	450
400	380	457	547	575	665	710	755	500
500	430	517	620	660	765	814	870	600
600	480	577	691	738	857	912	970	680
700	525	632	756	813	942	1003	1065	760
750	545	655	785	846	981	1044	1118	800
800	565	680	815	879	1020	1085	1150	840
900	605	728	872	941	....	....	....	920
1000	650	782	936	1001	1163	1238	1332	1000
1250	740	890	1066	1131	....	....	....	......
1500	815	980	1174	1261	1452	....	....	1360
1750	890	1070	1282	1370	....	....	....	......
2000	960	1155	1383	1472	1713	....	....	1670

Correction Factor for Room Temperatures Over 30° C

C	F	Rubber Type R	Rubber Type RP	Rubber Type RHT Type RH	Synthetic Type SNA / Asbestos Var-Cam Type AVB / Var-Cam Type V	Asbestos Var-Cam Type AVA	Impregnated Asbestos Type AI	Asbestos Type A	Slow-burning Type SB / Weatherproof Type W Type SBW / Paper
40	104	.71	.82	.88	.90	.94	.95	...	.....
45	113	.50	.71	.82	...	...	...	...	.....
50	122	.00	.58	.75	.80	.87	.89	...	.....
55	131	...	.41	.67	...	...	...	...	.....
60	140	...	.00	.58	.67	.79	.83	.97	.....
70	158	...	...	.35	.52	.71	.76	.93	.....
75	167	...	...	.00	...	...	...	...	.....
80	176	...	...	...	.30	.61	.69	.89	.....
90	194	...	...	...	...	.50	.61	.86	.....
100	212	...	...	...	...	...	.51	.82	.....
120	248	...	...	...	...	...	...	.72	.....
140	284	...	...	...	...	...	...	.63	.....

* Taken, by permission, from the National Electrical Code, 1940, as adopted by the National Fire Protection Association.

For explanation of Type Letters, see Table VIII.

TABLE VIII

CONDUCTOR INSULATIONS—600 VOLTS*

Trade Name	Type Letter	Maximum Operating Temperature	Insulation	Outer Covering	Use
Code	R	50 C (122 F)	Code Grade Rubber	Moisture Resistant Flame-Retardant Fibrous Covering	General Use
Moisture Resistant	RW	50 C (122 F)	Moisture Resistant Rubber	Moisture Resistant Flame-Retardant Fibrous Covering	General Use or in Wet Locations
Performance	RP	60 C (140 F)	Performance Grade Rubber	Moisture-Resistant Flame-Retardant Fibrous Covering	General Use
Heat-Resistant	RH	75 C (167 F)	Heat-Resistant Grade Rubber	Moisture Resistant Flame-Retardant Fibrous Covering	General Use
Small Diameter Building Wire (Heat-Resistant)	RHT	75 C (167 F)	Heat-Resistant Grade Rubber	Moisture-Resistant Flame-Retardant Fibrous Covering	General Use
Small Diameter Building Wire (Performance)	RPT	60 C (140 F)	Performance Grade Rubber	Moisture-Resistant Flame-Retardant Fibrous Covering	Rewiring Existing Raceways
Type RU Wire (See Note)	RU	60 C (140 F)	90 Per Cent Unmilled Grainless Rubber	Moisture-Resistant Flame-Retardant Fibrous Covering	Rewiring Existing Raceways
Solid Synthetic (See Note)	SN	60 C (140 F)	Solid, Flame-Retardant Moisture Resistant Synthetic Compound	None	Rewiring Existing Raceways

Asbestos Synthetic	SNA	90 C (194 F)	Synthetic and Felted Asbestos	Cotton Braid Thickness, 20 Mils	Switchboard Wiring
Varnished Cambric	V	85 C (185 F)	Varnished Cambric	Fibrous Covering or Lead Sheath	Dry Locations Only. Unless Lead Sheathed. Smaller than No. 6 by Special Permission
Asbestos Varnished Cambric	AVA	110 C (230 F)	Impregnated Asbestos and Varnished Cambric	Asbestos Braid	General Use Dry Locations
Asbestos Varnished Cambric	AVB	90 C (194 F)	Same as Type AVA	Flame-Retardant Cotton Braid	General Use Dry Locations
Asbestos Varnished Cambric	AVL	110 C (230 F)	Same as Type AVA	Lead Sheath	General Use Wet Locations
Asbestos	A	200 C (392 F)	Felted Asbestos	With or Without Asbestos Braid	Dry Locations Only. Not for General Conduit Installation. In Raceways, only as Leads to or within Apparatus. If without Braid or Moisture-resistant Treatment, Limited to 300 Volts
Impregnated Asbestos	AI	125 C (257 F)	Impregnated Felted Asbestos	With or Without Impregnated Asbestos Braid	
Paper		85 C (185 F)	Paper	Lead Sheath	As Permitted by 2304-b, or by Special Permission
Slow Burning	SB	90 C (194 F)	3 Braids Impregnated Fire-Retardant Thread	Outer Cover Finished Smooth and Hard	For Use Only in Dry Locations Where the Room Temperature Exceeds 85 C (185 F)
Slow Burning Weatherproof	SBW	90 C (194 F)	2 Layers Impregnated Cotton Thread	Outer Fire-Retardant Coating	For Use Only in Dry Locations where the Room Temperature Exceeds 85 C (183 F)
Weatherproof	WP	80 C (176 F)	At Least Three Cotton Braids Impregnated		May be used for Interior Wiring Only by Special Permission

Rubber-covered conductors of the lead-sheathed or multiple-conductor type do not require a flame-retardant, moisture-resistant outer covering over the individual conductors, but all such conductors shall have a fibrous covering.

Finished conductors of Type RPT and Type RU shall be equivalent mechanically and electrically to Type R conductor.

Synthetic insulation may stiffen at temperatures below freezing and care should be used in its installation at these temperatures. (It is a plastic material that exhibits many of the properties of rubber.)

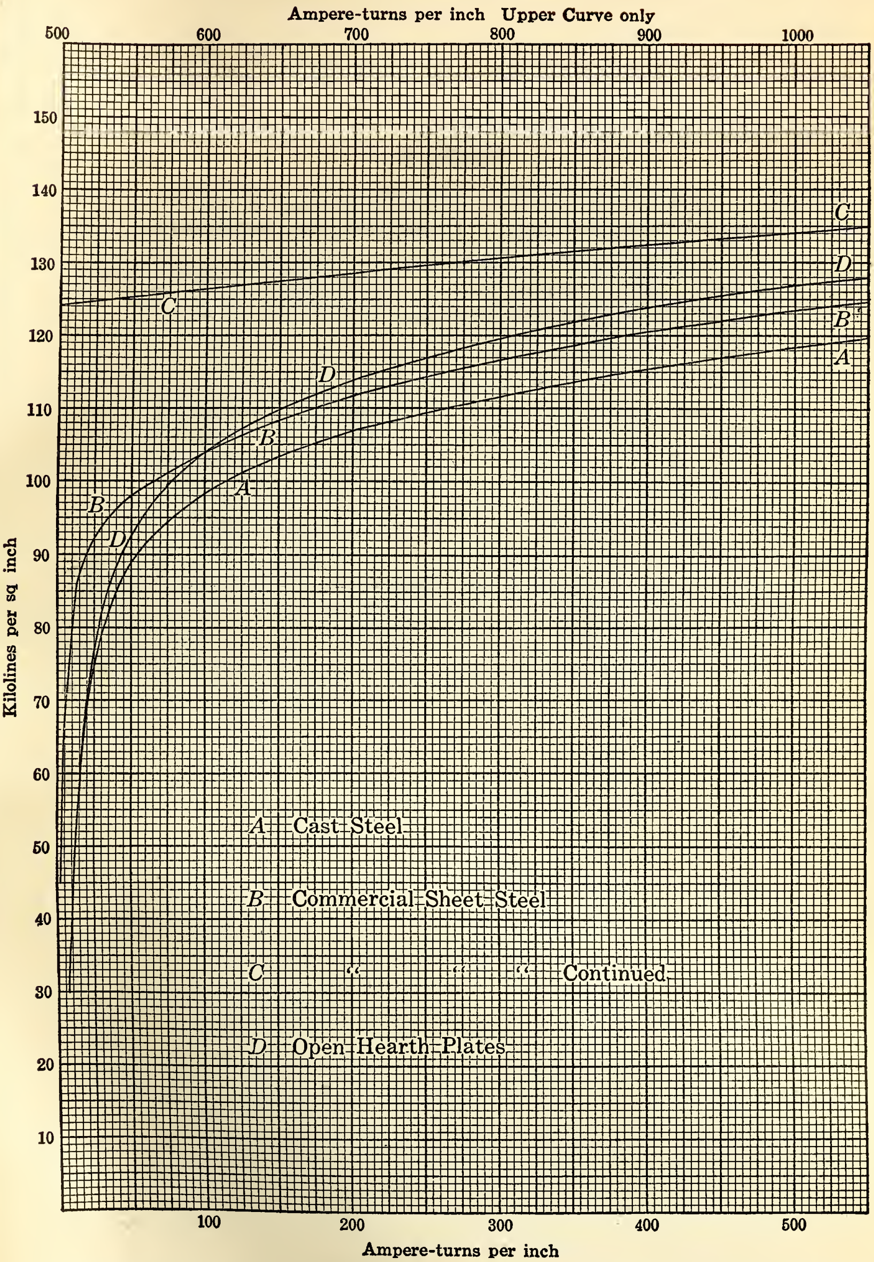
* Adapted, by permission, from the National Electrical Code, 1940, as adopted by the National Fire Protection Association.

TABLE IX

Current Density Allowable in Armature Conductors

Size of machine in kilowatts	Current density in amperes per square inch	Cross-section of copper in circular mils per ampere	Size of machine in kilowatts	Current density in amperes per square inch	Cross-section of copper in circular mils per ampere
Very large	1500	850	1 to 50	3000	425
Over 100	2000	635	Fractional	3500	363
50 to 100	2500	510			

TABLE X



Magnetization, or B–H, curves, for the materials almost universally used in modern machines, with the scales in kilolines per square inch and ampere-turns per inch of length of the magnetic circuit.

TABLE XI

Values of the Constants of Dushman's Equation, A and b_0 for Various Elements and Approximate Values for Composite Emitters

The values for the composite emitters vary considerably with the exact method of preparation; average values are given here for comparison with pure metal emitters.

Material	A	b_0
Molybdenum	60.2	51,500
Tantalum	60.2	47,200
Thorium	60.2	38,900
Tungsten	60.2	52,400
Thoriated tungsten	3.0	30,500
Oxide coated	0.008	14,400

INDEX

A

B

C

D

F

N

O

Date Due

23Jul'48		MAR 4 '60	MAR [illegible] '65
31 Jul'48	23Jul'48		MAR 2[illegible] '65
13Aug'48		OCT 28'60	APR [illegible]7 '65
3 Sep'48		NOV 18'60	~~OCT 2[illegible] '65~~
18Oct'48	5Oct'48	MAR 31 '61	~~OCT 4 1966~~
11Nov'48	10Nov'48	APR 14 '61	~~NOV 2[illegible] '66~~
Jan 21 '49		MAY 5 '61	~~OCT 30 '67~~
Jan 24 '49		6-2-61	~~MAY 8 1968~~
Jan 31 '49			~~OCT 2 [illegible] 1969~~
Dec 23 '49		MAR 28 '62	
JAN 29 '52	MAR 16 '55	APR 9 '62	
NOV 21 '53	OCT 31 '55		
DEC 8 '53	JAN 24 '58	RENEWED	
JAN 22 '54	NOV 21 '58	NOV 25 '63	
JUN 3 '54	OCT 14 '59	FEB 22 '65	
JAN 29 '55	DEC 4 '59	JUN 11 '65	
APR 9 '55	JAN 18 '60		
APR 27 '55	JAN 25 '60		

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